Answers

1. Mathematical logic

Exercise : 1.1

Sentences (ii), (x), (xiii), (xvi), (xvii), (xviii), (xix), (xx), (xxiv) are statements and their truth value is T.

Sentences (i), (v), (vi), (xi), (xii), (xiv), (xv) are statements and their truth value is F.

Sentences (iii), (iv), (vii), (viii), (ix), (xxi), (xxii), (xxiii), (xxv) are not statements in logic.

Exercise : 1.2

1) i) $p \vee q$ ii) $p \wedge q$ iii) $p \vee q$
iv) $p \wedge q$ v) $p \wedge q$

2) truth values are
i) F ii) F iii) F
iv) T

Exercise : 1.3

1) i) Some men are not animals.
ii) -3 is not a natural number.
iii) Nagpur is capital of Maharashtra
iv) $2 + 3 = 5$

2) Truth values are
i) F ii) F iii) T

Exercise : 1.4

1) i) $p \rightarrow q$ ii) $\sim P$ iii) $\sim P \wedge q$
iv) $p \leftrightarrow \sim q$ v) $p \leftrightarrow q$ vi) $p \rightarrow q$

2) Truth values are
i) T ii) F iii) F
iv) T v) T

3) Truth values are
i) F ii) T iii) T
iv) F v) T vi) F

4) Truth values are
i) F ii) T iii) F

5) i) He swims if and only if water is not warm.
ii) It is not true that he swims or water is warm.
iii) If water is warm then he swims.
iv) water is warm and he does not swim.

Exercise : 1.5

1) i) $\exists\, x \in N$, such that $x^2 + 3x - 10 = 0$
It is true statement, since $x = 2 \in N$ satisfies it.

ii) $\exists\, x \in N$, such that $3x - 4 < 9$
It is a true statement, since $x = 2, 3, 4 \in N$ satisfy $3x - 4 < 9$.

iii) $\forall\, n \in N$, $n^2 \geq 1$
It is true statement, since all $n \in N$ satisfy it.

iv) $\exists\, n \in N$, such that $2n - 1 = 5$
It is true statement, since $n = 3 \in N$ satisfy $2n - 1 = 5$.

v) $\exists\, y \in N$, such that $y + 4 > 6$
It is true statement, since $y = 3, 4, \ldots\ldots \in N$ satisfy $y + 4 > 6$.

v) $\exists\, y \in N$, such that $3y - 2 \leq 9$

It is true statement, since y =1, 2, 3, ... $\in$ N satisfy it.

2) Truth value are

i) F ii) T iii) F

iv) F v) F

Exercise : 1.6

1) i) TFTT ii) FFTT

iii) TTTTTTTT iv) TTFTFTFT

2) i) tatology ii) contradiction

iii) contigency iv) tautology

Exercise : 1.7

1) i) $(p \wedge q) \wedge r$

ii) $\sim (p \wedge q) \vee [p \wedge \sim (q \vee \sim r)]$

iii) $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$

iv) $\sim (p \vee q) \equiv \sim p \wedge \sim q$

2) i) 13 is a prime number or India is a democratic country.

ii) Karina is very good and everybody likes her.

iii) Radha or Sushmita can not read Urdu.

iv) A number is real number or the square of the numbers is non negative.

Exercise : 1.8

1) i) Some stars are shining and it is night.

ii) $\exists$ n $\in$ N, such that n + 1 $\leq$ 0

iii) $\forall$ n $\in$ N, $(n^2 + 2)$ is not odd number

iv) All continuous functions are not differentiable.

2) i) $(p \wedge \sim r) \vee \sim q$

ii) $(\sim p \wedge \sim q) \wedge \sim r$

iii) $(p \vee \sim q) \vee (q \wedge r)$

3) i) Converse : If they do not drive the car then it snows.

Inverse : If it does not snow then they drive the car.

Contrapositive : If they drive the car then it does not snow.

ii) Converse : If he will go to college then he studies.

Inverse : If he does not study, then he will not go to college.

Contrapositive : If he will not go to college then he does not study.

4) i) $(p \wedge \sim q) \wedge (p \wedge \sim r)$

ii) $(p \wedge \sim q) \wedge r$

iii) $(p \wedge \sim q) \vee \sim r$

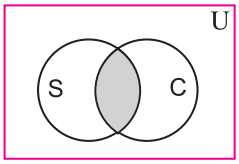
Exercise : 1.10

1) Venn diagrams.

i) U : The set of all students

S : The set of all hard working students.

O : The set of all obedient students.

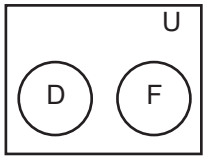


s$\cap$o $\neq \phi$

ii) U : The set of closed geometrical figures in plane.

D : The set of all polygons.

F : The set of all circles.

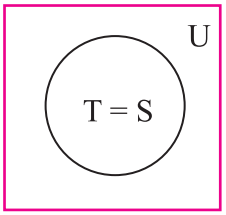


D $\cap$ F = ϕ

iii) U : The set of all human beigns.

T : The set of all teachers.

S : The set of all scholars.



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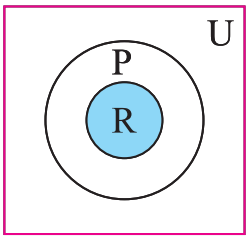
iv) U : The set of all quadrilaterals.

P : The set of all parallelograms.

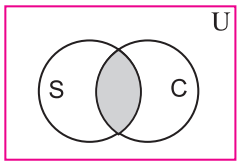
R : The set of all rhombus.



$R \subset P$

2) Venn diagrams

i)



$S \cap C \neq \phi$

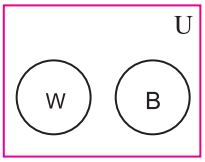
Where

U : The set of all human beings.

S : The set of all share brokers.

C : The set of all chartered accountants.

ii)



$W \cap B = \phi$

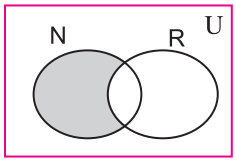
Where

U : The set of all human beings.

W : The set of all wicket keepers.

B : The set of all bowlers.

