6 Definite Integration

Let's Study

- Definite Integral
- Properties of Definite Integral

Introduction

We know that if $f(x)$ is a continuous function of x, then there exists a function $\phi(x)$ such that $\phi'(x) = f(x)$. In this case, $\phi(x)$ is an integral of $f(x)$ with respect to x and we denote it by $\int f(x)\,dx = \phi(x) + c$. Now, if we restrict the domain of $f(x)$ to (a, b), then the difference $\phi(b) - \phi(a)$ is called definite integral of $f(x)$ w.r.t. x on the interval [a, b] and is denoted by $\int_a^b f(x)dx$.

Thus $\int_a^b f(x)dx = \phi(b) - \phi(a)$

The numbers a and b are called limits of integration, 'a' is referred to as the lower limit of integral and b is the upper limit of integral.

Note that the domain of the variable x is restircted to the interval (a, b) and a, b are finite numbers.

Let's Learn

6.1 Fundamental theorem of Intergral Calculus.

Let f be a continuous function defined on (a, b)

$$\int f(x)\,dx = \phi(x) + c.$$

Then $\int_a^b f(x)dx = [\phi(x) + c]_a^b$

$= [\phi(b) + c] - [\phi(a) + c]$

$= \phi(b) - \phi(a)$

There in no need of taking the constant of integration c, because it gets eliminated.

SOLVED EXAMPLES

Ex 1 : Evaluate:

i) $\int_2^3 x^4 dx$

ii) $\int_0^1 \frac{1}{(2x+5)} dx$

iii) $\int_0^1 \frac{1}{\sqrt{1+x}+\sqrt{x}} dx$

Solution:

i) Here $f(x) = x^4$, $\phi(x) = \frac{x^5}{5} + c$

$\int_2^3 f(x)\,dx = [\phi(x)]_2^3$

$\int_2^3 x^4 dx = \left[\frac{x^5}{5}\right]_2^3 = \frac{3^5}{5} - \frac{2^5}{5}$

$= \frac{243}{5} - \frac{32}{5} = \frac{211}{5}$

ii) $\int_0^1 \frac{1}{(2x+5)} dx = \frac{1}{2}[\log|2x+5|]_0^1$

$= \frac{1}{2}[\log 7 - \log 5]$

$= \frac{1}{2}\log\frac{7}{5}$

iii) $\int_0^1 \frac{1}{\sqrt{1+x}+\sqrt{x}} dx$

$= \int_0^1 \frac{\sqrt{1+x}-\sqrt{x}}{\left(\sqrt{1+x}+\sqrt{x}\right)\left(\sqrt{1+x}-\sqrt{x}\right)} dx$

$= \int_0^1 \frac{\sqrt{1+x}-\sqrt{x}}{1+x-x} dx$

$$= \int_0^1 \left(\sqrt{1+x} - \sqrt{x}\right) dx$$

$$= \left[\frac{2}{3}(1+x)^{\frac{3}{2}} - \frac{2}{3}x^{\frac{3}{2}}\right]_0^1$$

$$= \left[\frac{2}{3}(1+1)^{\frac{3}{2}} - \frac{2}{3}(1)^{\frac{3}{2}}\right] - \left[\frac{2}{3}(1+0)^{\frac{3}{2}} - \frac{2}{3}(0)^{\frac{3}{2}}\right]$$

$$= \frac{2}{3}\left[2^{\frac{3}{2}} - 1\right] - \frac{2}{3}[1-0]$$

$$= \frac{2}{3}\left[2^{\frac{3}{2}} - 2\right]$$

$$= \frac{2}{3}\left[2\sqrt{2} - 2\right]$$

$$= \frac{4}{3}\left[\sqrt{2} - 1\right]$$

Ex 2 : Evaluate:

i) If $\int_0^1 (3x^2 + 2x + a)\, dx = 0$; find a.

