1 Decision Procedure

DO YOU KNOW THAT ....

- One can determine whether the statement form is tautology or not in a single row.
- One can determine the validity of many complicated arguments by merely constructing a shorter truth table.
- As in geometry, so in logic, one can decide that a statement form is a tautology by showing the impossibility of its opposite.

1.1 Decision procedure

I.M. Copi defines logic as **"The study of the methods and principles used to distinguish good (correct) from bad (incorrect) reasoning."** The two main functions in logic are - (i) To decide whether an argument is valid or invalid; and (ii) To decide whether a given statement form (truth functional form) is a tautology, contradiction or contingency. A procedure (or method) for deciding these, is called a decision procedure. The main requirement of a decision procedure is that it must be effective. To be an effective decision procedure, it must satisfy 3 conditions – reliable, mechanical and finite.

1.2 Need for shorter truth table method

We have already studied Truth Table as an effective decision procedure. Though, truth table is a simple and easy method for deciding whether a statement form is tautology or not and an argument is valid or invalid, but it has certain limitations. Truth table becomes inconvenient when a statement form involves many variables i.e. with four variables the truth table will have sixteen rows, five variables thirty two rows and so on. With the increase in number of propositional variables in a given expression, the number of rows in the truth table also increases. At such times the application of the method becomes complicated and difficult to manage and the truth table becomes very long, tedious and time consuming. We may make errors while constructing it so lot of carefulness is required. Hence we need shorter and accurate method for determining whether a statement form is tautology or not. Hence shorter truth table method is introduced.

The shorter Truth Table procedure can be carried out in a single line. In fact this is the main advantage of the shorter truth table as a decision procedure. Shorter truth table method is a quick and easy method. As it helps us to decide whether an argument is valid and whether a given statement form is tautology.

1.3 Nature of shorter truth table method

Shorter truth table is a decision procedure –

Shorter truth table method is an effective decision procedure as is satisfies all the conditions of an effective decision procedure. i.e. reliable, mechanical and finite.

The shorter truth table method is based on the principle of reductio-ad-absurdum. The principle of Reductio-ad-absurdum means to show that the **opposite of what is to be proved leads to an absurdity.** In the case of

Complete the following

p • q	p ∨ q	p ⊃ q	p ≡ q	∼ p	∼ p
T T ☐	☐ F F	T F ☐	☐ F T	☐ T	☐ F

argument we begin by assuming it to be invalid and if the assumption leads to an inconsistency then the argument is proved as valid otherwise it is invalid.

In the case of statement form we first assume it to be not a tautology and if the assumption leads to an inconsistency then the statement form is proved to be tautology or else it is not a tautology.

Since this method does not directly prove whether the argument is valid/invalid or whether the statement form is a tautology or not, it is called the **"Indirect method"**.

1.4 Shorter Truth Table Method as a test of Tautology –

The shorter truth table method is based on the basic truth tables of truth functional compound propositions.

Shorter truth table method is used to decide whether a statement form is tautology or not. Tautology is a truth functional statement form which is true under all truth possibilities of its components. While constructing shorter truth table, we assume that the statement form is not a tautology by placing the truth value 'F' under the main connective of the statement form. If we arrive at an inconsistency, then the assumption is wrong and given statement form is a tautology (tautologous). If we do not arrive at any inconsistency, then the assumption is correct and hence the given statement form is not a tautology. It is either contradictory or contingency.

This procedure involves the following steps –

(1) For determining whether a statement form is a tautology, one has to begin by assuming that it is not a tautology.

(2) For assuming statement form is not a tautology, one has to place 'F' under the main connective of the statement form.

(3) After assigning 'False' truth value under the main connective, with the help of basic truth tables, one can assign truth values to the various components of the statement form.

(4) Truth values are to be assigned to all the connectives and the variables of the statement form and every step is to be numbered.

(5) After assigning the truth value one has to check whether there is any inconsistency. Inconsistencies are of two types – (i) Violation of rules of basic truth table (ii) If a propositional variable gets both truth values i.e. True as well as False.

(6) An inconsistency will prove that the given statement form is a tautology. If there is no inconsistency, it will prove that the statement form is not a tautology.

(7) We mark the inconsistency with a cross "x" below it.

(8) Write whether the given statement form is a tautology or not a tautology.

Following example demonstrates the procedure.

