

Chapter 5: Application of Definite Integration

EXERCISE 5.1 [PAGE 187]

Exercise 5.1 | Q 1.1 | Page 187

Find the area of the region bounded by the following curves, X-axis and the given lines:
 $y = 2x$, $x = 0$, $x = 5$

SOLUTION

$$\begin{aligned}\text{Required area} &= \int_0^5 y \cdot dx, \text{ where } y = 2x \\ &= \int_0^5 2x \cdot dx \\ &= \left[\frac{2x^2}{2} \right]_0^5 \\ &= 25 - 0 \\ &= 25 \text{ sq units.}\end{aligned}$$

Exercise 5.1 | Q 1.2 | Page 187

Find the area of the region bounded by the following curves, X-axis and the given lines:
 $x = 2y$, $y = 0$, $y = 4$

SOLUTION

$$\begin{aligned}\text{Required area} &= \int_0^4 x \cdot dy, \text{ where } x = 2y \\ &= \int_0^4 2y \cdot dy \\ &= \left[\frac{2y^2}{2} \right]_0^4 \\ &= 16 - 0 \\ &= 16 \text{ sq units.}\end{aligned}$$

Exercise 5.1 | Q 1.3 | Page 187

Find the area of the region bounded by the following curves, X-axis and the given lines :
 $x = 0$, $x = 5$, $y = 0$, $y = 4$

SOLUTION

$$\begin{aligned}\text{Required area} &= \int_0^5 y \cdot dx, \text{ where } y = 4 \\ &= \int_0^5 4 \cdot dx \\ &= [4x]_0^5 \\ &= 20 - 0 \\ &= 20 \text{ sq units.}\end{aligned}$$

Exercise 5.1 | Q 1.4 | Page 187

Find the area of the region bounded by the following curves, X-axis and the given lines :
 $y = \sin x$, $x = 0$, $x = \pi/2$

SOLUTION

$$\begin{aligned}\text{Required area} &= \int_0^{\frac{\pi}{2}} y \cdot dx, \text{ where } y = \sin x \\ &= \int_0^{\frac{\pi}{2}} \sin x \cdot dx \\ &= [-\cos x]_0^{\frac{\pi}{2}} \\ &= -\cos \frac{\pi}{2} + \cos 0 \\ &= 0 + 1 \\ &= 1 \text{ sq unit.}\end{aligned}$$

Exercise 5.1 | Q 1.5 | Page 187

Find the area of the region bounded by the following curves, X-axis and the given lines:
 $xy = 2$, $x = 1$, $x = 4$

SOLUTION

For $xy = 2$, $y = \frac{2}{x}$.

Required area = $\int_1^4 y \cdot dx$, where $y = \frac{2}{x}$

$$= \int_1^4 \frac{2}{x} \cdot dx$$

$$= [2 \log|x|]_1^4$$

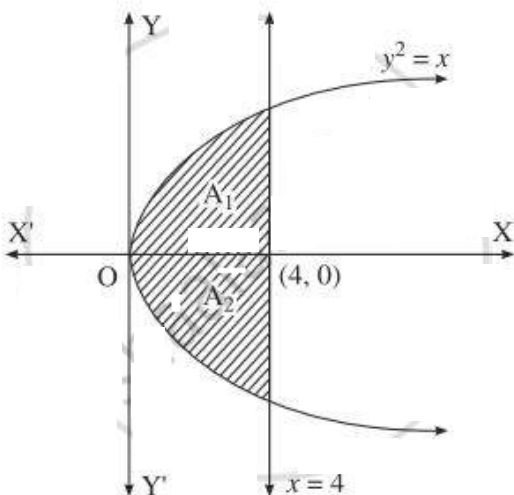
$$= 2 \log 4 - 2 \log 1$$

$$= 2 \log 4 - 0$$

$$= 2 \log 4 \text{ sq units.}$$

Exercise 5.1 | Q 1.6 | Page 187

Find the area of the region bounded by the following curves, X-axis and the given lines :
 $y^2 = x$, $x = 0$, $x = 4$

SOLUTION

The required area consists of two bounded regions A_1 and A_2 which are equal in areas.

For $y^2 = x$, $y = \sqrt{x}$

Required area = $A_1 + A_2 = 2A_1$

$$= 2 \int_0^4 y \cdot dx, \text{ where } y = \sqrt{x}$$

$$= 2 \int_0^4 \sqrt{x} \cdot dx$$

$$= 2 \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^4$$

$$= 2 \left[\frac{2}{3} (4)^{\frac{3}{2}} - 0 \right]$$

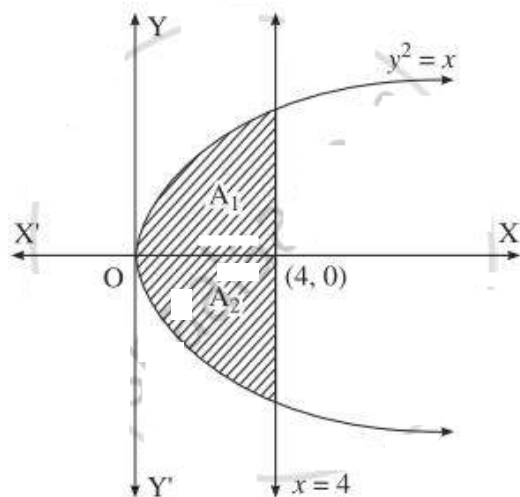
$$= 2 \left[\frac{2}{3} (2^2)^{\frac{3}{2}} \right]$$

$$= \frac{32}{3} \text{ sq units.}$$

Exercise 5.1 | Q 1.7 | Page 187

Find the area of the region bounded by the following curves, X-axis and the given lines:
 $y^2 = 16x$, $x = 0$, $x = 4$

SOLUTION



The required area consists of two bounded regions A_1 and A_2 which are equal in areas.

For $y^2 = x$, $y = \sqrt{x}$

Required area = $A_1 + A_2 = 2A_1$

$$= 2 \int_0^4 y \cdot dx, \text{ where } y = \sqrt{x}$$

$$= 2 \int_0^4 \sqrt{x} \cdot dx$$

$$= 2 \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^4$$

$$= 2 \left[\frac{2}{3} (4)^{\frac{3}{2}} - 0 \right]$$

$$= 2 \left[\frac{2}{3} (2^2)^{\frac{3}{2}} \right]$$

$$= \frac{128}{3} \text{ sq units.}$$

Exercise 5.1 | Q 2.1 | Page 187

Find the area of the region bounded by the parabola: $y^2 = 16x$ and its latus rectum.

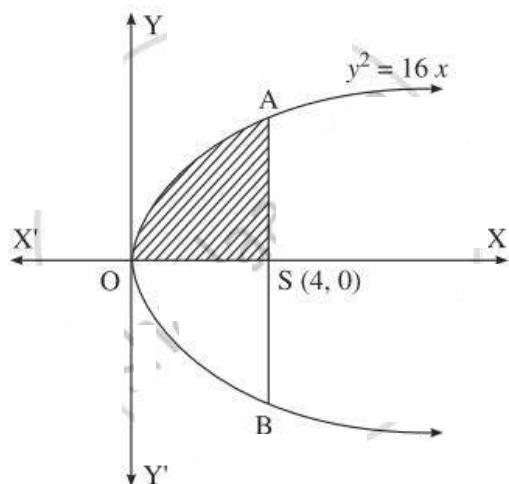
SOLUTION

Comparing $y^2 = 16x$ with $y^2 = 4ax$, we get

$$4a = 16$$

$$\therefore a = 4$$

$$\therefore \text{focus is } S(a, 0) = (4, 0)$$



For $y^2 = 16x$, $y = 4\sqrt{x}$

Required area = area of the region OBSAO

= 2[area of the region OSAO]

$$= 2 \int_0^4 y \cdot dx, \text{ where } y = 4\sqrt{x}$$

$$= 2 \int_0^4 4\sqrt{x} \cdot dx$$

$$= 8 \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^4$$

$$= 8 \left[\frac{2}{3} (4)^{\frac{3}{2}} - 0 \right]$$

$$= 8 \left[\frac{2}{3} (2^2)^{\frac{3}{2}} \right]$$

$$= \frac{128}{3} \text{ sq units.}$$

Exercise 5.1 | Q 2.2 | Page 187

Find the area of the region bounded by the parabola: $y = 4 - x^2$ and the X-axis.