ii) If $\int_0^a 3x^2 dx = 8$; find the value of a.

iii) $\int_a^b x^3 dx = 0$ and $\int_a^b x^2 dx = \frac{2}{3}$. Find the values of a and b.

iv) If $\int_0^a 4x^3 dx = 16$, find α

v) If $f(x)$ = a + bx + c x^2, show that

$$\int_0^1 f(x)\, dx = \frac{1}{6}\left[f(0) + 4f\left(\frac{1}{2}\right) + f(1)\right]$$

Solution:

i) $\int_0^1 (3x^2 + 2x + a)\, dx = 0$

Then $\left[3\frac{x^3}{3} + 2\frac{x^2}{2} + ax\right]_0^1 = 0$

$\left[x^3 + x^2 + ax\right]_0^1 = 0$

$(1+1+a) - 0 = 0$

$2 + a - 0 = 0$

a = –2

ii) $\int_0^a 3x^2\, dx = 8$

$3\left[\frac{x^3}{3}\right]_0^a = 8$

$(a^3 - 0^3) = 8$

$a^3 = 8$

$a = 2$

iii) $\int_a^b x^3\, dx = 0$ and $\int_a^b x^2\, dx = \frac{2}{3}$

$\therefore \left[\frac{x^4}{4}\right]_a^b = 0$ and $\left[\frac{x^3}{3}\right]_a^b = \frac{2}{3}$

$\therefore \frac{1}{4}\left(b^4 - a^4\right) = 0$ and $\frac{1}{3}(b^3 - a^3) = \frac{2}{3}$

$\therefore b^4 - a^4 = 0$ and $b^3 - a^3 = 2$

$\therefore b^4 = a^4 \;\therefore\; b = \pm a$

But b = a does not satisfy $b^3 - a^3 = 2$

$\therefore b \neq a$

$\therefore b = -a$

Substituting b = –a in $b^3 - a^3 = 2$

We get $(-a)^3 - a^3 = 2$, $-2a^3 = 2$

We get a = –1

$\therefore b = -a = 1$

$\therefore a = -1, \quad b = 1$

iv) $\int_0^a 4x^3 dx = 16$

$\therefore 4\left[\frac{x^4}{4}\right]_0^a = 16$

$\frac{4}{4}\left[a^4 - 0\right] = 16$

$a^4 = 16$

$\therefore \quad a = 2$

v) $\int_0^1 f(x)dx$

$= \int_0^1 (a + bx + cx^2)dx$

$= a\int_0^1 1\,dx + b\int_0^1 x\,dx + c\int_0^1 x^2 dx$

$= \left[ax + \frac{bx^2}{2} + \frac{cx^3}{3}\right]_0^1$

$= a + \frac{b}{2} + \frac{c}{3}$............(1)

Now $f(0) = a + b(0) + c(0)^2 = a$

$f(1/2) = a + b(1/2) + c(1/2)^2 = a + b/2 + c/4$

and $f(1) = a + b + c$

$\therefore \quad \frac{1}{6}\left[f(0) + 4f\left(\frac{1}{2}\right) + f(1)\right]$

$= \frac{1}{6}\left[a + 4(a + \frac{b}{2} + \frac{c}{4}) + (a + b + c)\right]$

$= \frac{1}{6}[a + 4a + 2b + c + a + b + c]$

$= \frac{1}{6}[6a + 3b + 2c]$

$= a + \frac{b}{2} + \frac{c}{3}$.............(2)

From (1) and (2)

$= \int_0^1 f(x)dx = \frac{1}{6}\left[f(0) + 4f\left(\frac{1}{2}\right) + f(1)\right]$