3) i)



$N - R \neq \phi$

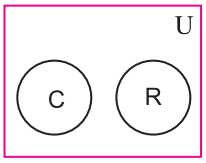
Where

U : The set of all human beings.

N : The set of all non resident Indians.

R : The set of all rich people.

ii)



$C \cap R = \phi$

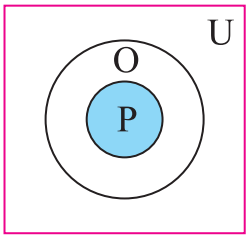
Where

U : The set of all geometrical polygons.

C : The set of all circles.

R : The set of all rectangles.

iiii)



$P \subset O$

Where

U : The set of all real numbers.

P : The set of all prime numbers and $n \neq 2$.

O : The set of all odd numbers.

MISCELLANEOUS EXERCISE - 1

I. 1) d 2) a 3) d 4) b
5) c 6) c 7) b 8) d
9) c 10) a 11) b 12) d
13) b 14) c 15) c

II. i) Converse ii) $p \wedge q$ iii) F
iv) No men are animals v) F
vi) $\sim p \rightarrow \sim q$ vii) different
viii) If the problem is not easy then it is not challenging.
ix) T

III. i) False ii) True iii) False
iv) False v) False vi) True
vii) True vii) False ix) False
x) True.

IV. 1) sentence (i), (ii), (iv), (v), (vi), (ix), (x), (xi) are statements. in logic
sentence (iii), (vii), (viii), (xii), (xiii) are not statements in logic

2) sentence (ii), (iii), (iv), (vi), (vii), (ix) are statement and truth value of each is T.
Sentence (x) is a statement and its truth value is F.
Sentence (i), (v), (viii) are not statement.

3) i) $p \wedge q$ ii) $p \wedge q$ iii) $p \leftrightarrow q$
iv) $p \wedge q$ v) $p \rightarrow q$ vi) $p \leftrightarrow q$
vii) $p \leftrightarrow q$ viii) $p \leftrightarrow q$ ix) $p \rightarrow q$
x) $p \rightarrow q$ xi) $\sim(p \wedge q)$ xii) $q \rightarrow p$
xiii) $\sim p$ xiv) $p \rightarrow q$

4) i) $p \wedge \sim q$ ii) $p \rightarrow q$ iii) $\sim(p \wedge q)$
iv) $q \leftrightarrow p$

5) i) Sachin wins the match or he is the member of Rajya Sabha or Sachin is happy.
ii) If Sachin wins the match then he is happy.
iii) Sachin does not win the match or he is the member of Rajya Sabha.
iv) If sachin wins the match, then he is the member of Rajyasabha or he is happy.
v) Sachin wins the match if and only if he is happy.
vi) Sachin wins the match and he is the member of Rajyasabha but he is not happy.
vii) It is false that sachin wins the match or he is the member of Rajyasabha but he is happy.

6) i) F ii) T iii) F
iv) T

7) i) T ii) T iii) F
iv) T

8) i) Demand does not fall or price does not increase.
ii) Price increase or demand does not falls.

9) i) F ii) F iii) F
iv) T v) T

10) i) ΔABC is not equilateral and it is equiangular.
ii) Ramesh is not intelligent or he is not hard working.
iii) An angle is a right angle and it is not of measure 90°, or an angle is of measure 90° and it is a right angles.
iv) Kanchanganga is not in India or Everest is not in Nepal.
v) $x \in (A \cap B)$ and $x \notin A$ or $x \notin B$.

ii) i) FTTF ii) FFFT

iii) TTTTTTTT iv) FTTTFTTT

v) TFTFTTFF

13) i) tautology ii) contradiction

iii) contradiction iv) contigency

v) contradiction

15) i) Converse : If 4 + 10 = 20, then 2 +5 = 10

Inverse : If 2 +5 ≠ 10, then 4 +10 ≠ 20

Contrapositive : If 4 + 10 ≠ 20, then 2 + 5 ≠ 10

ii) Converse : If a man is happy, then he is bachelor

Inverse : If a man is not bachelor, then he is not happy.

Contrapositive : If a man is not happy, then he is not bachelor.

iii) Converse : If I do not prosper, then I do not work hard.

Inverse : If I work hard then I prosper.

Contrapositive : If I prosper then I work hard.

16) i) $(p \vee \sim q) \wedge (\sim p \vee q) \equiv (p \wedge q) \vee \sim(p \vee q)$

ii) $p \wedge (q \wedge r) \equiv \sim[(p \vee q) \wedge (r \wedge s)]$

iii) 2 is even number and 9 is a perfect square.

17) i) A quadrilateral is not a rhombus or it is not a square.

ii) $10 - 3 \neq 7$ or $10 \times 3 \neq 30$

iii) It does not rain or the principal declares a holiday.

18) i) $(\sim p \vee q) \wedge (p \vee \sim q) \wedge (\sim p \vee \sim q)$

ii) $(p \vee q) \vee r \equiv p \vee (q \vee r)$

iii) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (q \wedge r)$

iv) $\sim(p \wedge q) \equiv \sim p \vee \sim q$

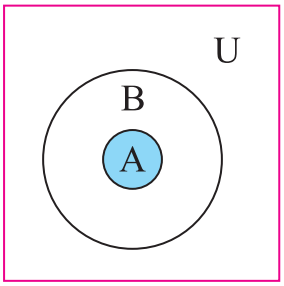
19) Statement (i) and (iii) are identical

Statement (ii) and iv) are identical

20) i) U : The set of all human being

A : The set of all men

B : The set of all mortal

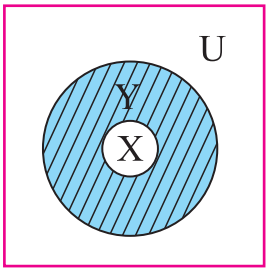


$A \subset B$

ii) U : The set of all human beings.

X : The set of all persons.

Y : The set of all politician

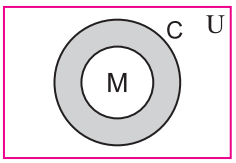


$Y - X \neq \phi$

iii) U : The set of all human beings.

X : The set of all members of the present Indian cricket.

Y : The set of all committed members of the present Indian cricket.

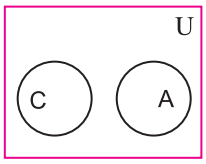


$C - M \neq \phi$

iv) U : Set of all human beings.