SOLUTION

The equation of the parabola is $y = 4 - x^2$

$\therefore x^2 = 4 - y$, i.e. $(x - 0)^2 = -(y - 4)$

It has vertex at P(0, 4)

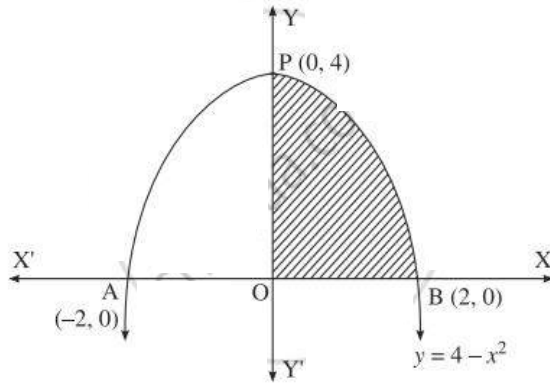
For points of intersection of the parabola with X-axis,
we put $y = 0$ in its equation.

$$\therefore 0 = 4 - x^2$$

$$\therefore x^2 = 4$$

$$\therefore x = \pm 2.$$

$\therefore$ the parabola intersect the X-axis at A (- 2, 0) and B(2, 0)



Required area = area of the region APBOA
= 2[area of the region OPBO]

$$= 2 \int y \cdot dx, \text{ where } y = 4 - x^2$$

$$= 2 \int_0^2 (4 - x^2) \cdot dx$$

$$= 8 \int_0^2 1 \cdot dx - 2 \int_0^2 x^2 \cdot dx$$

$$= 8[x]_0^2 - 2 \left[\frac{x^3}{3} \right]_0^2$$

$$= 8(2 - 0) - \frac{2}{3}(8 - 0)$$

$$= 16 - \frac{16}{3}$$

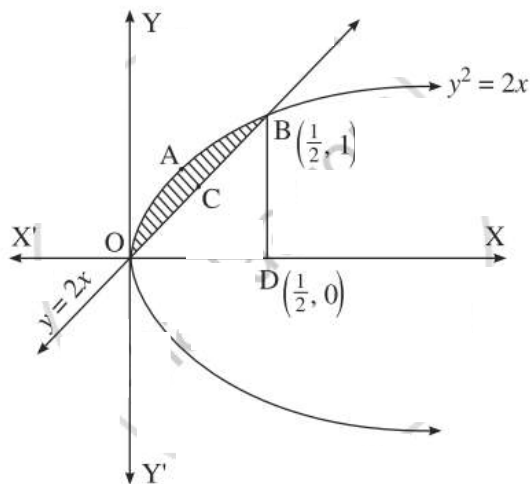
$$= \frac{32}{3} \text{ sq units.}$$

Exercise 5.1 | Q 3.1 | Page 187

Find the area of the region included between: $y^2 = 2x$ and $y = 2x$

SOLUTION

The vertex of the parabola $y^2 = 2x$ is at the origin $O = (0, 0)$.



To find the points of intersection of the line and the parabola, equating the values of $2x$ from both the equations we get,

$$\therefore y^2 = y$$

$$\therefore y^2 - y = 0$$

$$\therefore y(y - 1) = 0$$

$$\therefore y = 0 \text{ or } y = 1$$

$$\text{When } y = 0, x = \frac{0}{2} = 0$$

$$\text{When } y = 1, x = \frac{1}{2}$$

$$\therefore \text{the points of intersection are } O(0, 0) \text{ and } B\left(\frac{1}{2}, 1\right)$$

Required area = area of the region OABCO

= area of the region OABDO – area of the region OCBDO

Now, area of the region OABDO

= area under the parabola $y^2 = 2x$ between $x = 0$ and $x = \frac{1}{2}$

$$= \int_0^{\frac{1}{2}} y \cdot dx, \text{ where } y = \sqrt{2x}$$

$$= \int_0^{\frac{1}{2}} \sqrt{2x} dx$$

$$= \sqrt{2} \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^{\frac{1}{2}}$$

$$= \sqrt{2} \left[\frac{2}{3} \left(\frac{1}{2} \right)^{\frac{3}{2}} - 0 \right]$$

$$= \sqrt{2} \left[\frac{2}{3} \cdot \frac{1}{2\sqrt{2}} \right]$$

$$= \frac{1}{3}$$

Area of the region OCBDO

= area under the line y

= 2x between x

$$= 0 \text{ and } x = \frac{1}{2}$$

$$= \int_0^{\frac{1}{2}} y \cdot dx, \text{ where } y = 2x$$

$$= \int_0^{\frac{1}{2}} 2x \cdot dx$$

$$= \left[\frac{2x^2}{2} \right]_0^{\frac{1}{2}}$$

$$= \frac{1}{4} - 0$$

$$= \frac{1}{4}$$

$\therefore$ required area

$$= \frac{1}{3} = \frac{1}{4}$$

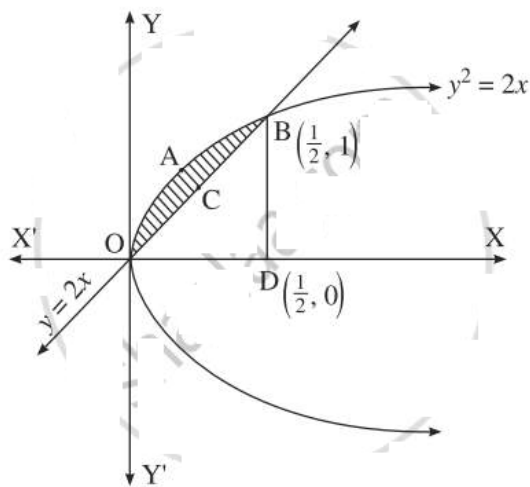
$$= \frac{1}{12} \text{ sq unit.}$$

Exercise 5.1 | Q 3.2 | Page 187

Find the area of the region included between: $y^2 = 4x$, and $y = x$

SOLUTION

The vertex of the parabola $y^2 = 4x$ is at the origin $O = (0, 0)$.



To find the points of intersection of the line and the parabola, equating the values of $4x$ from both the equations we get,

$$\therefore y^2 = y$$

$$\therefore y^2 - y = 0$$

$$\therefore y(y - 1) = 0$$

$$\therefore y = 0 \text{ or } y = 1$$

$$\text{When } y = 0, x = \frac{0}{2} = 0$$

$$\text{When } y = 1, x = \frac{1}{2}$$

$\therefore$ the points of intersection are $O(0, 0)$ and $B\left(\frac{1}{2}, 1\right)$

Required area = area of the region OABCO

= area of the region OABDO – area of the region OCBDO

Now, area of the region OABDO

= area under the parabola $y^2 = 4x$ between $x = 0$ and $x = \frac{1}{2}$

$$= \int_0^{\frac{1}{2}} y \cdot dx, \text{ where } y = \sqrt{4x}$$

$$= \int_0^{\frac{1}{2}} \sqrt{4x} dx$$

$$= \sqrt{4} \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^{\frac{1}{2}}$$

$$= \sqrt{4} \left[\frac{2}{3} \left(\frac{1}{2} \right)^{\frac{3}{2}} - 0 \right]$$

$$= \sqrt{4} \left[\frac{2}{3} \cdot \frac{1}{2\sqrt{2}} \right]$$

$$= \frac{1}{3}$$

Area of the region OCBDO

= area under the line y

= $2x$ between x

$$= 0 \text{ and } x = \frac{1}{2}$$

$$= \int_0^{\frac{1}{2}} y \cdot dx, \text{ where } y = x$$

$$= \int_0^{\frac{1}{2}} 2x \cdot dx$$

$$= \left[\frac{2x^2}{2} \right]_0^{\frac{1}{2}}$$

$$= \frac{4}{1} - 0$$

$$= \frac{4}{3}$$

$\therefore$ required area

$$= \frac{4}{1} = \frac{4}{3}$$

$$= \frac{8}{3} \text{ sq units.}$$

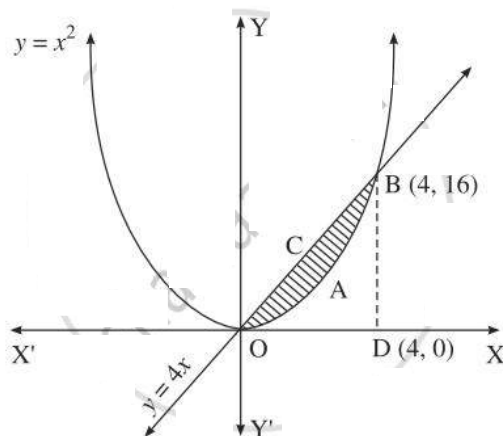
Exercise 5.1 | Q 3.3 | Page 187

Find the area of the region included between: $y = x^2$ and the line $y = 4x$

SOLUTION

The vertex of the parabola $y = x^2$ is at the origin $O(0, 0)$

To find the points of the intersection of the line and the parabola.



