

# ANSWERS

## 1. DIFFERENTIATION

### EXERCISE 1.1

(1) (i)  $5(3x^2 - 2)(x^3 - 2x - 1)^4$

(ii)  $\frac{5}{2}(3\sqrt{x} - 4\sqrt[3]{x})(2x^{\frac{3}{2}} - 3x^{\frac{4}{3}} - 5)^{\frac{3}{2}}$

(iii)  $\frac{x+2}{\sqrt{x^2 + 4x - 7}}$

(iv)  $\frac{x(2\sqrt{x^2 + 1} + 1)}{2\sqrt{x^2 + 1} \cdot \sqrt{x^2 + \sqrt{x^2 + 1}}}$

(v)  $-\frac{4x-7}{(2x^2 - 7x - 5)^{\frac{8}{3}}}$

(vi)  $\frac{15(3x-4)}{2(3x-5)^{\frac{3}{2}}} \left( \sqrt{3x-5} - \frac{1}{\sqrt{3x-5}} \right)^4$

(2) (i)  $-2x \sin(x^2 + a^2)$  (ii)  $\frac{3e^{(3x+2)}}{2\sqrt{e^{(3x+2)} + 5}}$

(iii) cosec  $x$  (iv)  $\frac{\sec^2 \sqrt{x}}{4\sqrt{x} \cdot \sqrt{\tan \sqrt{x}}}$   
 (v)  $\frac{-9 \operatorname{cosec}^2[\log(x^3)] \cdot \cot^2[\log(x^3)]}{x}$

(vi)  $3 \sin^2 x \cdot \cos x \cdot 5^{\sin^3 x + 3} \cdot \log 5$

(vii)  $\frac{\sin x \operatorname{cosec} \sqrt{\cos x} \cdot \cot(\sqrt{\cos x})}{2 \sqrt{\cos x}}$

(viii)  $-3x^2 \tan(x^3 - 5)$

(ix)  $5 \sin 2x \cdot e^{3 \sin^2 x - 2 \cos^2 x}$

(x)  $\frac{-2x \cdot \sin[2 \log(x^2 + 7)]}{x^2 + 7}$

(xi)  $-\sec^2[\cos(\sin x)] \cdot \sin(\sin x) \cdot \cos x$

(xii)  $4x^3 \cdot \sec^2(x^4 + 4) \cdot \sec[\tan(x^4 + 4)] \cdot \tan[\tan(x^4 + 4)]$

(xiii)  $\frac{2 \log x}{x} - \frac{2}{x}$

(xiv)  $\frac{\cos \sqrt{\sin \sqrt{x}} \cdot \cos \sqrt{x}}{4\sqrt{x} \cdot \sqrt{\sin \sqrt{x}}}$

(xv)  $2x \cdot e^{x^2} [\tan(e^{x^2})]$  (xvi)  $\frac{1}{2x \log x}$

(xvii)  $\frac{2 [\log[\log(\log x)]]}{x \log x \cdot \log(\log x)}$

(xviii)  $4x \sin(2x^2)$

(3) (i)  $6(x+2)(x^2 + 4x + 1)^2 + 4(3x^2 - 5)$   
 $(x^3 - 5x - 2)^3$

(ii)  $8(1-2x)(1+4x)^5(3+x-x^2)^7$   
 $+ 20(1+4x)^4(3+x-x^2)^8$

(iii)  $\frac{14-3x}{2(7-3x)^{\frac{3}{2}}}$  (iv)  $\frac{6x^2(x^3+15)(x^3-5)^4}{(x^3+3)^6}$

(v)  $\sin 2x(1+\sin^2 x)(1+\cos^2 x)^2(1-5\sin^2 x)$

(vi)  $-\frac{\sin x}{2\sqrt{\cos x}} - \frac{\sin \sqrt{x}}{4\sqrt{x} \cdot \sqrt{\cos \sqrt{x}}}$

(vii)  $3 \sec 3x$  (viii)  $\frac{\pi \cos x^\circ}{90(1-\sin x^\circ)^2}$

(ix)  $-\frac{\operatorname{cosec}^2\left(\frac{\log x}{2}\right)}{2x} + \tan x \cdot \operatorname{cosec}^2 x$

(x)  $\frac{8e^{4x}}{(e^{4x}+1)^2}$  (xi)  $-\frac{e^{\sqrt{x}}}{\sqrt{x}(e^{\sqrt{x}}-1)^2}$