C : Set of all child.

A : Set of all Adult.



$C \cap A = \phi$

21) i) T ii) F iii) T
iv) F

22) i) 7 is not prime number orTajmahal is not in Agra
ii) $10 < 5$ or $3 > 8$
iii) I will have not tea and cofee.
iv) $\exists\, n \in N$, such that $n + 3 \leq 9$
v) $\forall\, x \in A, x + 5 \geq 11$.

2. Matrices

Exercise : 2.1

1) i) $A = \begin{bmatrix} 0 & \frac{1}{4} \\ \frac{1}{3} & 0 \\ 2 & \frac{1}{2} \end{bmatrix}$ ii) $A = \begin{bmatrix} -2 & -5 \\ -1 & -4 \\ 0 & -3 \end{bmatrix}$

iii) $A = \frac{1}{5}\begin{bmatrix} 8 & 27 \\ 27 & 64 \\ 64 & 125 \end{bmatrix}$

2) i) Upper Triangular Matrix
ii) Column Matrix
iii) Row Matrix
iv) Scalar Matrix
v) Lower Triangular Matrix
vi) Diagonal Matrix
vii) Identity Matrix

3) i) Singular Matrix
ii) Singular Matrix
iii) Non Singular Matrix
iv) Non singular Matrix

4) i) $k = \frac{-6}{7}$ ii) $k = 6$

ii) $k = \frac{49}{8}$

Exercise : 2.2

2) $A - 2B + 6I = \begin{bmatrix} 5 & 4 \\ -3 & 23 \end{bmatrix}$

3) $C = \begin{bmatrix} -10 & -1 & 1 \\ 7 & -9 & 3 \\ -4 & 6 & 2 \end{bmatrix}$

4) $X = \begin{bmatrix} -1 & \frac{2}{5} \\ \frac{6}{5} & \frac{19}{5} \\ \frac{19}{5} & \frac{26}{5} \end{bmatrix}$

5) $(A^T)^T = A$ 6) $(A^T)^T = A$

7) $a = -4, b = \frac{3}{5}$, $c = -7$

8) $x = \frac{-3}{2}$, $y = 5i$, $z = \sqrt{2}$

9) i) $A = A^T$ $\therefore$ A is a symmetric matrix,
ii) Neither $A = A^T$ nor $A = -A^T$ $\therefore$ A is neither symmetric nor skew symmetric matrix.
iii) $A = -A^T$ $\therefore$ A is a skew symmetric matrix.

10) $A = \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$

$\therefore$ A is a skew symmetric matrix.

11) $X = \begin{bmatrix} \frac{3}{8} & \frac{-1}{4} \\ \frac{-3}{8} & \frac{1}{2} \end{bmatrix}$, $Y = \begin{bmatrix} \frac{1}{8} & \frac{1}{4} \\ \frac{-1}{8} & \frac{1}{2} \end{bmatrix}$

12) $A = \begin{bmatrix} 3 & \frac{-14}{3} & \frac{-8}{3} \\ -2 & 1 & 3 \end{bmatrix}$,

$B = \begin{bmatrix} 0 & \frac{-10}{3} & \frac{-16}{3} \\ 0 & 0 & 5 \end{bmatrix}$

13) $x = \frac{-1}{4}$, $y = \frac{9}{2}$

14) $a = 1,\ b = 0,\ c = \frac{2}{5}$, $d = \frac{9}{5}$

15) i) Suresh book shop : Rs. 1050/- in Physics Rs. 305/- in Chemistry and Rs. 405/- in Maths.

Ganesh book shop : Rs. 350/- in Physics Rs. 445/- in Chemistry and Rs. 1295/- in Maths.

ii) The profit for Suresh book shop are Rs. 665/- in Physics Rs. 705.50/- in Chemistry and Rs. 890.50/- in Maths.

For Ganesh book shop are Rs. 700/- in Physics Rs. 750/- in Chemistry and Rs. 1020/- in Maths.

Exercise : 2.3

1) i) $\begin{bmatrix} 6 & -12 & 9 \\ 4 & -8 & 6 \\ 2 & -4 & 3 \end{bmatrix}$ ii) [8]

2) $AB = \begin{bmatrix} 2 & 1 & -1 \\ 13 & 2 & 14 \\ -6 & 3 & -1 \end{bmatrix}$ and $BA = \begin{bmatrix} 4 & -7 & 6 \\ -1 & -3 & 5 \\ 4 & 4 & 2 \end{bmatrix}$

$\therefore$ $AB \neq BA$

7) $(A + I)(A - I) = \begin{bmatrix} 9 & 6 & 4 \\ 15 & 32 & -2 \\ 35 & -7 & 29 \end{bmatrix}$

9) $k = -7$ 11) $a = 2,\ b = -1$

12) $k = 1$ 13) $x = 19, y = 12$

14) $x = -3, y = 1, z = -1$

15) Jay Rs. 104 and Ram Rs. 150

Exercise : 2.4

1) i) $A^T = \begin{bmatrix} 1 & -4 \\ 3 & 5 \end{bmatrix}$

ii) $A^T = \begin{bmatrix} 2 & -4 \\ -6 & 0 \\ 1 & 5 \end{bmatrix}$

2) $A = \begin{bmatrix} 0 & -2 & -4 \\ 2 & 0 & -2 \\ 4 & 2 & 0 \end{bmatrix}$ and $A^T = \begin{bmatrix} 0 & 2 & 4 \\ -2 & 0 & 2 \\ -4 & -2 & 0 \end{bmatrix}$

$\therefore$ Both are skew symmetric.