Equating the values of y from the two equations, we get

$$x^2 = 4x$$

$$\therefore x^2 - 4x = 0$$

$$\therefore x(x - 4) = 0$$

$$\therefore x = 0, x = 4$$

$$\text{When } x = 0, y = 4(0) = 0$$

$$\text{When } x = 4, y = 4(4) = 16$$

$\therefore$ the points of intersection are $O(0, 0)$ and $B(4, 16)$

Required area = area of the region OABCO

= (area of the region ODBCO) – (area of the region ODBAO)

Now, area of the region ODBCO

= area under the line $y = 4x$ between $x = 0$ and $x = 4$

$$= \int_0^4 y \cdot dx, \text{ where } y = 4x$$

$$= \int_0^4 4x \cdot dx$$

$$= 4 \int_0^4 x \cdot dx$$

$$= 4 \left[\frac{x^2}{2} \right]_0^4$$

$$= 2(16 - 0)$$

$$= 32$$

Area of the region ODBAO

= area under the parabola $y = x^2$ between $x = 0$ and $x = 4$

$$= \int_0^4 y \cdot dx, \text{ where } y = x^2$$

$$= \int_0^4 x^2 \cdot dx$$

$$\begin{aligned}
 &= \left[\frac{x^3}{3} \right]_0^4 \\
 &= \frac{1}{3}(64 - 0) \\
 &= \frac{64}{3}
 \end{aligned}$$

$\therefore$ required area

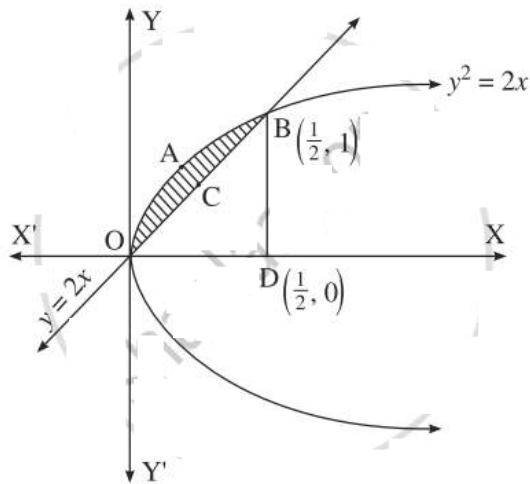
$$\begin{aligned}
 &= 32 - \frac{64}{3} \\
 &= \frac{32}{3} \text{ sq units.}
 \end{aligned}$$

Exercise 5.1 | Q 3.4 | Page 187

Find the area of the region included between: $y^2 = 4ax$ and the line $y = x$

SOLUTION

The vertex of the parabola $y^2 = 4ax$ is at the origin $O = (0, 0)$.



To find the points of intersection of the line and the parabola, equating the values of $4ax$ from both the equations we get,

$$\begin{aligned}
 \therefore y^2 &= y \\
 \therefore y^2 - y &= 0 \\
 \therefore y(y - 1) &= 0 \\
 \therefore y &= 0 \text{ or } y = 1
 \end{aligned}$$

$$\text{When } y = 0, x = \frac{0}{2} = 0$$

$$\text{When } y = 1, x = \frac{1}{2}$$

$\therefore$ the points of intersection are $O(0, 0)$ and $B\left(\frac{1}{2}, 1\right)$

Required area = area of the region OABCO

= area of the region OABDO – area of the region OCBDO

Now, area of the region OABDO

= area under the parabola $y^2 = 4ax$ between $x = 0$ and $x = \frac{1}{2}$

$$= \int_0^{\frac{1}{2}} y \cdot dx, \text{ where } y = \sqrt{2x}$$

$$= \int_0^{\frac{1}{2}} \sqrt{2x} dx$$

$$= \sqrt{2} \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^{\frac{1}{2}}$$

$$= \sqrt{2} \left[\frac{2}{3} \left(\frac{1}{2} \right)^{\frac{3}{2}} - 0 \right]$$

$$= \sqrt{2} \left[\frac{2}{3} \cdot \frac{1}{2\sqrt{2}} \right]$$

$$= \frac{1}{3}$$

Area of the region OCBDO

= area under the line y

= $4ax$ between x

$$= 0 \text{ and } x = \frac{1}{4ax}$$

$$= \int_0^{\frac{1}{2}} y \cdot dx, \text{ where } y = x$$

$$= \int_0^{\frac{1}{2}} 2x \cdot dx$$

$$= \left[\frac{2x^2}{2} \right]_0^{\frac{1}{2}}$$

$$= \frac{4}{3} - 0$$

$$= \frac{2a^2}{1}$$

∴ required area

$$= \frac{4}{3} = \frac{2a^2}{1}$$

$$= \frac{8a^2}{3} \text{ sq units.}$$

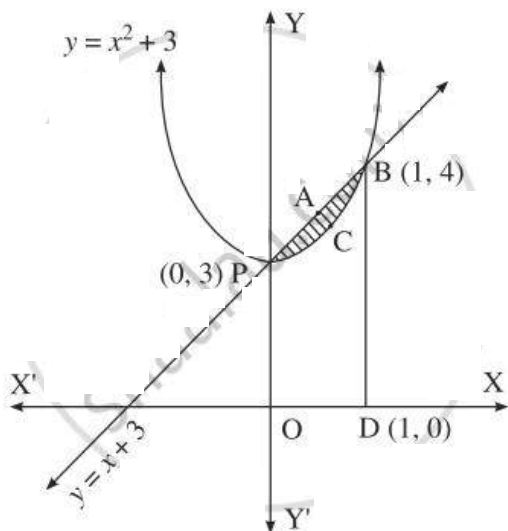
Exercise 5.1 | Q 3.5 | Page 187

Find the area of the region included between: $y = x^2 + 3$ and the line $y = x + 3$

SOLUTION

The given parabola is $y = x^2 + 3$, i.e. $(x - 0)^2 = y - 3$

∴ its vertex is P(0, 3).