Ex 3 : Evaluate:

i) $\int_0^2 \frac{1}{4 + x - x^2}dx$

ii) $\int_0^4 \frac{dx}{\sqrt{x^2 + 2x + 3}}dx$

Solution:

i) $\int_0^2 \frac{1}{4 + x - x^2}dx$

$= \int_0^2 \frac{1}{-x^2 + x + 4}dx$

$= \int_0^2 \frac{-1}{x^2 - x + \frac{1}{4} - \frac{1}{4} - 4}dx$

$= \int_0^2 \frac{-1}{\left(x - \frac{1}{2}\right)^2 - \left(\frac{\sqrt{17}}{2}\right)^2}dx$

$= \int_0^2 \frac{1}{\left(\frac{\sqrt{17}}{2}\right)^2 - \left(x - \frac{1}{2}\right)^2}dx$

$= \frac{1}{\sqrt{17}}\left[log\left|\frac{\frac{\sqrt{17}}{2} + \left(x - \frac{1}{2}\right)}{\frac{\sqrt{17}}{2} - \left(x - \frac{1}{2}\right)}\right|\right]_0^2$

$= \frac{1}{\sqrt{17}}\left[log\left|\frac{\sqrt{17} + 2x - 1}{\sqrt{17} - 2x + 1}\right|\right]_0^2$

$= \frac{1}{\sqrt{17}}\left\{log\left(\frac{\sqrt{17} + 4 - 1}{\sqrt{17} - 4 + 1}\right) - log\left(\frac{\sqrt{17} - 1}{\sqrt{17} + 1}\right)\right\}$

$= \frac{1}{\sqrt{17}}\left\{log\left(\frac{\sqrt{17} + 3}{\sqrt{17} - 3}\right) - log\left(\frac{\sqrt{17} - 1}{\sqrt{17} + 1}\right)\right\}$

$= \frac{1}{\sqrt{17}}log\left(\frac{\sqrt{17} + 3}{\sqrt{17} - 3} \times \frac{\sqrt{17} + 1}{\sqrt{17} - 1}\right)$

$= \frac{1}{\sqrt{17}}log\left(\frac{20 + 4\sqrt{17}}{20 - 4\sqrt{17}}\right)$

$= \frac{1}{\sqrt{17}}log\left(\frac{5 + \sqrt{17}}{5 - \sqrt{17}}\right)$

ii) $\int_0^4 \frac{dx}{\sqrt{x^2+2x+3}}$

$= \int_0^4 \frac{1}{\sqrt{x^2+2x+1-1+3}}dx$

$= \int_0^4 \frac{1}{\sqrt{(x+1)^2+(\sqrt{2})^2}}\,dx$

$= \left[\log\left|(x+1)+\sqrt{(x+1)^2+(\sqrt{2})^2}\right|\right]_0^4$

$= \left[\log\left|(x+1)+\sqrt{x^2+2x+3}\right|\right]_0^4$

$= \log(5+\sqrt{16+8+3})-\log(1+\sqrt{3})$

$= \log(5+3\sqrt{3})-\log(1+\sqrt{3})$

$= \log\left[\frac{5+3\sqrt{3}}{(1+\sqrt{3})}\right]$

Ex. 4: Evaluate:

i) $\int_1^2 log\, x\, dx$

ii) $\int_1^2 \frac{log\, x}{x^2}dx$

Solution:

i) $I = \int_1^2 log\, x\, dx$

$I = \int_1^2 \log x.1.dx$

$I = [\log x.x]_1^2 - \int_1^2 \frac{1}{x}x\,dx$

$= [x\log x - x]_1^2$

$= [(2log2-1log1)]-[2-1]$

$=$ (log4 – 0) – 1

$= log\,4-1$

ii) $\int_1^2 \frac{\log x}{x^2}dx$

$= \int_1^2 \log x.\frac{1}{x^2}.dx$

$= \int \log x.\frac{1}{x^2}.dx = \left[(\log x\left(\frac{-1}{x}\right)\right]_1^2 - \int_1^2 \frac{1}{x}\left(\frac{x^{-1}}{-1}\right)dx$