Example 1 (p • p) ⊃ p

(1) One has to assume that the given statement form is 'not a tautology' by writing 'F' under the main connective '⊃'. We mark the assumption 'F' with a star as shown below.

(p • p)	⊃	p
	F	
	*	

(2) The next step is to assign values by using basic truth tables. Since in the example, implication is assumed to be false, the antecedent has to be true and consequent has to be false. So we assign values as follows and number the steps.

(p • p)	⊃	p
T	F	F
(1)	*	(1)

(3) In the next step one has to assign truth values to the component statements of the antecedent. The antecedent is ‘p • p’ is true. Conjunction is true when both its conjuncts are true. So one has to assign values as follows and number them.

(p • p) ⊃ p

T T T F F

(2) (1) (2) * (1)

(4) Next step is to find out whether these assumption leads to any inconsistency. In the above example one gets inconsistent values for ‘p’. We indicate inconsistency by ‘x’ mark as shown below.

(p • p) ⊃ p

T T T F F

(2) (1) (2) * (1)

x x x

In the above example there is inconsistency in step number 1 and 2. So the assumption is wrong. Hence the given statement form is a tautology.

Example 2 (p • ∼ q) ∨ (q ⊃ p)

(1) To begin with, one has to assume that the given statement form is ‘not a tautology’, by writing ‘F’ below the main connective ‘∨’ (Disjunction). We mark the assumption “F” with a star as shown below.

(p •∼ q) ∨ (q ⊃ p)

F

*

(2) The next step is to assign truth values by using basic truth tables. Since in the example disjunction is assumed to be false, both the disjuncts will be false.

(p • ∼ q) ∨ (q ⊃ p)

F F F

(1) * (1)

(3) The next step is to assign truth values to the components of both the disjuncts and number them. In case of 1st disjunct “•” (conjunction) is the main connective and it is false. Conjunction is false under three possibilities, so we should not assign values to its components. We try to get truth values of the second disjunct which is “q ⊃ p”. Implication is false only under one condition i.e. when its antecedent is true and its consequent is false. So one has to assign values to its components and number them as shown below.

(p • ∼ q) ∨ (q ⊃ p)

F F T F F

(1) * (2) (1) (2)

(4) Since one knows the truth values of both ‘p’ and ‘q’, the same truth values can be assigned to the components of the left disjunct, as shown below and number them.

(p • ∼ q) ∨ (q ⊃ p)

F F F T F T F F

(3) (1) (5) (4) * (2) (1) (2)

(5) Next step is to see whether these truth values lead to any inconsistency. In the above example, there is no inconsistency. The assumption is correct. Hence the given statement form is not a tautology.

Example 3 (p ⊃ ∼ q) ≡ ∼ (q • p)

One has to assume that the given statement form is ‘not a tautology’ by writing ‘F’ under the main connective ‘≡’ (equivalence). Equivalent statement is false under two possibilities. – (1) The first component is true and the second is false. And (2) The first compoment is false and second is true. We have to solve the example by assuming both the possibilities.

1st possibility

(1) Considering the first possibility, values are assigned in the given example as follows.

$(p \supset \sim q) \equiv \sim (q \bullet p)$

T F F

1 * 1

(2) The next step is to assign truth values to the components of equivalence and number them. In case of first compoment "⊃" is the main connective and it is true. Implication is true under three possibilities, so we should not assign values to its components. We try to get truth values of the second compoment which is '∼ (q • p)'. We already placed 'F' below '∼'. When negation is false, conjunction has to be true. Accordingly one has to assign values to its components as shown below.

$(p \supset \sim q) \equiv \sim (q \bullet p)$

T F F T T T

1 * 1 3 2 3

(3) Since one knows the truth values of both 'p' and 'q', the same truth values can be assigned to the variables in the first component and also to the negation of the variable 'q' as shown below.

$(p \supset \sim q) \equiv \sim (q \bullet p)$

T T F T F F T T T

4 1 6 5 * 1 3 2 3

x

(4) There is inconsistency in step number 1 as it violates the rule of implication. So the assumption is wrong. Hence the given statement form is a tautology, in the case of first possibility.

Now let's consider the second possiblity

2 nd possibility

(1) $(p \supset \sim q) \equiv \sim (q \bullet p)$

F F T

1 * 1

Considering the second possiblity, truth values are assigned as follows.