To find the points of intersection of the line and the parabola. Equating the values of y from both the equations, we get

$$\begin{aligned}x^3 + 3 &= x + 3 \\ \therefore x^2 - x &= 0 \\ \therefore x(x - 1) &= 0 \\ \therefore x &= 0 \text{ or } x = 1\end{aligned}$$

When $x = 0$, $y = 0 + 3 = 3$

When $x = 1$, $y = 1 + 3 = 4$

$\therefore$ the points of intersection are $P(0, 3)$ and $B(1, 4)$ Required area = area of the region PABCP

= area of the region OPABDO – area of the region OPCBDO

Now, area of the region OPABDO

= area under the line $y = x + 3$ between $x = 0$ and $x = 1$

$$= \int_0^1 y \cdot dx, \text{ where } y = x + 3$$

$$= \int_0^1 (x + 3) \cdot dx$$

$$= \int_0^1 x \cdot dx + 3 \int_0^1 1 \cdot dx$$

$$\begin{aligned}
 &= \left[\frac{x^2}{2} \right]_0^1 + 3[x]_0^1 \\
 &= \left(\frac{1}{2} - 0 \right) + 3(1 - 0) \\
 &= \frac{7}{2}
 \end{aligned}$$

Area of the region OPCBDO

= area under the parabola $y = x^2 + 3$ between $x = 0$ and $x = 1$

$$= \int_0^1 y \cdot dx, \text{ where } y = x^2 + 3$$

$$= \int_0^1 (x^2 + 3) \cdot dx$$

$$= \int_0^1 x^2 \cdot dx + 3 \int_0^1 1 \cdot dx$$

$$= \left[\frac{x^3}{3} \right]_0^1 + 3[x]_0^1$$

$$= \left(\frac{1}{3} - 0 \right) + 3(1 - 0)$$

$$= \frac{10}{3}$$

$$\therefore \text{required area} = \frac{7}{2} - \frac{10}{3}$$

$$= \frac{21 - 20}{6}$$

$$= \frac{1}{6} \text{ sq unit.}$$

MISCELLANEOUS EXERCISE 5 [PAGES 188 - 190]

Miscellaneous Exercise 5 | Q 1.01 | Page 188

Choose the correct option from the given alternatives :

The area bounded by the regional $\leq x \leq 5$ and $2 \leq y \leq 5$ is given by

1. 12 sq units
2. 8 sq units
3. 25 sq units
4. 32 sq units

SOLUTION

12 sq units.

Miscellaneous Exercise 5 | Q 1.02 | Page 188

Choose the correct option from the given alternatives :

The area of the region enclosed by the curve $y = \frac{1}{x}$, and the lines $x = e$, $x = e^2$ is given by

- 1 sq unit
- $\frac{1}{2}$ sq unit
- $\frac{3}{2}$ sq units
- $\frac{5}{2}$ sq units

SOLUTION

1 sq unit.

Miscellaneous Exercise 5 | Q 1.03 | Page 188

Choose the correct option from the given alternatives :

The area bounded by the curve $y = x^3$, the X-axis and the lines $x = -2$ and $x = 1$ is

- 9 sq units
- $-\frac{15}{4}$ sq units
- $\frac{15}{4}$ sq units
- $\frac{17}{4}$ sq units

SOLUTION

$\frac{15}{4}$ sq units.

Choose the correct option from the given alternatives :

The area enclosed between the parabola $y^2 = 4x$ and line $y = 2x$ is

$$\frac{2}{3} \text{ sq units}$$

$$\frac{1}{3} \text{ sq unit}$$

$$\frac{1}{4} \text{ sq unit}$$

$$\frac{4}{3} \text{ sq unit}$$

SOLUTION

$$\frac{1}{3} \text{ sq unit.}$$

Choose the correct option from the given alternatives :

The area of the region bounded between the line $x = 4$ and the parabola $y^2 = 16x$ is

$$\frac{128}{3} \text{ sq units}$$

$$\frac{108}{3} \text{ sq units}$$

$$\frac{118}{3} \text{ sq units}$$

$$\frac{218}{3} \text{ sq units}$$

SOLUTION

$$\frac{128}{3} \text{ sq units.}$$

Choose the correct option from the given alternatives :

The area of the region bounded by $y = \cos x$, Y-axis and the lines $x = 0$, $x = 2\pi$ is

1 sq unit

2 sq units

3 sq units

4 sq units

SOLUTION

4 sq units.

Choose the correct option from the given alternatives :

The area bounded by the parabola $y^2 = 8x$, the X-axis and the latus rectum is

$\frac{31}{3}$ sq units

$\frac{32}{3}$ sq units

$\frac{32\sqrt{2}}{3}$ sq units

$\frac{16}{3}$ sq units

SOLUTION

$\frac{32}{3}$ sq units.

Choose the correct option from the given alternatives :

The area under the curve $y = 2\sqrt{x}$, enclosed between the lines $x = 0$ and $x = 1$ is

$$4 \text{ sq units}$$

$$\frac{3}{4} \text{ sq unit}$$

$$\frac{2}{3} \text{ sq unit}$$

$$\frac{4}{3} \text{ sq units}$$

SOLUTION

$$\frac{4}{3} \text{ sq units.}$$

Miscellaneous Exercise 5 | Q 1.09 | Page 189

Choose the correct option from the given alternatives :

The area of the circle $x^2 + y^2 = 25$ in first quadrant is

$$\frac{25\pi}{4} \text{ sq units}$$

$$5\pi \text{ sq units}$$

$$5 \text{ sq units}$$

$$3 \text{ sq units}$$

SOLUTION

$$\frac{25\pi}{4} \text{ sq units.}$$

Miscellaneous Exercise 5 | Q 1.1 | Page 189

Choose the correct option from the given alternatives :

The area of the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

$$ab \text{ sq units}$$

$$\pi ab \text{ sq units}$$

$$\frac{\pi}{ab} \text{ sq units}$$

$$\pi a^2 \text{ sq units}$$

SOLUTION

π ab sq units.

Miscellaneous Exercise 5 | Q 1.11 | Page 189

Choose the correct option from the given alternatives :

The area bounded by the parabola $y^2 = x$ and the line $2y = x$ is

$\frac{4}{3}$ sq unit

1 sq unit

$\frac{2}{3}$ sq unit

$\frac{1}{3}$ sq unit

SOLUTION

$\frac{4}{3}$ sq unit.

Miscellaneous Exercise 5 | Q 1.12 | Page 189

Choose the correct option from the given alternatives :

The area enclosed between the curve $y = \cos 3x$, $0 \leq x \leq \frac{\pi}{6}$ and the X-axis is

$\frac{1}{2}$ sq unit

1 sq unit

$\frac{2}{3}$ sq unit

$\frac{1}{3}$ sq unit

SOLUTION

$\frac{1}{3}$ sq unit.

Choose the correct option from the given alternatives :

The area bounded by $y = \sqrt{x}$ and the $x = 2y + 3$, X-axis in first quadrant is

$$2\sqrt{3} \text{ sq units}$$

$$9 \text{ sq units}$$

$$\frac{34}{3} \text{ sq units}$$

$$18 \text{ sq units}$$

SOLUTION

9 sq units.

Choose the correct option from the given alternatives :

The area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the line $\frac{x}{a} + \frac{y}{b} = 1$ is

$$(\pi ab - 2ab) \text{ sq units}$$

$$\left(\frac{\pi ab}{4} - \frac{ab}{2} \right) \text{ sq units}$$

$$(\pi ab - ab) \text{ sq units}$$

$$\pi ab \text{ sq units}$$

SOLUTION

$$\left(\frac{\pi ab}{4} - \frac{ab}{2} \right) \text{ sq units.}$$

Choose the correct option from the given alternatives :

The area bounded by the parabola $y = x^2$ and the line $y = x$ is

$$\frac{1}{2} \text{ sq unit}$$

$$\frac{1}{3} \text{ sq unit}$$

$$\frac{1}{6} \text{ sq unit}$$

$$\frac{1}{12} \text{ sq unit}$$

SOLUTION

$$\frac{1}{6} \text{ sq unit.}$$

Miscellaneous Exercise 5 | Q 1.16 | Page 189

Choose the correct option from the given alternatives :

The area enclosed between the two parabolas $y^2 = 4x$ and $y = x$ is

$$\frac{16}{3} \text{ sq units}$$

$$\frac{32}{3} \text{ sq units}$$

$$\frac{8}{3} \text{ sq units}$$

$$\frac{4}{3} \text{ sq units}$$

SOLUTION

$$\frac{8}{3} \text{ sq units.}$$

Miscellaneous Exercise 5 | Q 1.17 | Page 190

Choose the correct option from the given alternatives :

The area bounded by the curve $y = \tan x$, X-axis and the line $x = \frac{\pi}{4}$ is

$$\frac{1}{2} \log 2 \text{ sq units}$$

$$\log 2 \text{ sq units}$$

$$2 \log 2 \text{ sq units}$$

$$3 \cdot \log 2 \text{ sq units}$$

SOLUTION

$$\frac{1}{2} \log 2 \text{ sq units.}$$

Miscellaneous Exercise 5 | Q 1.18 | Page 190

Choose the correct option from the given alternatives :