$= \left[\log x\left(\frac{-1}{x}\right)-\frac{1}{x}\right]_1^2$

$= \left(\frac{-1}{2}\log 2+.\log 1\right)-\left(\frac{1}{2}-\frac{1}{1}\right)$

$= \frac{-1}{2}\log 2+\frac{1}{2}$

$= \frac{1}{2}(-\log 2+1)$

$= \frac{1}{2}(-\log 2+\log e)$

$= \frac{1}{2}\log\frac{e}{2}$

Ex. 5: Evaluate:

i) $\int_1^2 \frac{1}{(x+1)(x+3)}dx$

ii) $\int_1^3 \frac{1}{x(1+x^2)}dx$

Solution:

i) $\int_1^2 \frac{1}{(x+1)(x+3)}dx$

Let $\frac{1}{(x+1)(x+3)} = \frac{A}{(x+1)}+\frac{B}{(x+3)}$

$1 = A(x+3) + B(x+1)$ .... (1)

Putting $x + 1 = 0$

i.e. $x=-1$ in equation (i) we get $A=\frac{1}{2}$

Putting $x + 3 = 0$

i.e. $x=-3$ in equation (i) we get $B=\frac{-1}{2}$

$$\frac{1}{(x+1)(x+3)}=\frac{\frac{1}{2}}{(x+1)}+\frac{-\frac{1}{2}}{(x+3)}$$

$$\int_1^2 \frac{1}{(x+1)(x+3)}\,dx$$

$$=\frac{1}{2}\int_1^2 \frac{dx}{x+1}-\frac{1}{2}\int_1^2\frac{dx}{x+3}$$

$$=\frac{1}{2}\left[\log|x+1|-\log|x+3|\right]_1^2$$

$$=\frac{1}{2}(\log 3-\log 2)-\frac{1}{2}(\log 5-\log 4)$$

$$=\frac{1}{2}\left[\log\frac{3}{2}-\log\frac{5}{4}\right]$$

$$=\frac{1}{2}\log\left[\frac{6}{5}\right]$$

ii) $\int_1^3 \frac{1}{x(1+x^2)}dx$

Let $\frac{1}{x(1+x^2)}=\frac{A}{x}+\frac{Bx+c}{1+x^2}$

$1=A(1+x^2)+(Bx+c)x$ ....(1)

Putting x = 0 in equation (i) we get A = 1 Comparing the coefficient of x^2 and x, we get A + B = 0, B = –1 & C = 0

$$\frac{1}{x(1+x^2)}=\frac{1}{x}-\frac{x}{1+x^2}$$

$$\int_1^3\frac{dx}{x(1+x^2)}$$

$$=\int_1^3\frac{1}{x}\,dx-\int_1^3\frac{x}{1+x^2}\,dx$$

$$=\left[\log|x|\right]_1^3-\frac{1}{2}\left[\log|1+x^2|\right]_1^3$$

$$=(\log 3-\log 1)-\frac{1}{2}(\log 10-\log 2)$$

$$=\log\left(\frac{3}{1}\right)-\frac{1}{2}\log\left(\frac{10}{2}\right)$$

$$=(\log 3-\frac{1}{2}\log 5)$$

$$=\log 3-\log 5^{1/2}=\log 3-\log\sqrt{5}$$

$$=\log\left(\frac{3}{\sqrt{5}}\right)$$

EXERCISE 6.1

Evaluate the following definite intergrals:

1. $\int_4^9 \frac{1}{\sqrt{x}}dx$
2. $\int_{-2}^3 \frac{1}{x+5}dx$
3. $\int_2^3 \frac{x}{x^2-1}dx$
4. $\int_0^1 \frac{x^2+3x+2}{\sqrt{x}}dx$
5. $\int_2^3 \frac{x}{(x+2)(x+3)}dx$
6. $\int_1^2 \frac{dx}{x^2+6x+5}dx$
7. If $\int_0^a (2x+1)\,dx=2$, find the real value of a.
8. If $\int_1^a (3x^2+2x+1)\,dx=11$, find a.
9. $\int_0^1 \frac{1}{\sqrt{1+x}+\sqrt{x}}dx$
10. $\int_1^2 \frac{3x}{(9x^2-1)}dx$
11. $\int_1^3 \log x\,dx$

6.2 Properties of definite integrals

In this section we will study some properties of definite integrals which are very useful in evaluating integrals.