The next step is to assign truth values to the components of equivalence. In case of first component '⊃' is false. So truth values are assigned as follows.

(2) $(p \supset \sim q) \equiv \sim (q \bullet p)$

T F F T F T

2 1 2 3 * 1

'∼ q' is 'F' so 'q' will be 'T'

Since one knows the truth values of both 'p' and 'q', the same truth values can be assigned to the variables in the second component as shown below.

(3) $(p \supset \sim q) \equiv \sim (q \bullet p)$

T F F T F T T F T

2 1 2 3 * 1 5 4 6

x

There is inconsistency in step number 4 as it violates the rule of conjunction. So the assumption is wrong. Hence the given statement form is a tautology in the case of second possibility as well

In above example we get inconsistency in both the possiblities. So in both the possiblities it is a tautology and therefore, the given statement form is a tautology. It should be noted that if one of the possibilities is not a tautology, then the statement form is not a tautology. To be tautology, the statement form must be tautology under every possibility.

Example 4 $(p \vee \sim q) \bullet (\sim p \supset q)$

One has to begin by assuming the above statement form to be 'not a tautology' by writing 'F' below '•'. Conjunction is false under three possibilities. –

(1) First conjunct is True and second conjunct is False;

(2) First conjunct is False and second conjunct is True; and

(3) Both the conjuncts are false.

This problem is to be solved considering all the three possibilities.

1st possibility

(p ∨ ∼ q) • (∼ p ⊃ q)

F T T F F   T F F F

4 1 6 5 * 2 3 1 2

There is no inconsistency. The assumption is correct. Hence in this possiblity the given statement form is not a tautology.

2nd possibility

(p ∨ ∼ q) • (∼ p ⊃ q)

F F F T F   T F T T

2 1 2 3 * 6 4 1 5

There is no inconsistency. The assumption is correct. Hence in this possibility too the given statement form is not a tautology.

3rd possibility

(p ∨ ∼ q) • (∼ p ⊃ q)

F F F T F   T F F T

2 1 2 3 * 6 4 1 5

x

There is an inconsistency in step number 1 as it violates the rule of implication. So the assumption is wrong and a statement form is a tautology in case of this possibility. Out of three possibilities, the statement form is not a tautology in the case of two possibilities and is a tautology in the case of one possibility. Hence, the given statement form is not a tautology.

If we get 'not a Tautology' in the first possibility, then the whole expression will be 'not a Tautology' and there is no need to check further possibilities.

Example 5 (p • q) ∨ (p ∨ q)

F F F   F   F F F

3 1 3 * 2 1 2

There is no inconsistency, therefore the given statement form is not a tautology.

Example 6 (p • ∼ q) ⊃ ∼ q

T T T F   F F T

3 1 3 4 * 1 2

x   x

There is inconsistency in step Number 2 and 4, therefore the given statement form is a tautology.

Example 7 [(p ⊃ q) • q] ⊃ ∼ p

T T T T T F F T

4 3 5 1 3 * 1 2

There is no inconsistency. Therefore the given statement form is not a tautology.

Example 8 (p ⊃ q) ⊃ [(p ∨ r) ⊃ (q ∨ r)]

T T F   F   T T F   F   F F F

6 1 7 * 5 2 4 1 3 2 3

x

Since there is inconsistency in step number 1. Therefore the given statement form is a tautology.

Example 9 ∼ (∼ p ∨ q) ∨ (q ∨ ∼ p)

F   F T T F   F   F F F T

1   7 5 2 6 * 3 1 3 4

x x x

> Assign the correct truth value
>
> (1) (p ⊃ q) ⊃ [(p ⊃ r) ⊃ q]
>
> ☐   F   ☐
>
> *
>
> (2) ∼ [(∼ p ∨ q) • (∼ q • r)]
>
> F   ☐

Summary

- Shorter truth table method is a decision procedure.
- It is an effective decision procedure because it is reliable, finite and mechanical.
- It is a convenient method.
- It is used to test whether a statement form is a tautology or not a tautology.
- It is an indirect method.
- It is based on the principle of reductio-ad-absurdum.
- It is based on the basic truth tables of truth functional compound statements.