The area of the region bounded by $x^2 = 16y$, $y = 1$, $y = 4$ and $x = 0$ in the first quadrant, is

$$\frac{7}{3} \text{ sq units}$$

$$\frac{8}{3} \text{ sq units}$$

$$\frac{64}{3} \text{ sq units}$$

$$\frac{56}{3} \text{ sq units}$$

SOLUTION

$$\frac{56}{3} \text{ sq units.}$$

Miscellaneous Exercise 5 | Q 1.19 | Page 190

Choose the correct option from the given alternatives :

The area of the region included between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, ($a > 0$) is given by

$$\frac{16a^2}{3} \text{ sq units}$$

$$\frac{8a^2}{3} \text{ sq units}$$

$$\frac{64}{3} \text{ sq units}$$

$$\frac{56}{3} \text{ sq units}$$

SOLUTION

$$\frac{16a^2}{3} \text{ sq units.}$$

Choose the correct option from the given alternatives :

The area of the region included between the line $x + y = 1$ and the circle $x^2 + y^2 = 1$ is

$$\left(\frac{\pi}{2} - 1\right) \text{ sq units}$$

$$(\pi - 2) \text{ sq units}$$

$$\left(\frac{\pi}{4} - \frac{1}{2}\right) \text{ sq units}$$

$$\left(\pi - \frac{1}{2}\right) \text{ sq units}$$

SOLUTION

$$\left(\frac{\pi}{4} - \frac{1}{2}\right) \text{ sq units}$$

Miscellaneous Exercise 5 | Q 2.01 | Page 190

Solve the following :

Find the area of the region bounded by the following curve, the X-axis and the given lines : $0 \leq x \leq 5, 0 \leq y \leq 2$

SOLUTION

$$\text{Required area} = \int_0^5 y \cdot dx, \text{ where } y = 2$$

$$= \int_0^5 2 \cdot dx = [2x]_0^5$$

$$= 2 \times 5 - 0$$

$$= 10 \text{ sq units.}$$

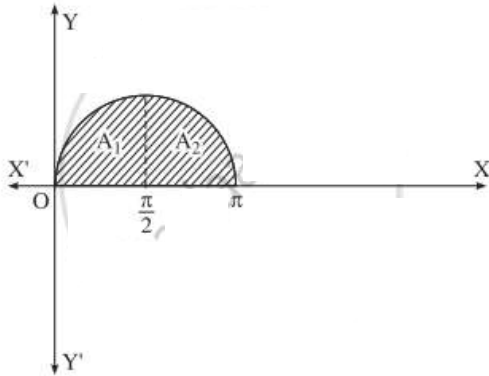
Miscellaneous Exercise 5 | Q 2.01 | Page 190

Solve the following :

Find the area of the region bounded by the following curve, the X-axis and the given lines : $y = \sin x, x = 0, x = \pi$

SOLUTION

The curve $y = \sin x$ intersects the X-axis at $x = 0$ and $x = \pi$ between $x = 0$ and $x = \pi$.



Two bounded regions A_1 and A_2 are obtained. Both the regions have equal areas.

$\therefore$ required area $= A_1 + A_2 = 2A_1$

$$= 2 \int_0^{\frac{\pi}{2}} y \cdot dx, \text{ where } y = \sin x$$

$$= 2 \int_0^{\frac{\pi}{2}} \sin x \cdot dx$$

$$= 2[-\cos x]_0^{\frac{\pi}{2}}$$

$$= 2\left[-\cos \frac{\pi}{2} - \cos 0\right]$$

$$= 2(-0 + 1)$$

$$= 2 \text{ sq units.}$$

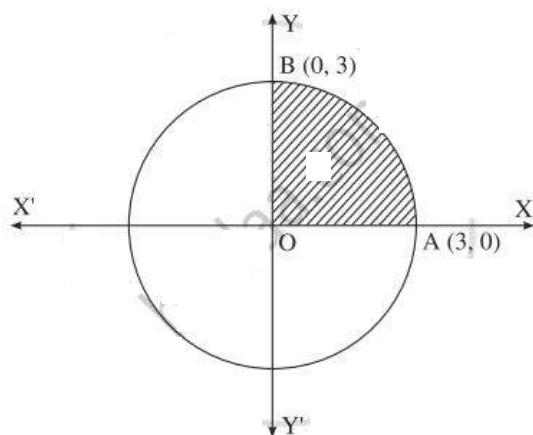
Miscellaneous Exercise 5 | Q 2.02 | Page 190

Solve the following :

Find the area of the circle $x^2 + y^2 = 9$, using integration.

SOLUTION

By the symmetry of the circle, its area is equal to 4 times the area of the region OABO. Clearly for this region, the limits of integration are 0 and 3.



From the equation of the circle, $y^2 = 9 - x^2$.

In the first quadrant, $y > 0$

$$\therefore y = \sqrt{9 - x^2}$$

$\therefore$ area of the circle = 4 (area of the region OABO)

$$= 4 \int_0^3 y \cdot dx = 4 \int_0^3 \sqrt{9 - x^2} \cdot dx$$

$$= 4 \left[\frac{x}{2} \sqrt{9 - x^2} + \frac{9}{2} \sin^{-1} \left(\frac{x}{3} \right) \right]_0^3$$

$$= 4 \left[\frac{3}{2} \sqrt{9 - 9} + \frac{9}{2} \sin^{-1} \left(\frac{3}{3} \right) \right] - 4 \left[\frac{0}{2} \sqrt{9 - 0} + \frac{9}{2} \sin^{-1}(0) \right]$$

$$= 4 \cdot \frac{9}{2} \cdot \frac{\pi}{2}$$

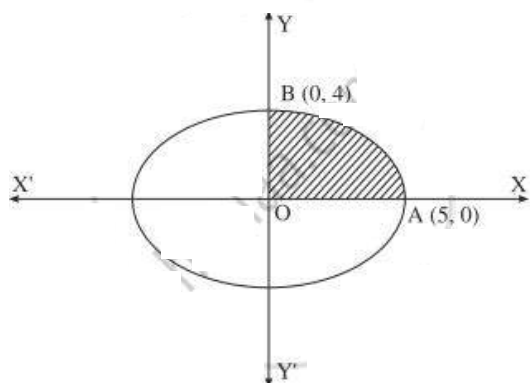
$$= 9\pi \text{ sq units.}$$

Miscellaneous Exercise 5 | Q 2.03 | Page 190

Solve the following :

Find the area of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ using integration

SOLUTION



By the symmetry of the ellipse, its area is equal to 4 times the area of the region OABO. Clearly for this region, the limits of integration are 0 and 5.

From the equation of the ellipse

$$\frac{y^2}{16} = 1 - \frac{x^2}{25} = \frac{25 - x^2}{25}$$

$$\therefore y^2 = \frac{16}{25} (25 - x^2)$$

In the first quadrant $y > 0$

$$\therefore y = \frac{4}{5} \sqrt{25 - x^2}$$

$\therefore$ area of the ellipse = 4 (area of the region OABO)

$$= 4 \int_0^5 y \cdot dx$$

$$= \int_0^5 \frac{4}{5} \sqrt{25 - x^2} \cdot dx$$

$$= \frac{16}{5} \int_0^5 \sqrt{25 - x^2} \cdot dx$$

$$= \frac{16}{5} \left[\frac{x}{2} \sqrt{25 - x^2} + \frac{25}{2} \sin^{-1} \left(\frac{x}{5} \right) \right]_0^5$$

$$= \frac{16}{5} \left(\frac{5}{2} \sqrt{25 - 25} + \frac{25}{2} \sin^{-1}(1) \right) - \frac{16}{5} \left[\frac{5}{2} \sqrt{25 - 0} + \frac{25}{2} \sin^{-1}(0) \right]$$

$$= \frac{16}{5} \times \frac{25}{2} \times \frac{\pi}{2}$$

$$= 20\pi \text{ sq units.}$$

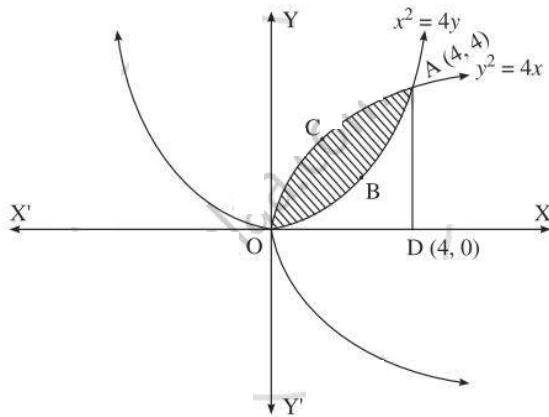
Miscellaneous Exercise 5 | Q 2.04 | Page 190