Property 1 : $\int_a^a f(x)\, dx = 0$

Property 2 : $\int_a^b f(x)\, dx = -\int_b^a f(x)\, dx$

Property 3 : $\int_a^b f(x)\, dx = \int_a^b f(t)\, dt$

Property 4 : $\int_a^b f(x)\, dx = \int_a^c f(x)\, dx$

$+\int_c^b f(x)\, dx$, where $a < c < b$

Property 5 : $\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$

Property 6 : $\int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx$

Property 7 : $\int_0^{2a} f(x)\, dx = \int_0^a f(x)\, dx + \int_0^a f(2a-x)\, dx$

[If $f(-x) = f(x)$, $f(x)$ is an even function. If $f(-x) = -f(x)$, $f(x)$ is an odd function.]

Property 8 : $\int_{-a}^a f(x)\, dx = 2\int_0^a f(x)\, dx$ if f is an even function

$= 0$ if f is an odd function

SOLVED EXAMPLES

Ex. Evaluate the following integrals:

1. $\int_{-1}^1 f(x)\, dx$ where $f(x) = \begin{cases} 1-2x; x \le 0 \\ 1+2x; x \ge 0 \end{cases}$

2. $\int_0^a x(1-x)^n\, dx$

3. $\int_0^3 \frac{\sqrt[3]{x+4}}{\sqrt[3]{x+4}+\sqrt[3]{7-x}}\, dx$

4. $\int_a^b \frac{f(x)}{f(x)+f(a+b+x)}\, dx$

5. $\int_4^7 \frac{(11-x^2)}{x^2+(11-x^2)}\, dx$

Solution:

1. $$\int_{-1}^1 f(x)\, dx = \int_{-1}^0 f(x)\, dx + \int_0^1 f(x)\, dx$$

$$= \int_{-1}^0 (1-2x)\, dx + \int_0^1 (1+2x)\, dx$$

$$= \left[x - x^2\right]_{-1}^0 + \left[x + x^2\right]_0^1$$

$$= \left[0-(-1-1)\right] + \left[(1+1)-0\right]$$

$$= 2 + 2 = 4$$

2. $\int_0^1 x(1-x)^n\, dx$

By property $\int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx$

$$I = \int_0^1 (1-x)\left[1-(1-x)\right]^n$$

$$= \int_0^1 (1-x)x^n\, dx$$

$$= \int_0^1 (x^n - x^{n+1})\, dx$$

$$= \left[\frac{x^{n+1}}{n+1}\right]_0^1 - \left[\frac{x^{n+2}}{n+2}\right]_0^1$$

$$= \frac{1}{n+1} - \frac{1}{n+2} = \frac{(n+2)-(n+1)}{(n+1)(n+2)}$$

$$= \frac{1}{(n+1)(n+2)}$$

3. Let $I = \int_0^3 \frac{\sqrt[3]{(x+4)}\ \ dx}{\sqrt[3]{(x+4)}+\sqrt[3]{(7-x)}}$ .... (1)

By property $\int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx$

$$I = \int_0^3 \frac{\sqrt[3]{(3-x)+4}}{\sqrt[3]{(3-x)+4}+\sqrt[3]{7-(3.-x)}}\,dx$$

$$= \int_0^3 \frac{\sqrt[3]{7-x}}{\sqrt[3]{7-x}+\sqrt[3]{x+4}}\,dx \quad \text{................. (2)}$$