(xii)  $6 \operatorname{cosec} 2x + 4 \cot x + \frac{14x}{x^2+7}$

(xiii)  $3 \operatorname{cosec} 3x$  (xiv)  $-\frac{5}{2} \operatorname{cosec}\left(\frac{5x}{2}\right)$

$$(xv) -\sec x$$

$$(xvi) 2 \log 4 + \frac{3x}{x^2+5} - \frac{9x^2}{2(2x^3-4)}$$

$$(xvii) 2x - \frac{6}{5-4x} + \frac{2}{7-6x}$$

$$(xviii) -\sin x \log a - \frac{6x}{x^2-3} - \frac{1}{x \log x}$$

$$(xix) 0 \quad (xx) \frac{x(x^2+2)^3(7x^2+38)}{(x^2+5)^{\frac{3}{2}}}$$

$$(4) (i) -16 \quad (ii) 35 \quad (iii) -20 \quad (iv) 28$$

$$(5) -5 \quad (6) \frac{12}{5} \quad (7) x=0 \text{ or } \frac{2\pi}{3} \text{ or } 2\pi$$

$$(8) e^{2x} + 6e^x + 14, e^{x^2+5}, 2x, e^x, f' [g(x)] \cdot g'(x), 2e^{2x} + 6e^x, 8, g'[f(x)] \cdot f'(x), 2xe^{x^2+5}, -2e^6.$$

$$(6) (i) \frac{1}{x[1+(\log x)^2]} \quad (ii) \frac{e^x}{\sqrt{1-e^{2x}}} \\ (iii) -\frac{3x^2}{1+x^6} \quad (iv) -\frac{4^x \log 4}{1+4^{2x}}$$

$$(v) \frac{1}{2\sqrt{x}(1+x)} \quad (vi) \frac{x}{\sqrt{1-x^4}} \\ (vii) \frac{2}{\sqrt{2-x^2}} \quad (viii) \frac{3\sqrt{x}}{2\sqrt{1-x^3}}$$

$$(ix) 9x^8 \quad (x) 2x$$

$$(7) (i) 2xe^{x^2} \quad (ii) -5^x \log 5 \quad (iii) \frac{1}{2} \\ (iv) -x \quad (v) -\frac{1}{2} \quad (vi) -6 \\ (vii) -\frac{1}{6} \quad (viii) -\frac{3}{2} \quad (ix) -\frac{7}{2} \\ (x) -\frac{1}{2} \quad (xi) -\frac{1}{2} \quad (xii) \frac{2}{3}$$

### EXERCISE 1.2

$$(1) (i) \frac{1}{2\sqrt{x}} \quad (ii) -\frac{1}{4\sqrt{x}\sqrt{2-\sqrt{x}}}$$

$$(iii) \frac{1}{3\sqrt[3]{(x-2)^2}}, \text{ for } x > 2 \quad (iv) \frac{2}{2x-1}$$

$$(v) 2 \quad (vi) e^x$$

$$(vii) 2e^{2x-3} \quad (viii) \frac{1}{x \log 2}$$

$$(2) (i) \frac{1}{x \cdot e^x (x+2)} \quad (ii) \frac{1}{\cos x - x \sin x}$$

$$(iii) \frac{1}{7^x(x \log 7 + 1)} \quad (iv) \frac{x}{2x^2 + 1}$$

$$(v) \frac{1}{1 + \log x}$$

$$(3) (i) \frac{1}{14} \quad (ii) \frac{1}{4} \quad (iii) \frac{1}{12} \quad (iv) \frac{1}{5}$$

$$(4) 1$$

(5) (i) and (ii) derivative proved.

$$(8) (i) 1 \quad (ii) 1 \quad (iii) \frac{1}{2\sqrt{x}}$$

$$(iv) 3 \quad (v) e^x \quad (vi) 2^x \log 2$$

$$(9) (i) \frac{2}{1+x^2} \quad (ii) \frac{2}{1+x^2}$$

$$(iii) -\frac{2}{1+x^2} \quad (iv) \pm \frac{2}{\sqrt{1-x^2}}$$

$$(v) -\frac{3}{\sqrt{1-x^2}} \quad (vi) -\frac{2e^x}{1+e^{2x}}$$

$$(vii) \frac{2 \cdot 3^x \log 3}{1+3^{2x}}$$

$$(viii) \frac{2 \cdot 4^x \log 4}{1+4^{2x}} \text{ or } \left( \frac{4^{x+\frac{1}{2}} \log 4}{1+4^{2x}} \right)$$