Solve the following :

Find the area of the region lying between the parabolas :

$$y^2 = 4x \text{ and } x^2 = 4y$$

SOLUTION



For finding the points of intersection of the two parabolas, we equate the values of y^2 from their equations.

$$\text{From the equation } x^2 = 4y, y = \frac{x^2}{4}$$

$$\therefore y = \frac{x^4}{16}$$

$$\therefore \frac{x^4}{16} = 4x$$

$$\therefore x^4 - 64x = 0$$

$$\therefore x(x^3 - 64) = 0$$

$$\therefore x = 0 \text{ or } x^3 = 64$$

$$\text{i.e. } x = 0 \text{ or } x = 4$$

$$\text{When } x = 0, y = 0$$

When $x = 4$, $y = \frac{4^2}{4} = 4$

$\therefore$ the points of intersection are $O(0, 0)$ and $A(4, 4)$.

Required area = area of the region OBACO

$$= [\text{area of the region ODACO}] - [\text{area of the region ODABO}]$$

Now, area of the region ODACO

= area under the parabola $y^2 = 4x$,

i.e. $y = 2\sqrt{x}$ between $x = 0$ and $x = 4$

$$= \int_0^4 2\sqrt{x} \cdot dx$$

$$= \left[2 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^4$$

$$= 2 \times \frac{2}{3} \times 4^{\frac{3}{2}} - 0$$

$$= \frac{4}{3} \times (2^3)$$

$$= \frac{32}{3}$$

Area of the region ODABO

= area under the parabola $x^2 = 4y$,

i.e. $y = \frac{x^2}{4}$ between $x = 0$ and $x = 4$

$$= \int_0^4 \frac{1}{4} x^2 \cdot dx$$

$$\begin{aligned}
 &= \frac{1}{4} \left[\frac{x^3}{3} \right]_0^4 \\
 &= \frac{1}{4} \left(\frac{64}{3} - 0 \right) \\
 &= \frac{16}{3}
 \end{aligned}$$

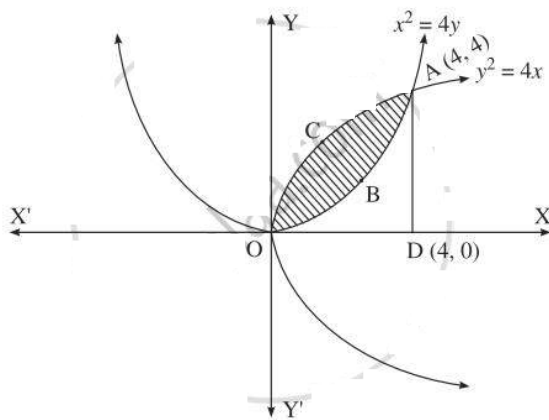
$$\begin{aligned}
 \therefore \text{required area} &= \frac{32}{3} - \frac{16}{3} \\
 &= \frac{16}{3} \text{ sq units.}
 \end{aligned}$$

Miscellaneous Exercise 5 | Q 2.04 | Page 190

Solve the following :

Find the area of the region lying between the parabolas : $y^2 = x$ and $x^2 = y$.

SOLUTION



For finding the points of intersection of the two parabolas, we equate the values of y^2 from their equations.

From the equation $x^2 = y$, $y = \frac{x^2}{y}$

$$\therefore y = \frac{x^2}{y}$$

$$\therefore \frac{x^2}{y} = x$$

$$\therefore x^2 - y = 0$$

$$\therefore x(x^3 - y) = 0$$

$$\therefore x = 0 \text{ or } x^3 = y$$

$$\text{i.e. } x = 0 \text{ or } x = 4$$

$$\text{When } x = 0, y = 0$$

$$\text{When } x = 4, y = \frac{4^2}{4} = 4$$

$\therefore$ the points of intersection are $O(0, 0)$ and $A(4, 4)$.

Required area = area of the region OBACO

$$= [\text{area of the region ODACO}] - [\text{area of the region ODABO}]$$

Now, area of the region ODACO

= area under the parabola $y^2 = 4x$,

i.e. $y = 2\sqrt{x}$ between $x = 0$ and $x = 4$

$$= \int_0^4 2\sqrt{x} \cdot dx$$

$$= \left[2 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^4$$

$$= 2 \times \frac{2}{3} \times 4^{\frac{3}{2}} - 0$$

$$= \frac{4}{3} \times (2^3)$$

$$= \frac{32}{3}$$

Area of the region ODABO

= area under the parabola $x^2 = 4y$,

i.e. $y = \frac{x^2}{4}$ between $x = 0$ and $x = 4$

$$= \int_0^4 \frac{1}{4} x^2 \cdot dx$$

$$= \frac{1}{4} \left[\frac{x^3}{3} \right]_0^4$$

$$= \frac{1}{4} \left(\frac{64}{3} - 0 \right)$$

$$= \frac{16}{3}$$

$$\therefore \text{required area} = \frac{32}{3} - \frac{16}{3}$$

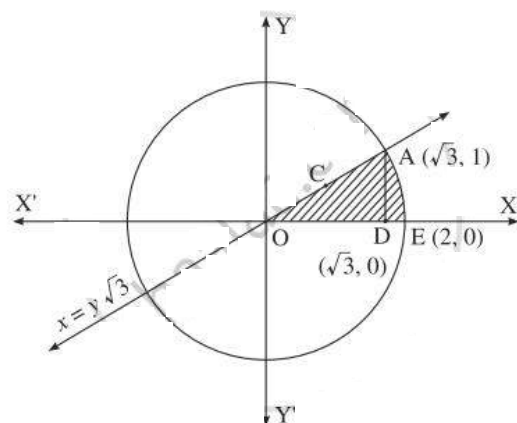
$$= \frac{16}{3} \text{ sq units.}$$

Miscellaneous Exercise 5 | Q 2.05 | Page 190

Solve the following :

Find the area of the region in first quadrant bounded by the circle $x^2 + y^2 = 4$ and the X-axis and the line $x = y\sqrt{3}$.

SOLUTION



For finding the point of intersection of the circle and the line, we solve

$$x^2 + y^2 = 4 \quad \dots(1)$$

$$\text{and } x = y\sqrt{3} \quad \dots(2)$$

$$\text{From (2), } x^2 = 3y$$

$$\text{From (1), } x^2 = 4 - y^2$$

$$\therefore 3y^2 = 4 - y^2$$

$$\therefore 4y^2 = 4$$

$$\therefore y^2 = 1$$

$$\therefore y = 1 \text{ in the first quadrant.}$$

$$\text{When } y = 1, x = 1 \times \sqrt{3} = \sqrt{3}$$

$\therefore$ the circle and the line intersect at $A(\sqrt{3}, 1)$ in the first quadrant

Required area = area of the region OCAEDO

= area of the region OCADO + area of the region DAED

Now, area of the region OCADO

= area under the line $x = y\sqrt{3}$

i.e. $y = \frac{x}{\sqrt{3}}$ between $x = 0$ and $x = \sqrt{3}$

$$\begin{aligned}
&= \int_0^{\sqrt{3}} \frac{x}{\sqrt{3}} \cdot dx \\
&= \left[\frac{x^2}{2\sqrt{3}} \right]_0^{\sqrt{3}} \\
&= \frac{3}{2\sqrt{3}} - 0 \\
&= \frac{\sqrt{3}}{2}
\end{aligned}$$