On adding equations (1) and (2)

$$2I = \int_0^3 \left[\frac{\sqrt[3]{x+4}}{\sqrt[3]{7-x}+\sqrt[3]{x+4}} + \frac{\sqrt[3]{7-x}}{\sqrt[3]{7-x}+\sqrt[3]{x+4}}\right] dx$$

$$= \int_0^3 \frac{\sqrt[3]{x+4}+\sqrt[3]{(7-x)}}{\sqrt[3]{x+4}+\sqrt[3]{(7-x)}}\,dx$$

$$= \int_0^3 1\,dx$$

$$= [x]_0^3$$

2I = 3

$$I = \frac{3}{2}$$

$$\therefore \int_0^3 \frac{\sqrt[3]{(x+4)}}{\sqrt[3]{(x+4)}+\sqrt[3]{(7-x)}}\,dx = \frac{3}{2}$$

4. $$\int_a^b \frac{f(x)}{f(x)+f(a+b+x)}\,dx \quad \text{....... (1)}$$

By property $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

$$I = \int_a^b \frac{f(a+b-x)}{f(a+b-x)+f[(a+b-(a+b-x)]}\,dx$$

$$I = \int_a^b \frac{f(a+b-x)}{f(a+b-x)+f(x)}\,dx \quad \text{.............. (2)}$$

Adding equations (1) and (2) we get,

$$2I = \int_a^b \frac{f(x)}{f(x)+f(a+b-x)}\,dx + \int_a^b \frac{f(a+b-x)}{f(a+b-x)+f(x)}\,dx$$

$$= \int_a^b \frac{f(x)+f(a+b+x)}{f(x)+f(a+b-x)}\,dx$$

$$= \int_a^b 1\,dx$$

$$= [x]_a^b$$

$$2I = b-a$$

$$I = \frac{b-a}{2}$$

$$\therefore \int_a^b \frac{f(x)}{f(x)+f(a+b-x)}\,dx = \frac{b-a}{2}$$

5. $$\int_4^7 \frac{(11-x^2)}{x^2+(11-x^2)}\,dx \quad \text{......(1)}$$

$$= \int_4^7 \frac{x^2}{(11-x^2)+x^2}\,dx \quad \text{......(2)}$$

By Property

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

Adding equations (1) and (2)

$$2I = \int_4^7 \frac{(11-x^2)}{x^2+(11-x^2)}\,dx + \int_4^7 \frac{x^2}{(11-x^2)+x^2}\,dx$$

$$= \int_4^7 \frac{(x^2)+(11-x^2)}{(x^2)+(11-x^2)}\,dx$$

$$= \int_4^7 1\,dx$$

$$= [x]_4^7$$

2I = 3

$$I = \frac{3}{2}$$

$$\therefore \int_4^7 \frac{(11-x^2)}{x^2+(11-x^2)}\,dx = \frac{3}{2}$$

EXERCISE 6.2

Evaluate the following integrals:

1) $\int_{-9}^{9} \frac{x^3}{4-x^2}\,dx$

2) $\int_{0}^{a} x^2(a-x)^{3/2}\,dx$

3) $\int_{1}^{3} \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}}\,dx$

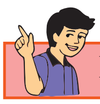
4) $\int_{2}^{5} \frac{\sqrt{x}}{\sqrt{x}+\sqrt{7-x}}\,dx$

5) $\int_{1}^{2} \frac{\sqrt{x}}{\sqrt{3-x}+\sqrt{x}}\,dx$

6) $\int_{2}^{7} \frac{\sqrt{x}}{\sqrt{x}+\sqrt{9-x}}\,dx$

7) $\int_{0}^{1} \log\left(\frac{1}{x}-1\right)dx$

8) $\int_{0}^{1} x(1-x)^5\,dx$



Let's Remember

- **Rules for evaluating definite integrals.**

1) $\int_a^b [f(x) \pm g(x)]dx = \int_a^b [f(x)dx \pm \int_a^b g(x)dx$

2) $\int_a^b k\,f(x)dx = k\int_a^b f(x)\,dx$

- **Properties of definite integrals**

1) $\int_a^a f(x)\,dx = 0$

2) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

3) $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

4) $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$

5) $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

6) $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

7) $\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx + \int_0^a f(2a-x)\,dx$

8) $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$, if f is even function,

$= 0$, if f is odd function

MISCELLANEOUS EXERCISE - 6

I) Choose the correct alternative.