6) $C^T = \begin{bmatrix} -16 & 14 \\ -6 & -10 \end{bmatrix}$

7) i) $\begin{bmatrix} 7 & 8 \\ -5 & 8 \\ 12 & -18 \end{bmatrix}$ ii) $\begin{bmatrix} 35 & -10 \\ 25 & 15 \\ -15 & 10 \end{bmatrix}$

11) i) $\begin{bmatrix} 4 & \frac{1}{2} \\ \frac{1}{2} & -5 \end{bmatrix} + \begin{bmatrix} 0 & \frac{-5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}$

ii) $\frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}$

Exercise : 2.5

1) i) $\begin{bmatrix} 2 & 2 \\ 3 & -4 \end{bmatrix}$ ii) $\begin{bmatrix} 4 & 2 \\ -5 & 1 \end{bmatrix}$

iii) $\begin{bmatrix} 3 & 1 & -1 \\ 3 & 9 & 3 \\ -1 & 1 & 3 \end{bmatrix}$ and $\begin{bmatrix} 3 & -11 & -1 \\ 1 & -1 & 1 \\ -1 & 5 & 3 \end{bmatrix}$

2) $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & \frac{-1}{3} \\ 0 & 0 & \frac{-1}{3} \end{bmatrix}$

3) i) $\begin{bmatrix} -8 & -5 \\ -2 & 1 \end{bmatrix}$ ii) $\begin{bmatrix} -3 & -1 & 5 \\ -1 & 19 & -21 \\ 22 & -12 & -2 \end{bmatrix}$

4) i) $\begin{bmatrix} 5 & 3 \\ -3 & 2 \end{bmatrix}$ ii) $\begin{bmatrix} -3 & -1 & -11 \\ -12 & 3 & -9 \\ 6 & 2 & 1 \end{bmatrix}$

5) i) $\begin{bmatrix} 1 & -1 \\ 2 & -3 \end{bmatrix}$ ii) $\frac{1}{18}\begin{bmatrix} 5 & 2 \\ -4 & 2 \end{bmatrix}$

iii) $\frac{1}{10}\begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$

6) i) $\frac{1}{5}\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ ii) $\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

7) $\frac{1}{6}\begin{bmatrix} 4 & -2 & 2 \\ -3 & 0 & 3 \\ 3 & 2 & -2 \end{bmatrix}$ 8) $\begin{bmatrix} 13 & 2 & -7 \\ -3 & -1 & 2 \\ -2 & 0 & 1 \end{bmatrix}$

9) $\frac{1}{6}\begin{bmatrix} 4 & 4 & 2 \\ 11 & 8 & -5 \\ 10 & 10 & 2 \end{bmatrix}$ 10) $\begin{bmatrix} -\frac{1}{3} \\ -\frac{7}{3} \\ 2 \end{bmatrix}$

Exercise : 2.6

1) i) $x = 0, y = 1$ ii) $x = 4, y = -3$

iii) $x = 1, y = 2, z = 1$

iv) $x = \frac{5}{2}, y = \frac{-1}{2}, z = -1$

2) i) $x = \frac{1}{2}, y = \frac{1}{2}$ ii) $x = 1, y = 2$

iii) $x = 3, y = 2, z = 1$

iv) $x = -2, y = 0, z = 3$

3) Cost price T.V. Rs. 3000 and cost price of V.C. Rs. 13000. Selling price of T.V. Rs. 4000 and Selling price of V.C.R. Rs. 13500.

4) Cost of one Economics book is Rs. 300, Cost of one Co-operation book is Rs. 60 and Cost of one Account book is Rs. 60.

MISCELLANEOUS EXERCISE - 2

I.

1)	c	2)	b	3)	d	4)	c
5)	a	6)	a	7)	b	8)	d
9)	c	10)	a	11)	b	12)	b
13)	d	14)	c	15)	b		

II.

1)	Column	2)	2×3	3)	2
4)	-1	5)	3	6)	-2
7)	$\|A\|$	8)	A	9)	-1

10) $\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$

III. 1) True 2) False 3) True
4) False 5) False 6) False
7) False 8) False 9) False
10) True

IV. 1) $k = \frac{15}{7}$ 2) $x = 3,\ y = 5,\ z = 5$

4) $A - 4B + 7I = \begin{bmatrix} 5 & -23 \\ 15 & 14 \end{bmatrix}$

9) $a = \frac{-2}{7}$, $b = \frac{-2}{7}$

10) $A^3 = \begin{bmatrix} -9 & 22 \\ -11 & 13 \end{bmatrix}$

11) $x = -9,\ y = -3,\ z = 0$

14) i) Shantaram Kantaram

$$\begin{pmatrix} \text{Rs.}33000 & \text{Rs.}39000 \\ \text{Rs.}28000 & \text{Rs.}31500 \\ \text{Rs.}2e000 & \text{Rs.}24000 \end{pmatrix} \begin{matrix} \text{Rice} \\ \text{Wheat} \\ \text{Groundnut} \end{matrix}$$

ii) Shantaram Kantaram

$$\begin{pmatrix} \text{Rs.}3000 & \text{Rs.}3000 \\ \text{Rs.}2000 & \text{Rs.}1500 \\ \text{Rs.}0 & \text{Rs.}8000 \end{pmatrix} \begin{matrix} \text{Rice} \\ \text{Wheat} \\ \text{Groundnut} \end{matrix}$$

15) i) Invertible ii) Not Invertible
iii) Invertible iv) Not Invertible

16) i) $\begin{pmatrix} \frac{3}{5} & \frac{1}{5} \\ \frac{-2}{5} & \frac{1}{5} \end{pmatrix}$ ii) $\begin{pmatrix} 4 & -1 \\ -7 & 2 \end{pmatrix}$

iii) $\begin{bmatrix} \frac{-2}{5} & 0 & \frac{3}{5} \\ \frac{-1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & \frac{-2}{5} \end{bmatrix}$

iv) $\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

17) $A^{-1} = \frac{1}{40}\begin{bmatrix} 19 & 5 & -27 \\ -2 & 10 & -14 \\ -3 & -5 & 19 \end{bmatrix}$