Area of the region DAED

$$\begin{aligned}
&= \text{area under the circle } x^2 + y^2 = 4 \text{ i.e. } y = +\sqrt{4-x^2} \text{ (in the first quadrant) between } x = \sqrt{3} \text{ and } x = 2 \\
&= \int_{\sqrt{3}}^2 \sqrt{4-x^2} \cdot dx \\
&= \left[\frac{x}{2} \sqrt{4-x^2} + \frac{4}{2} \sin^{-1} \left(\frac{x}{2} \right) \right]_{\sqrt{3}}^2 \\
&= \left[\frac{2}{2} \sqrt{4-4} + 2 \sin^{-1}(1) \right] - \left[\frac{\sqrt{3}}{2} \sqrt{4-3} + 2 \sin^{-1} \frac{\sqrt{3}}{2} \right] \\
&= 0 + 2 \left(\frac{\pi}{2} \right) - \frac{\sqrt{3}}{2} - 2 \left(\frac{\pi}{3} \right) \\
&= \pi - \frac{\sqrt{3}}{2} - \frac{2\pi}{3} \\
&= \frac{\pi}{3} - \frac{\sqrt{3}}{2} \\
\therefore \text{required area} &= \frac{\sqrt{3}}{2} + \left(\frac{\pi}{3} - \frac{\sqrt{3}}{2} \right) \\
&= \frac{\pi}{3} \text{ sq units.}
\end{aligned}$$

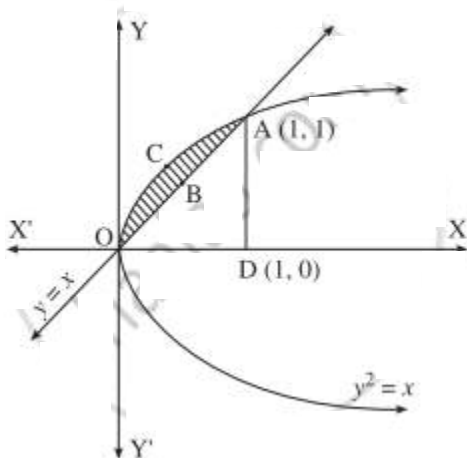
Miscellaneous Exercise 5 | Q 2.06 | Page 190

Solve the following :

Find the area of the region bounded by the parabola $y^2 = x$ and the line $y = x$ in the first quadrant.

SOLUTION

To obtain the points of intersection of the line and the parabola, we equate the values of x from both the equations.



$$\begin{aligned}\therefore y^2 &= y \\ \therefore y^2 - y &= 0 \\ \therefore y(y - 1) &= 0 \\ \therefore y &= 0 \text{ or } y = 1\end{aligned}$$

When $y = 0$, $x = 0$

When $y = 1$, $x = 1$

$\therefore$ the points of intersection are $O(0, 0)$ and $A(1, 1)$. Required area of the region $OCABO$
= area of the region $OCADO$ – area of the region $OBADO$

Now, area of the region $OCADO$

= area under the parabola $y^2 = x$ i.e. $y = \pm\sqrt{x}$ (in the first quadrant) between $x = 0$ and $x = 1$

$$\begin{aligned}&= \int_0^1 \sqrt{x} \cdot dx \\&= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^1 \\&= \frac{2}{3} \times (1 - 0) \\&= \frac{2}{3}\end{aligned}$$

Area of the region OBADO

= area under the line $y = x$ between $x = 0$ and $x = 1$

$$= \int_0^1 x \cdot dx$$

$$= \left[\frac{x^2}{2} \right]_0^1$$

$$= \frac{1}{2} - 0$$

$$= \frac{1}{2}$$

$$\therefore \text{required area} = \frac{2}{3} - \frac{1}{2}$$

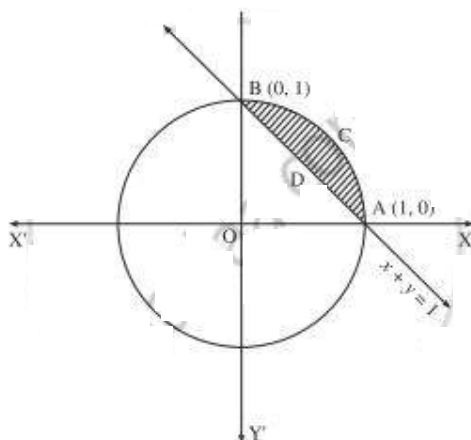
$$= \frac{1}{6} \text{ sq unit.}$$

Miscellaneous Exercise 5 | Q 2.07 | Page 190

Solve the following :

Find the area enclosed between the circle $x^2 + y^2 = 1$ and the line $x + y = 1$, lying in the first quadrant.

SOLUTION



Required area = area of the region ACBDA

= (area of the region OACBO) – (area of the region OADBO)

Now, area of the region OACBO

= area under the circle $x^2 + y^2 = 1$ between $x = 0$ and $x = 1$

$$= \int_0^1 \cdot dx, \text{ where } y^2 = 1 - x^2,$$

$$\text{i.e. } y = \sqrt{1 - x^2}, \text{ as } y > 0$$

$$= \int_0^1 \sqrt{1 - x^2} \cdot dx$$

$$= \left[\frac{x}{2} \sqrt{1 - x^2} + \frac{1}{2} \sin^{-1}(x) \right]_0^1$$

$$= \frac{1}{2} \sqrt{1 - 1} + \frac{1}{2} \sin^{-1} 1 - 0$$

$$= \frac{1}{2} \times \frac{\pi}{2}$$

$$= \frac{\pi}{4}$$

Area of the region OADBO

= area under the line $x + y = 1$ between $x = 0$ and $x = 1$

$$= \int_0^1 y \cdot dx, \text{ where } y = 1 - x$$

$$= \int_0^1 (1 - x) \cdot dx$$

$$= \left[x - \frac{x^2}{2} \right]_0^1$$

$$= 1 - \frac{1}{2} - 0$$

$$= \frac{1}{2}$$

$$\therefore \text{required area} = \left(\frac{\pi}{4} - \frac{1}{2} \right) \text{sq units.}$$

Miscellaneous Exercise 5 | Q 2.08 | Page 190

Solve the following :

Find the area of the region bounded by the curve $(y - 1)^2 = 4(x + 1)$ and the line $y = (x - 1)$.

SOLUTION

The equation of the curve is $(y - 1)^2 = 4(x + 1)$

This is a parabola with vertex at $A(-1, 1)$.

To find the points of intersection of the line $y = x - 1$ and the parabola.

Put $y = x - 1$ in the equation of the parabola, we get

$$(x - 1 - 1)^2 = 4(x + 1)$$

$$\therefore x^2 - 4x + 4 = 4x + 4$$

$$\therefore x^2 - 8x = 0$$

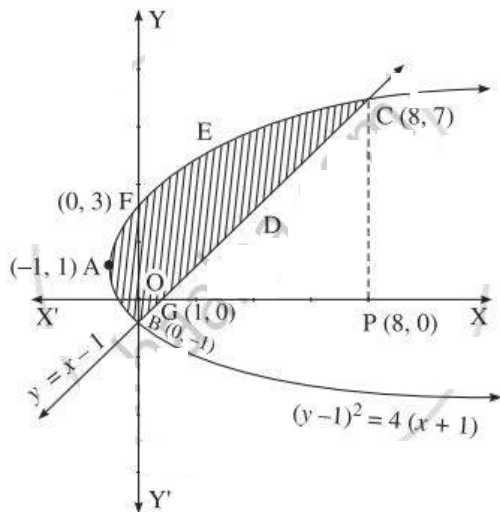
$$\therefore x(x - 8) = 0$$

$$\therefore x = 0, x = 8$$

$$\text{When } x = 0, y = 0 - 1 = -1$$

$$\text{When } x = 8, y = 8 - 1 = 7$$

$\therefore$ the points of intersection are $B(0, -1)$ and $C(8, 7)$



To find the points where the parabola $(y - 1)^2 = 4(x + 1)$ cuts the Y-axis.