1) $\int_{-9}^{9} \frac{x^3}{4-x^2}\,dx =$

a) 0 b) 3 c) 9 d) -9

2) $\int_{-2}^{3} \frac{dx}{x+5} =$

a) $-\log\left(\frac{8}{3}\right)$ b) $\log\left(\frac{8}{3}\right)$

c) $\log\left(\frac{3}{8}\right)$ d) $-\log\left(\frac{3}{8}\right)$

3) $\int_{2}^{3} \frac{x}{x^2-1}\,dx =$

a) $\log\left(\frac{8}{3}\right)$ b) $-\log\left(\frac{8}{3}\right)$

c) $\frac{1}{2}\log\left(\frac{8}{3}\right)$ d) $\frac{-1}{2}\log\frac{8}{3}$

4) $\int_4^9 \frac{dx}{\sqrt{x}} =$

a) 9 b) 4 c) 2 d) 0

5) If $\int_0^a 3x^2 dx = 8$ then a = ?

a) 2 b) 0 c) $\frac{8}{3}$ d) a

6) $\int_2^3 x^4\, dx =$

a) $\frac{1}{2}$ b) $\frac{5}{2}$ c) $\frac{5}{211}$ d) $\frac{211}{5}$

7) $\int_0^2 e^x\, dx =$

a) e – 1 b) 1 – e

c) 1 – e^2 d) e^2 – 1

8) $\int_a^b f(x)\, dx =$

a) $\int_b^a f(x)\, dx$ b) $-\int_a^b f(x)\, dx$

c) $-\int_b^a f(x)\, dx$ d) $\int_0^a f(x)\, dx$

9) $\int_{-7}^7 \frac{x^3}{x^2+7} dx =$

a) 7 b) 49 c) 0 d) $\frac{7}{2}$

10) $\int_2^7 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{9-x}} dx =$

a) $\frac{7}{2}$ b) $\frac{5}{2}$ c) 7 d) 2

II) Fill in the blanks.

1) $\int_0^2 e^x dx =$ ....

2) $\int_2^3 x^4 dx =$ ....

3) $\int_0^1 \frac{dx}{2x+5} =$ ....

4) If $\int_0^a 3x^2\, dx = 8$ then $a =$ ....

5) $\int_4^9 \frac{1}{\sqrt{x}} dx =$ ....

6) $\int_2^3 \frac{x}{x^2-1} dx =$ ....

7) $\int_{-2}^3 \frac{dx}{x+5} =$ ....

8) $\int_{-9}^9 \frac{x^3}{4-x^2} dx =$ ....

III) State whether each of the following is True or False

1) $\int_a^b f(x)\, dx = \int_{-b}^{-a} f(x)\, dx$

2) $\int_a^b f(x)\, dx = \int_a^b f(t)\, dt$

3) $\int_0^a f(x)\, dx = \int_a^0 f(a-x)\, dx$

4) $\int_a^b f(x)\, dx = \int_a^b f(x-a-b)\, dx$

5) $\int_{-5}^5 \frac{x^3}{x^2+7} dx = 0$

6) $\int_1^2 \frac{\sqrt{x}}{\sqrt{3-x}+\sqrt{x}}\,dx = \frac{1}{2}$

7) $\int_2^7 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{9-x}}\,dx = \frac{9}{2}$

8) $\int_4^7 \frac{(11-x)^2}{(11-x)^2+x^2}\,dx = \frac{3}{2}$

IV Solve the following.