18) i) $x = \frac{26}{7}$, $y = \frac{30}{7}$

ii) $x = 3, y = 1, z = 2$

iii) $x = 2, y = -1, z = 1$

19) i) $x = 4, y = -3$

ii) $x = \frac{-5}{7}$, $y = \frac{6}{7}$, $z = 2$

iii) $x = \frac{1}{6}$, $y = -\frac{1}{3}$, $z = \frac{5}{6}$

20) Three number $x = 1, y = 2, z = 3$

3. Differentiation

Exercise : 3.1

I. 1) $\frac{1}{2}\left(x + \frac{1}{x}\right)^{\frac{-1}{2}}\left(1 - \frac{1}{x^2}\right)$

2) $\frac{2x}{3}\left(a^2 + x^2\right)^{\frac{-2}{3}}$

3) $9(5x^3 - 4x^2 - 8x)^8(15x^2 - 8x - 8)$

II. 1) $\frac{1}{x.\log x}$

2) $\frac{(40x^3 + 15x^2 - 6x)}{(10x^4 + 5x^3 - 3x^2 + 2)}$

3) $\frac{2ax + b}{ax^2 + bx + c}$

III. 1) $(10x-2).e^{5x^2-2x+4}$

2) $a^{(1+logx)}loga.\frac{1}{x}$

3) $5^{(x+logx)}log5.\left(1+\frac{1}{x}\right)$

Exercise : 3.2

I. 1) $\frac{1}{10+50x}$

2) $\frac{x-4}{18x-71}$

3) $\frac{1+x^2}{25x^2+2x+25}$

II. 1) $\frac{e^x}{1-x}$

2) $\frac{(x^2+1)^2}{1-4x-x^2}$

3) $\frac{-(2x-10)^2}{68}$

Exercise : 3.3

I. 1) $x^{x^{2x}}x^{2x}logx\left[2(1+logx)+\frac{1}{x.logx}\right]$

2) $x^{e^x}e^x\left[\frac{1}{x}+logx\right]$

3) $e^{x^x}x^x\left[1+logx\right]$

II. 1) $\left(1+\frac{1}{x}\right)^x\left[log\left(1+\frac{1}{x}\right)-\frac{1}{1+x}\right]$

2) $(2x+5)^x\left[\log(2x+5)+\frac{2x}{2x+5}\right]$

3) $\frac{1}{3}\sqrt[3]{\frac{(3x-1)}{(2x+3)(5-x)^2}}\left[\frac{3}{3x-1}-\frac{2}{2x+3}+\frac{2}{5-x}\right]$

III. 1) $(logx)^x\left[\frac{1}{logx}+\log(logx)\right]\times x^{logx}\left[\frac{2logx}{x}\right]$

2) $x^x(1+logx)+a^xloga$

3) $10^{x^x}.x^x.log10(1+logx)+10^{x^{10}}$

$(10.x^9)log10+10^{10^x}.10^x(log10)^2$

Exercise : 3.4

I. 1) $-\sqrt{\frac{y}{x}}$ 2) $-\frac{3x^2(1+4y)}{3y^2+4x^3}$

3) $-\frac{3x^2+2xy+y^2}{x^2+2xy+3y^2}$

II. 1) $-\frac{e^y+ye^x}{e^x+xe^y}$ 2) $\frac{logx}{(1+logx)^2}$

3) $-\frac{y}{x}$

III. 1) $\frac{y}{x}$ 2) $-\frac{y^2}{x^2}$

3) $-e^{y-x}$

Exercise : 3.5

I. 1) $\frac{1}{t}$ 2) t^2 3) $\frac{4}{3}e^{t+5}$

II. 1) $\frac{ylog2}{2\sqrt{x}}$ 2) $\frac{2}{\sqrt{1+u^2}}$

3) $x.5^x(log5)$

Exercise : 3.6

I. 1) $-\frac{1}{4}x^{\frac{-3}{2}}$ 2) $20x^3$ 3) $56x^{-9}$

II. 1) e^x 2) $4.e^{(2x+1)}$ 3) 0

MISCELLANEOUS EXERCISE - 3

I.

1) a 2) c 3) a 4) b 5) c

6) b 7) b 8) a 9) d 10) c

II.

1) -1 2) y 3) x 4) y 5) $\frac{1}{x}$

6) $\frac{-1}{x^2}$ 7) $\frac{y^2}{y^2-1}$ 8) axy

9) y 10) my

III.

1)	False	2)	True	3)	False
4)	True	5)	False	6)	True
7)	False	8)	True	9)	False

IV.

1) $10\,(6x^3-3x^2-9x)^4(18x^2-6x-9)$

2) $\frac{4}{5}(3x^2+8x+5)^{\frac{-1}{5}}(6x+8)$

3) $\frac{2\log[\log(\log x)]}{x.\log x.\log(\log x)}$ 4) $\frac{1}{30-2x}$

5) $-\frac{(2x-13)^2}{79}$ 6) $x^x.(1+\log x)$

7) $2^{x^x}x^x.\log 2.(1+\log x)$

8) $\sqrt{\frac{(3x-4)^3}{(x+1)^4(x+2)}}\frac{1}{2}\left[\frac{9}{3x-4}-\frac{4}{x+1}-\frac{1}{x+2}\right]$

9) $x^x(1+\log x)+(7x-1)^x\left[\log(7x-1)+\frac{7x}{7x-1}\right]$

10) $\frac{-3(x^2+y^2+2xy)}{(6xy+3x^2-1)}$ 11) $\frac{-(y+3x^2)}{(2y+x)}$

12) $\frac{x}{y}\left[\frac{2-3xy^3}{2+3x^3y}\right]$ 13) $\frac{1}{t}$ 14) $\frac{1}{6\sqrt{t}}e^{(\sqrt{t}-3t)}$

15) $\frac{2x}{a^x.\log a.(1+x^2)}$ 16) $\frac{e^{(4x+5)}}{10^{4x}.\log 10}$

17) $\frac{-1}{x^2}$ 18) $\frac{-1}{2at^3}$

19) $e^x(x^2+4x+2)$

4. Applications of Derivatives

Exercise : 4.1

1) i) $5x-y-2=0;\ x+5y-16=0$

ii) $2x+3y-5\ ;\ 3x-2y-1$

iii) $x+y=2\ ;\ x-y=0$

2) $4x-y+7=0$ & $x+4y-38=0$

3) $3x-y-8=0$ & $x+3y+14=0$

Exercise : 4.2

1) i) Increasing, $x \in R-\{2\}$

ii) Increasing, $x \in R, x \neq 0$

iii) Decreasing, $x \in R, x \neq 0$

2) i) $(-\infty, 2)\ U\ (3, \infty)$

ii) $x > -1$ i.e. $(-1, \infty)$

iii) $(-\infty, -3)\ U\ (8, \infty)$

3) i) $-3 < x < 8$

ii) $-\infty < x < \frac{3}{2}$

iii) $-2 < x < 7$

Exercise : 4.3

1) i) maximum at $x = 1$, max value $= -3$ & minimum at $x = 6$, min value $= -128$

ii) minimum at $x = \frac{1}{e}$, min value $= \frac{-1}{e}$

ii) minimum at $x = 2$, min value $= 12$

2) first part = 10, second part = 10

3) Length = breath = 9 cm

4) 30

Exercise : 4.4

1) Decreasing function 2) $D < 20$

3) $x > 100$

4) i) $x < 120$ ii) $x < 118$

5) i) $x < 27$ iii) $x < 30$

6) i) $x < 10$, C_A increasing.