Basic Truth Table

Negation

~	p
F	T
T	F

Conjunction

p	•	q
T	T	T
T	F	F
F	F	T
F	F	F

Disjunction

p	∨	q
T	T	T
T	T	F
F	T	T
F	F	F

Implication

p	⊃	q
T	T	T
T	F	F
F	T	T
F	T	F

Equivalence

p	≡	q
T	T	T
T	F	F
F	F	T
F	T	F

Exercises

Q. 1. Fill in the blanks with suitable words from those given in the brackets :

(1) Shorter truth table is an ………. method. *(direct/indirect)*

(2) ………. method is based on the principle of reductio-ad-absurdum. *(Truth table/ Shorter Truth Table)*

(3) If both the antecedent and the consequent of an implicative statement are false then the statement is ………. . *(true/false)*

(4) If inconsistency is obtained after assuming the given statement form to be false, then the statement form is proved to be ……….. *(tautology/ not a tautology)*

(5) When both the components of a disjunctive statement are false then the truth value of the statement is ………. . *(true/ false)*

(6) When we deny tautology, we get ………. . *(contradiction/ contingency)*

(7) If 'p' is true then '∼p' is ………. . *(true/ false)*

(8) Shorter truth table is a ………. . *(decision procedure/ deductive proof)*

(9) Equivalence is ………. when both its components are false. *(true/ false)*

(10) ………. is a symbol used for negative statement. (• / ∼)

Q. 2. State whether the following statements are true or false.

(1) A negative statement is false when its component statement is true.

(2) If a conjunctive proposition is false both its components must be false.

(3) '•' is a monadic connective.

(4) Inconsistency in a shorter truth table is obtained when a rule of basic truth table is violated.

(5) Shorter truth table method is inconvenient than truth table method.

(6) Truth table is based on the principle of reductio-ad-absurdum.

(7) Shorter truth table does not directly prove whether a statement form is a tautology or not.

(8) Contingency is always true.

(9) If the consequent is true then the implicative statement must be true.

(10) Contradictory statement form is always false.

(11) ' p ∨ ∼ p' is a tautology.

Q. 3. Match the columns :

	(A)		(B)
(1)	Shorter Truth Table	(a)	Always true
(2)	Truth Table	(b)	Always false
(3)	Contradiction	(c)	Direct Method
(4)	Tautology	(d)	Reductio-ad-absurdum

Q. 4. Give logical terms for the following :

(1) A statement form which is always true.

(2) A decision procedure based on reductio-ad-absurdum.

(3) A statement form which is true under all truth possibilities of its components.

(4) A decision procedure which is an indirect method.

(5) Statement having antecedent and consequent as its components.

(6) A statement form which is false under all possibilities.

(7) A statement form which is true under some possibilities and false under some possibilities.

Q. 5. Use shorter truth table method to test whether the following statement forms are tautologous.

(1) $[(p \supset \sim q) \cdot q] \supset \sim p$

(2) $(\sim p \cdot \sim q) \cdot (p \equiv q)$

(3) $(p \supset q) \supset (\sim q \supset \sim p)$

(4) $(p \cdot q) \vee (q \supset p)$

(5) $(p \cdot p) \vee \sim p$

(6) $(q \supset \sim p) \vee \sim q$

(7) $(\sim p \supset q) \cdot (\sim p \cdot \sim q)$

(8) $[(\sim p \vee \sim q) \cdot q] \supset \sim p$

(9) $(p \supset \sim q) \vee (\sim q \supset p)$

(10) $\sim p \vee (p \supset q)$

(11) $(p \supset q) \equiv (\sim p \vee q)$

(12) $(\sim p \cdot \sim q) \supset (q \supset \sim p)$

(13) $(p \vee q) \supset \sim (p \cdot q)$

(14) $\sim (p \vee q) \equiv (\sim p \cdot \sim q)$

(15) $(\sim p \cdot q) \supset (q \supset p)$

(16) $(q \supset p) \cdot \sim p$

(17) $\sim (p \cdot q) \vee (p \supset \sim q)$

(18) $(\sim p \supset q) \cdot (\sim q \supset p)$

(19) $p \supset [(r \supset p) \supset p]$

(20) $p \supset (p \vee q)$

(21) $(p \vee p) \equiv \sim p$

(22) $\sim (p \supset \sim q) \supset (q \cdot p)$

(23) $p \cdot \sim (p \supset \sim p)$

(24) $\sim [p \supset (\sim q \vee p)]$

(25) $(p \cdot q) \equiv (\sim p \supset \sim q)$

❖ ❖ ❖