$$(ix) -\frac{10}{1+25x^2} \quad (x) -\frac{3\sqrt{x}}{1+x^3}$$

$$(xi) \frac{5x\sqrt{x}}{1+x^5} \quad (xii) \frac{1}{2\sqrt{x}(1+x)}$$

$$(10) (i) \frac{3}{1+9x^2} + \frac{5}{1+25x^2}$$

$$(ii) \frac{7}{1+49x^2} - \frac{5}{1+25x^2}$$

$$(iii) \frac{1}{2\sqrt{x}} \left( \frac{3}{1+9x} - \frac{1}{1+x} \right)$$

$$(iv) 2^x \log 2 \left( \frac{3}{1+9(2^{2x})} + \frac{1}{1+2^{2x}} \right)$$

$$(v) 2^x \log 2 \left( \frac{2}{1+4(2^{2x})} - \frac{1}{1+2^{2x}} \right)$$

$$(vi) \frac{3a}{a^2+9x^2} + \frac{2a}{a^2+4x^2} \quad (vii) 1$$

$$(viii) \frac{2}{1+(2x+1)^2} - \frac{3}{1+(3x-4)^2}$$

$$(ix) \frac{2}{1+(2x+3)^2} + \frac{1}{1+(x-1)^2}$$

### EXERCISE 1.3

$$(1) (i) \frac{(x+1)^2}{(x+3)^3(x+3)^4} \left[ \frac{2}{x+1} - \frac{3}{x+2} - \frac{4}{x+3} \right]$$

$$(ii) \frac{1}{3} \sqrt[3]{\frac{4x-1}{(2x+3)(5-2x)^2}} \left( \frac{4}{4x-1} - \frac{2}{2x+3} + \frac{4}{5-2x} \right)$$

$$(iii) (x^2+3)^{\frac{3}{2}} \cdot \sin^3 2x \cdot 2^{x^2} \left[ \frac{3x}{x^2+3} + 6 \cot 2x + 2x \log 2 \right]$$

$$(iv) \frac{(x^2+2x+2)^{\frac{3}{2}}}{(\sqrt{x}+3)^3(\cos x)^x} \left[ \frac{3(x+1)}{x^2+2x+2} - \frac{3}{2\sqrt{x}(\sqrt{x}+3)} + x \tan x - \log(\cos x) \right]$$

$$(v) \frac{x^5 \cdot \tan^3 4x}{\sin^2 3x} \left[ \frac{5}{x} + 24 \operatorname{cosec} 8x - 6 \cot 3x \right] \quad (vi) x^{\tan^{-1} x} \left[ \frac{\tan^{-1} x}{x} + \frac{\log x}{1+x^2} \right]$$

$$(vii) \sin^x x \quad [x \cot x + \log(\sin x)] \quad (viii) \cos(x^x) \cdot x^x (1 + \log x)$$

$$(2) (i) ex^{e-1} + e^x + x^x (1 + \log x) \quad (ii) x^{x^x} \cdot x^x \cdot \log x \left[ 1 + \log x + \frac{1}{x \log x} \right] + e^{x^x} \cdot x^x (1 + \log x)$$

$$(iii) (\log x)^x \left[ \frac{1}{\log x} + \log(\log x) \right] + (\cos x)^{\cot x} [1 + \operatorname{cosec}^2 x \log(\cos x)]$$

$$(iv) x^{e^x} \cdot e^x \left[ \frac{1}{x} + \log x \right] + (\log x)^{\sin x} \left[ \frac{\sin x}{x \log x} + \cos x \log(\log x) \right]$$

$$(v) \sec^2 x \cdot e^{\tan x} + (\log x)^{\tan x} \left[ \frac{\tan x}{x \log x} + \sec^2 x \log(\log x) \right]$$

$$(vi) (\sin x)^{\tan x} [1 + \sec^2 x \log(\sin x)] + (\cos x)^{\cot x} [1 + \operatorname{cosec}^2 x \log(\cos x)]$$

$$(vii) 10^{x^x} x^x \log 10 (1 + \log x) + x^{x^{10}} \cdot x^9 (1 + 10 \log x) + x^{10^x} \cdot 10^x \left( \frac{1}{x} + \log x \cdot \log 10 \right)$$