ii) $x > 10$, C_A decreasing

7) i) $R_A = 36$ ii) $P = 42$

iii) $\eta = 3$

8) 3.6 9) $P = \frac{3}{2}$

10) i) $\eta = 6.5$ elastic ii) $\frac{7}{20}$ inelastic

11) i) $\eta = 2$ elastic ii) $\frac{18}{41}$ inelastic

12) i) R increasing, $x < 60$

ii) Profit increasing, $x < 59$

ii) $\eta = 2$

13) i) MPC = 0.675, MPS = 0.325

ii) APC = 0.375, APS = 0.625

MISCELLANEOUS EXERCISE - 4

I.

1) a 2) a 3) c 4) b 5) a 6) c

II.

1) gradient 2) $6(x - 1)$

3) $14x^{-3}$ 4) $x = 27, y = 27$

5) $\frac{-1}{e}$

III.

1) True 2) False 3) True

4) True

IV.

1) i) $\left(y - \frac{c}{t}\right) = \frac{1}{t^2}(x - ct)$;

$\left(y - \frac{c}{t}\right) = -t^2 (x - ct)$

ii) for $(-3, -3)$ $2x + y + 9 = 0$; $x - 2y - 3 = 0$ and for $(-1,-3)$ $2x - y - 1 = 0$; $x + 2y + 7 = 0$

iii) $10x + 2y - 8 = 0$; $2x - 10y + 14 = 0$

iv) $16x - y + 19 = 0$; $x + 16y + 210 = 0$

v) $x - 2y - 2 = 0$

vi) max at 2 and min at 4

5. Integration

Exercise : 5.1

i) $\frac{2}{15}\left((5x-4)^{3/2} + (5x-2)^{3/2}\right) + c$

ii) $x + \frac{x^2}{2} + \frac{x^3}{6}$

iii) $x^3 - 4\sqrt{x}$

iv) $\left(\frac{9x^5}{5}\right) - 10x^3 + 25x + c$

v) $log\left|\frac{x-1}{x}\right| + c$

vi) $f(x) = x^2 + 5$ and $f(0) = -1$ $f(x) = \frac{x^3}{3} + 5x + c$

If $x = 0$, then $f(0) =$ c $\Leftrightarrow$ c $= -1$ Hence

$f(x) = \frac{x^3}{3} + 5x - 1$.

vii) $f(x) = x^4 - x^3 + x^2 + 2x + 1$

viii) $f(x) = \frac{x^3}{6} - \frac{x^2}{2} + x + 2$

Exercise : 5.2

i) $\frac{1}{3}(1+x^2)^{\frac{3}{2}}+c$

ii) $\frac{1}{2}\sqrt{(1+x^4)}+c$

iii) $\frac{(e^x+e^{-x})^3}{3}+c$

iv) $\log\left|xe^x+1\right|+c$

v) $\frac{(x+2)^{10}}{10}-\frac{(x+2)^8}{8}+c$

vi) $log\left|logx\right|+c$

vii) $\frac{1}{4}(x^1+1)^2-(x^2+1)+\frac{1}{2}log(x^2+1)+c$

viii) $2\sqrt{x^2+6x+3}+c$

ix) $2log\left|\sqrt{x}+1\right|+c$

x) $\frac{1}{6}log\left|\frac{x^6}{x^6+1}\right|+c$

Exercise : 5.3

i) $-t+\frac{7}{8}log\left|4e^{2t}-5\right|+c$

ii) $5x+log\left|3e^x-4\right|+c$

iii) $\frac{-1}{2}x+2log\left|2e^x-8\right|+c$

iv) $5x-8log\left|2e^x+1\right|+c$

Exercise : 5.4

i) $\frac{1}{4}log\left|\frac{2x-1}{2x+1}\right|+c$

ii) $\frac{1}{6}log\left|\frac{x-1}{x+5}\right|+c$

iii) $\frac{1}{8\sqrt{2}}log\left|\frac{2x-5-2\sqrt{2}}{2x-5+2\sqrt{2}}\right|+c$

iv) $\frac{1}{4\sqrt{13}}log\left|\frac{4x^2-1-\sqrt{13}}{4x^2-1+\sqrt{13}}\right|+c$