1) $\int_2^3 \frac{x}{(x+2)(x+3)}\,dx$

2) $\int_1^2 \frac{x+3}{x(x+2)}\,dx$

3) $\int_1^3 x^2 \log x\,dx$

4) $\int_0^1 e^{x^2} x^3\,dx$

5) $\int_1^2 e^{2x}\left(\frac{1}{x}-\frac{1}{2x^2}\right)dx$

6) $\int_4^9 \frac{1}{\sqrt{x}}\,dx$

7) $\int_{-2}^3 \frac{1}{x+5}\,dx$

8) $\int_2^3 \frac{x}{x^2-1}\,dx$

9) $\int_0^1 \frac{x^2+3x+2}{\sqrt{x}}\,dx$

10) $\int_3^5 \frac{dx}{\sqrt{x+4}+\sqrt{x-2}}$

11) $\int_2^3 \frac{x}{x^2+1}\,dx$

12) $\int_1^2 x^2\,dx$

13) $\int_{-4}^{-1} \frac{1}{x}\,dx$

14) $\int_0^1 \frac{1}{\sqrt{1+x}+\sqrt{x}}\,dx$

15) $\int_0^4 \frac{1}{\sqrt{x^2+2x+3}}\,dx$

16) $\int_2^4 \frac{x}{x^2+1}\,dx$

17) $\int_0^1 \frac{1}{2x-3}\,dx$

18) $\int_1^2 \frac{5x^2}{x^2+4x+3}\,dx$

19) $\int_1^2 \frac{dx}{x(1+\log x)^2}$

20) $\int_0^9 \frac{1}{1+\sqrt{x}}\,dx$

Activities

1) Complete the following activity.

If $\int_a^b x^3\,dx = 0$ then

$$\left(\frac{x^4}{\square}\right)_a^b = 0$$

$$\therefore \quad \frac{1}{4}(\square - \square) = 0$$

$$\therefore \quad b^4 - \square = 0$$

$$\therefore \quad (b^2-a^2)(\square + \square) = 0$$

$$\therefore \quad b^2 - \square = 0 \quad as \quad a^2+b^2 \neq 0$$

$$\therefore \quad b = \pm \square$$

2) $\int_0^2 \frac{dx}{4+x-x^2}$

$$= \int_0^2 \frac{dx}{-x^2+\square+\square}$$

$$= \int_0^2 \frac{dx}{-x^2+x+\frac{1}{4}-\square-4}$$

$$= -\int_0^2 \frac{dx}{\left(x-\frac{1}{2}\right)^2-(\square)^2}dx$$

$$= \frac{1}{\sqrt{17}}\log\left(\frac{20+4\sqrt{17}}{20-4\sqrt{17}}\right)$$

3) $\int_0^1 \log\left(\frac{1}{x}-1\right)dx$

$$= \int_0^1 \log\left(\frac{1-x}{\square}\right)dx \quad \text{........(1)}$$

$$= \int_0^1 \log\left(\frac{1-(1-x)}{\square}\right)dx$$

$$= \int_0^1 \log\left(\frac{\square}{1-x}\right)dx \quad \text{............(2)}$$

Adding (i) and (ii), we get

$$2I = \int_0^1 \log\left[\frac{1-x}{x}\times\frac{x}{\square}\right]dx$$

$$= \int_0^1 \log \square \, dx = \int_0^1 o\, dx =$$

4) $\int_{-8}^{8} \frac{x^5}{1-x^2}dx$

$$f(x) = \frac{x^5}{1-x^2}$$

$$f(-x) = \frac{(-x)^5}{1-x^2} = \frac{\square}{1-x^2}$$

Hence f is $\square$ function

$$\therefore \int_{-8}^{8} \frac{x^5}{1-x^2}dx = \square$$

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