Put $x = 0$ in the equation of the parabola, we get

$$(y - 1)^2 = 4(0 + 1) = 4$$

$$\therefore y - 1 = \pm 2$$

$$\therefore y - 1 = 2 \text{ or } y - 1 = -2$$

$$\therefore y = 3 \text{ or } y = -1$$

$\therefore$ the parabola cuts the Y-axis at the points $B(0, -1)$ and $F(0, 3)$.

To find the point where the line $y = x - 1$ cuts the X-axis. Put $y = 0$ in the equation of the line, we get

$$x - 1 = 0$$

$$\therefore x = 1$$

$\therefore$ the line cuts the X-axis at the point G (1, 0).

Required area = area of the region BFAB + area of the region OGDCEFO + area of the region OBGO

Now, area of the region BFAB

= area under the parabola $(y - 1)^2 = 4(x + 1)$, Y-axis from $y = -1$ to $y = 3$

$$= \int_{-1}^3 x \cdot dy, \text{ where } x + 1 = \frac{(y - 1)^2}{4}, \text{ i.e. } x = \frac{(y - 1)^2}{4} - 1$$

$$= \int_{-1}^3 \left[\frac{(y - 1)^2}{4} - 1 \right] \cdot dy$$

$$= \left[\frac{1}{4} \cdot \frac{(y - 1)^3}{3} - y \right]_{-1}^3$$

$$= \left[\left\{ \frac{1}{12} (3 - 1)^3 - 3 \right\} - \left\{ \frac{1}{12} (-1 - 1)^3 - (-1) \right\} \right]$$

$$= \frac{8}{12} - 3 + \frac{8}{12} - 1$$

$$= \frac{16}{12} - 4$$

$$= \frac{4}{3} - 4$$

$$= -\frac{8}{3}$$

Since, area cannot be negative, area of the region BFAB

$$= \left| -\frac{8}{3} \right|$$

$$= \frac{8}{3} \text{ sq units.}$$

Area of the region OGDCEFO

= area of the region OPCEFO – area of the region GPCDG

$$= \int_0^8 y \cdot dx, \text{ where } (y-1)^2$$

$$= 4(x+1), \text{ i.e. } y = 2\sqrt{x+1} + 1 - \int_1^8 y \cdot dx, \text{ where } y = x - 1$$

$$= \int_0^8 [2\sqrt{x+1} + 1] \cdot dx - \int_1^8 (x-1) \cdot dx$$

$$= \left[\frac{2 \cdot (x+1)^{\frac{3}{2}}}{\frac{3}{2}} + x \right]_0^8 - \left[\frac{x^2}{2} - x \right]_1^8$$

$$= \left[\frac{4}{3}(9)^{\frac{3}{2}} + 8 - \frac{4}{3}(1)^{\frac{3}{2}} - 0 \right] - \left[\left(\frac{64}{2} - 8 \right) - \left(\frac{1}{2} - 1 \right) \right]$$

$$= \left(36 + 8 - \frac{4}{3} \right) - \left(24 + \frac{1}{2} \right)$$

$$= 44 - \frac{4}{3} - 24 - \frac{1}{2}$$

$$= 20 - \left(\frac{4}{3} + \frac{1}{2} \right)$$

$$= 20 - \frac{11}{6}$$

$$= \frac{109}{6} \text{ sq units.}$$

Area of region OBGO = $\int_0^1 y \cdot dx, \text{ where } y = x - 1$

$$\begin{aligned}
 &= \int_0^1 (x - 1) \cdot dx \\
 &= \left[\frac{x^2}{2} - x \right]_0^1 \\
 &= \frac{1}{2} - 1 - 0 \\
 &= -\frac{1}{2}
 \end{aligned}$$

Since, area cannot be negative,

$$\text{area of the region} = \left| -\frac{1}{2} \right| = \frac{1}{2} \text{ sq unit.}$$

$$\begin{aligned}
 \therefore \text{required area} &= \frac{8}{3} + \frac{109}{6} + \frac{1}{2} \\
 &= \frac{16 + 109 + 3}{6} \\
 &= \frac{128}{6} \\
 &= \frac{64}{3} \text{ sq units.}
 \end{aligned}$$

Miscellaneous Exercise 5 | Q 2.09 | Page 190

Solve the following :

Find the area of the region bounded by the straight line $2y = 5x + 7$, X-axis and $x = 2$, $x = 5$.

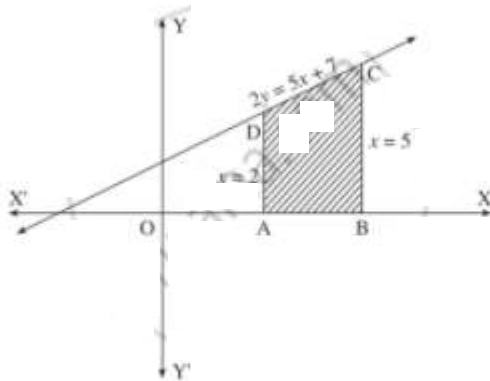
SOLUTION

The equation of the line is $2y = 5x + 7$,

$$\text{i.e., } y = \frac{5}{2}x + \frac{7}{2}$$

Required area = area of the region ABCDA

$$= \text{area under the line } y = \frac{5}{2}x + \frac{7}{2} \text{ between } x = 2 \text{ and } x = 5$$



$$= \int_2^5 \left(\frac{5}{2}x + \frac{7}{2} \right) \cdot dx$$

$$= \frac{5}{2} \cdot \int_2^5 x \cdot dx + \frac{7}{2} \int_2^5 1 \cdot dx$$

$$= \frac{5}{2} \left[\frac{x^2}{2} \right]_2^5 + \frac{7}{2} [x]_2^5$$

$$= \frac{5}{2} \left[\frac{25}{2} - \frac{4}{2} \right] + \frac{7}{2} [5 - 2]$$

$$= \frac{5}{2} \times \frac{21}{2} + \frac{21}{2}$$

$$= \frac{105}{4} + \frac{42}{4}$$

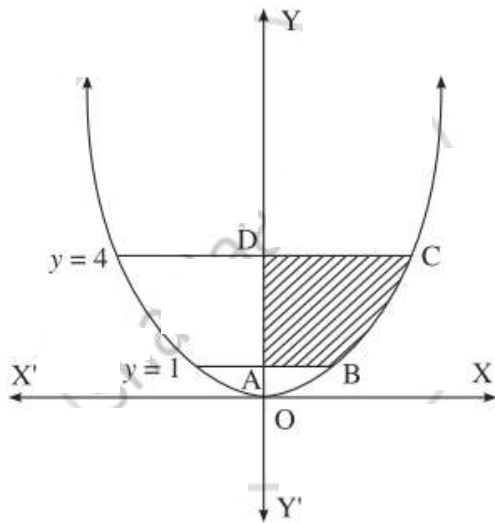
$$= \frac{147}{4} \text{ sq units.}$$

Miscellaneous Exercise 5 | Q 2.10 | Page 190

Solve the following :

Find the area of the region bounded by the curve $y = 4x^2$, Y-axis and the lines $y = 1$, $y = 4$.

SOLUTION



By symmetry of the parabola, the required area is 2 times the area of the region ABCD.

From the equation of the parabola, $x^2 = \frac{y}{4}$

the first quadrant, $x > 0$

$$\therefore x = \frac{1}{2} \sqrt{y}$$

$$\therefore \text{required area} = \int_1^4 x \cdot dy$$

$$= \frac{1}{2} \int_1^4 \sqrt{y} \cdot dy$$

$$= \frac{1}{2} \left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^4$$

$$= \frac{1}{2} \times \frac{2}{3} \left[4^{\frac{3}{2}} - 1^{\frac{3}{2}} \right]$$

$$= \frac{1}{3} \left[(2^2)^{\frac{3}{2}} - 1 \right]$$

$$= \frac{1}{3} [8 - 1]$$

$$= \frac{7}{3} \text{sq units.}$$