v) $\frac{1}{16}log\left|\frac{4x^4-5}{4x^4+5}\right|+c$

vi) $\frac{1}{2ab}log\left|\frac{a+bx}{a-bx}\right|+c$

vii) $\frac{1}{8}log\left|\frac{1+x}{7-x}\right|+c$

viii) $\frac{1}{\sqrt{3}}log\left|\sqrt{3}x+\sqrt{3x^2+8}\right|+c$

ix) $log\left|(x+2)+\sqrt{x^2+4x+29}\right|+c_1$

x) $\frac{1}{\sqrt{3}}log\left|\sqrt{3}x+\sqrt{3x^2-5}\right|+c$

xi) $\log\left|(x-4)+\sqrt{(x-4)^2-6^2}\right|+c$

xi) $log\left|(x-4)+\sqrt{x^2-8x-20}\right|+c$

Exercise : 5.5

i) $\frac{x^2}{2}logx-\frac{x^2}{4}+c$

ii) $\frac{e^{4x}}{4}\left[x^2-\frac{x}{2}+\frac{1}{8}\right]+c$

iii) $\frac{1}{3}x^2e^{3x}-\frac{2}{9}xe^{3x}+\frac{2}{27}e^{3x}+c$

iv) $\frac{1}{2}\left\{(x^2-1)e^{x^2}+c\right\}$

v) $e^x \frac{1}{x} + c$

vi) $e^x \frac{1}{x+1} + c$

vii) $e^x \frac{1}{(x+1)^2} + c$

viii) $e^x (logx)^2 + c$

ix) $\frac{x}{logx} + c$

x) $\frac{x}{1+logx} + c$

Exercise : 5.6

i) $\frac{1}{3} log|x+1| + \frac{5}{3} log|x-2| + c$

ii) $\frac{1}{4} log|x| - log|x-1| + \frac{3}{4} log|x-4| + c$

iii) $x - log|x+3| + log|x-2| + c$

iv) $\frac{2}{9} log\left|\frac{x-1}{x+2}\right| - \frac{1}{3(x-1)} + c$

v) $\frac{11}{4} log\left|\frac{x+1}{x+3}\right| + \frac{5}{2(x+1)} + c$

vi) $\frac{1}{5} log\left|\frac{x^5}{x^5+1}\right| + c$

vii) $\frac{1}{n} log\left|\frac{x^n}{x^n+1}\right| + c$

viii) $6 log|x| - log|x+1| - \frac{9}{x+1} + c$

MISCELLANEOUS EXERCISE - 5

I. 1) b 2) a 3) b 4) c 5) a
6) c 7) b 8) a 9) b 10) a

II. 1) $x^5 - \frac{5}{3}x^3 + 5x + c$

2) $x + 4\log(x-1) + c$

3) $f(x) = \log x + \frac{x^2}{2} + c$

4) $1 + \log x = t$

5) $p = \frac{1}{3}$

III. 1) True 2) False 3) True
4) True 5) False

IV. 1) i) $\frac{5x^2}{4} + \frac{3x}{4} + \frac{21}{8}\log|2x-3| + c$

ii) $\frac{9}{65}(5x+1)^{13/9} + c$

iii) $\frac{\log|2x+3|}{2} + c$

iv) $\frac{2}{3}(x+4)^{3/2} - 10\sqrt{x+4} + c$

v) $\frac{2x^{3/2}}{3} + \frac{4}{3}$

vi) $-\frac{x^2}{2} + c$

2) i) $-\log|e^{-x}+1| + c$

ii) $\frac{1}{2(ae^x - be^{-x})} + c$

iii) $\frac{\log|2+3\log x|}{3} + c$

iv) $2\log|1+\sqrt{x}| + c$

v) $-3x + \frac{7}{2}\log|4e^x + 1| + c$

3) i) $\frac{1}{2}\log\left|x + \sqrt{x^2 - \frac{5}{4}}\right| + c$

ii) $\frac{1}{4}\log\left|\frac{3+x}{1-x}\right| + c$

iii) $\frac{1}{30}\log\left|\frac{3x-5}{3x+5}\right|$

iv) $\log\left|e^x+2+\sqrt{e^{2x}+4e^x+13}\right|+c$

v) $\frac{1}{2\sqrt{5}}\log\left|\frac{\log x+2-\sqrt{5}}{\log x+2+\sqrt{5}}\right|+c$

vi) $\frac{1}{8\sqrt{5}}\log\left|\frac{\sqrt{5}+4x}{\sqrt{5}-4x}\right|+c$

vii) $\frac{1}{10}\log\left|\frac{5+\log x}{5-\log x}\right|$

viii) $\frac{1}{4}\log\left|\frac{e^x-1}{e^x+1}\right|+c$

4) i) $x(\log x)^2-2x\log x+2x+c$

ii) $\frac{e^x}{2+x}+c$

iii) $\frac{(2x-1)}{4}e^{2x}$

iv) $x\left[\log(x^2+x)\right]-2x+\log|x+1|+c$

v) $2\left(\sqrt{x}-1\right)e^{\sqrt{x}}+c$

vi) $\frac{x+1}{2}\sqrt{x^2+2x+5}+2\log\left|(x+1)+\sqrt{x^2+2x+5}\right|+c$

vii) $\frac{(x-4)}{2}\sqrt{x^2-8x+7}-\frac{9}{2}\log\left|(x-4)+\sqrt{x^2-8x+7}\right|+c$

5) i) $\frac{2}{3}\log|x-1|+\frac{5}{3}\frac{\log|2x+1|}{2}+c$

ii) $\frac{x^2}{2}-x+\log\left(\frac{x+2}{2x+5}\right)+c$

iii) $\frac{2}{7}\log(3+\log x)+\frac{1}{21}\log(2+3\log x)+c$

6. Definite Integrals

Exercise : 6.1

1) 2 2) $log\left(\frac{8}{3}\right)$ 3) $\frac{1}{2}log\left(\frac{8}{3}\right)$

4) $\frac{32}{5}$ 5) $log\left(\frac{3456}{3125}\right)$ 6) $\frac{1}{4}log\left(\frac{9}{7}\right)$

7) $a=-2$ or 1 8) $a=2$ 9) $\frac{4}{3}\left(\sqrt{2}-1\right)$

10) $\frac{1}{6}\log\left(\frac{35}{8}\right)$

11) $log\ 27-4$ or $3log3-4$

Exercise : 6.2

1) 0 2) $\frac{16}{315}a^{9/2}$ 3) 1

4) $\frac{3}{2}$ 5) $\frac{1}{2}$ 6) $\frac{5}{2}$

7) 0 8) $\frac{1}{4^2}$

MISCELLANEOUS EXERCISE - 6

I.

1) a 2) b 3) c 4) c

5) a 6) d 7) d 8) c

9) c 10) b

II.

1) i) e^2-1 2) $\frac{211}{5}$ 3) $\frac{1}{2}log\left(\frac{7}{2}\right)$

4) 2 5) 2 6) $\frac{1}{2}log\left(\frac{8}{3}\right)$

7) $log\left(\frac{8}{3}\right)$ 8) 0

III.

1) True 2) True 3) False

4) False 5) True 6) True

7) False 8) True

IV.

1) $3log|x+3|-2log|x+2|+c$

2) $\frac{log6}{2}$ 3) $9log3-\frac{26}{9}$

4) $\frac{1}{2}$ 5) $\frac{e^4}{4}-\frac{e^2}{2}$

6) 2 7) $log\frac{8}{3}$

8) $\frac{1}{2}log\frac{8}{3}$ 9) $\frac{32}{5}$

10) $\frac{1}{9}\left(28-3\sqrt{3}-7\sqrt{7}\right)$ 11) $log\sqrt{2}$

12) $\frac{7}{3}$ 13) $-log\,4$

14) $\frac{4}{3}\left(\sqrt{2}-1\right)$ 15) $log\left(\frac{5+3\sqrt{3}}{1+\sqrt{3}}\right)$

16) $\frac{1}{2}log\left(\frac{17}{5}\right)$ 17) $-\frac{1}{2}log\,3$

18) $5+\frac{1}{2}\left(5log3+85log2-45log2\right)$

19) $\frac{log2}{1+log2}$ 20) $6-4log2$

7. Applications of Definite Integral

Exercise : 7.1

1) i) $\frac{3124}{5}$ sq. units ii) $\frac{56}{3}$ sq. units

iii) 4π sq. units iv) 96 sq. units

v) 5 sq. units vi) 12 sq. units

vii) $\frac{10}{3}$ sq. units

2) $8\sqrt{3}$ sq. units

3) 25π sq. units

4) 10π sq. units

MISCELLANEOUS EXERCISE - 7

I. 1) a 2) c 3) c 4) b 5) c

II. 1) $\frac{3124}{5}$ sq. units

2) $49\,\pi$ sq. units

3) $\frac{56}{3}$ sq. units

4) $\frac{70}{3}$ sq. units

5) $\frac{28}{3}$ sq. units

III. 1) True 2) False

3) True 4) False

5) True

IV. 1) c^2log2 sq. units

2) $\frac{49}{3}$ sq. units

3) $\frac{40\sqrt{10}}{3}$ sq. units

4) $12\,\pi$ sq. units

5) 21 sq. units

6) $\frac{70}{3}$ sq. units

7) $A=2\int_0^5 y\,dx=2\int_0^5 5\sqrt{x}\,dx$

$=\frac{100\sqrt{5}}{3}$ sq. units

8. Differential Equations and Applictions

Exercise : 8.1

1.

	order	Degree
i	2	1
ii	2	2
iii	4	1
iv	3	2
v	1	5
vi	2	1
vii	3	1

Exercise : 8.2

1) i) $\frac{dy^2}{dx^2} = 9y$

ii) $x^2\frac{dy^2}{dx^2} + 2x\frac{dy}{dx} = 0$

iii) $\frac{dy^2}{dx^2} - 2\frac{dy}{dx} + y = 0$

iv) $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$

v) $\frac{dy}{dx} = \frac{3}{2}\sqrt[3]{y}$

2) $2xy\frac{dy}{dx} = y^2 - x^2$

3) $\frac{d^2y}{dx^2} = 0$

4) $2x^3 - y^3 + 3xy^2\frac{dy}{dx} = 0$

5) $x + 4y\frac{dy}{dx} = 0$

Exercise : 8.3

1. i) $\log y = \frac{x^3}{3} + x + c$

ii) $\theta - \theta_0 = e^{-kt+c}$

iii) $logx - logy = \frac{1}{x} + \frac{1}{y} + c$

iv) $2y^2\log|1+x| = -1 + 2y^2c$

2. i) $|1+x^2||1-y^2| = 5$

ii) $3x - 2e^y - 1 = 0$

iii) $exlogx = y$

iv) $\log\left|\frac{4x+y+5}{6}\right| = x + c$

Exercise : 8.4

1) $x^2 + 2y^2 = c$

2) $logx + \frac{1}{4}log\left|\frac{2y^2+xy}{x^2}\right| + \frac{3}{4}log\left|\frac{2y}{x+2y}\right| = c$

3) $\frac{x^3}{3y^3} = \log yc$

4) $\log\left|\frac{x+y}{x-y}\right| - \frac{1}{2}\log|x^2-y^2| + 2\log x = \log c$

5) $x^2 + y^2 = xc$

6) $x^2 + y^2 = cx^4$

7) $\frac{x+y}{x-y} = cx^2$

Exercise : 8.5

1) $ye^x = x + c$

2) $ye^x = 3e^x + c$

3) $yx^2 = logx.\frac{x^4}{4} - \frac{x^4}{16} + c$

4) $x + y + 1 = c.e^y$

5) $3xy = y^3 + c$

6) $ye^{x^2} = \frac{1}{2}e^{x^2} + c$

7) $y(x + a) = ax + c$

8) $ye^{2x} = 4e^{2x} + c$

Exercise : 8.6

1) 8 2) 73482

3) 45248 4) $30000\left(\frac{4}{3}\right)^{\frac{t}{40}}$

5) Rs. 628571

MISCELLANEOUS EXERCISE - 8

I. 1) a 2) c 3) b 4) a 5) d 6) c 7) d 8) b 9) c 10) a

II.

1) Order of the differential equation

2) Degree of the differential equation

3) Particular solution

4) Positive

5) e^{-x} 6) $\frac{d^2y}{dx^2} = 0$

III.

1) True 2) True 3) True

4) False 5) False 6) True

IV.

1) i) Order : 3 , Degree : 3

ii) Order : 1 , Degree : 3

2) $x\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

3) i) $\log|1 + y| = x + \frac{x^2}{2} + c$

ii) $y = x(logx - 1) + c$

iii) $log\ r = a\ log\ |1 + \theta| + c$

iv) $\frac{x^2d^2y}{dx^2} - 2x^2\frac{dy}{dx} - 2x\frac{dy}{dx} + x^2y + 2y + 2xy = 0$

4) $log\ |x + y| = y - x + \frac{1}{3}$

5) $log\ |x + y + 1| = cx$

6) $2xlogy = (logy)^2 + c$

7) $a^3 + x + y = ce^{\frac{y}{a^2}}$

8) $5x^2y = x^5 + c$

9) 50 years

10) Rs. 10,000

11) $xy^2 = c^2(x + 2y)$

12) $logy - \frac{x^3}{3y^3} = c$

13) $x^2 + y^2 = c^2x^4$

14) $x = y\ (c + y^2)$

15) $y = c.x - (1 + logx)$

16) $y = xlogx - x + c$

17) $xlogy = \frac{1}{2}(log\ y)^2 + c$

Notes

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$$

$$xdy + ydx = 0$$

$$\int e^x dx = e^x + C$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c,\ n \neq -1$$

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