$$(viii) 2$$

- (3) (i)  $-\sqrt{\frac{y}{x}}$  (ii)  $-\sqrt{\frac{x}{y}}$   
 (iii)  $-\frac{\sqrt{y}(2\sqrt{x} + \sqrt{y})}{\sqrt{x}(2\sqrt{y} + \sqrt{x})}$  (iv)  $-\frac{3x^2 + 2xy + y^2}{x^2 + 2xy + 3y^2}$   
 (v)  $-\frac{y}{x}$  (vi)  $-\frac{e^y + ye^x}{e^x + xe^y}$   
 (vii)  $\frac{\sin(x-y) + e^{x+y}}{\sin(x-y) - e^{x+y}}$  (viii)  $-\frac{1 + y \sin(xy)}{1 + x \sin(xy)}$   
 (ix)  $\frac{y(1 - xe^{x-y})}{x(1 - ye^{x-y})}$   
 (x)  $\frac{\sin(x-y) - \cos(x+y) - 1}{\sin(x-y) + \cos(x+y) - 1}$

### EXERCISE 1.5

- (1) (i)  $\frac{1}{t}$  (ii)  $\frac{b}{a} \cos \theta$  (iii)  $\frac{2}{\sqrt{a^2 + m^2}}$   
 (iv)  $\sec^3 \theta$  (v)  $\frac{b}{a} \tan\left(\frac{\theta}{2}\right)$   
 (vi)  $\frac{y(t^2 + 1) \log a}{axt}$  (vii)  $-\frac{1}{2}$  (viii)  $\frac{1}{3}$   
 (2) (i)  $\frac{3\sqrt{3}}{2}$  (ii)  $-\sqrt{3}$  (iii)  $-\frac{\pi}{6}$   
 (iv)  $1 - \sqrt{2}$  (v)  $3 + \pi$   
 (4) (i)  $\frac{x \cos x + \sin x}{\sec^2 x}$  (ii) 1  
 (iii)  $-\frac{1}{2}$  (iv) 2 (v)  $-x (\log x)^2 \cdot 3^x$   
 (vi)  $-\frac{x \sqrt{x^2 - 1}}{2}$   
 (vii)  $\frac{(1 + \log x) \cdot x^{x+1 - \sin x}}{\sin x + x \cos x \cdot \log x}$   
 (viii)  $\frac{\sqrt{1-x^2}}{4(1+x^2)}$

- (1) (i)  $40x^3 - 24x - \frac{12}{x^4}$   
 (ii)  $2e^{2x}(1 + \tan x) \cdot (2 + \tan x + \tan^2 x)$   
 (iii)  $-e^{4x}(9 \cos 5x + 40 \sin 5x)$   
 (iv)  $x(5 + 6 \log x)$  (v)  $-\frac{1 + \log x}{(x \log x)^2}$

- (iv)  $x^{x-1} + x^x(1 + \log x)^2$   
 (2) (i)  $-\frac{1}{4a} \operatorname{cosec}^4\left(\frac{\theta}{2}\right)$  (ii)  $-\frac{1}{4at^3}$   
 (iii) 6 (iv)  $-\frac{2\sqrt{2}b}{a^2}$

- (4) (i)  $\frac{d^n y}{dx^n} = \frac{m! a^n (ax+b)^{m-n}}{(m-n)!}$  if  $m > 0, m > n$ ,  
 $\frac{d^n y}{dx^n} = 0$  if  $m > 0, m < n$   
 $\frac{d^n y}{dx^n} = n! a^n$  if  $m > 0, m = n$   
 (ii)  $\frac{(-1)^n n!}{x^{n+1}}$  (iii)  $a^n e^{ax+b}$   
 (iv)  $p^n a^{px+q} (\log a)^n$   
 (v)  $\frac{(-1)^{n-1} (n-1)! a^n}{(ax+b)^n}$  (vi)  $\cos\left(\frac{n\pi}{2} + x\right)$   
 (vii)  $a^n \sin\left(\frac{n\pi}{2} + ax + b\right)$   
 (viii)  $(-2)^n \cos\left(\frac{n\pi}{2} + 3 - 2x\right)$   
 (ix)  $\frac{(-1)^{n-1} (n-1)! 2^n}{(2x+3)^n}$   
 (x)  $\frac{(-1)^n \cdot n! \cdot 3^n}{(3x-5)^{n+1}}$   
 (xi)  $e^{ax} (a^2 + b^2)^{\frac{n}{2}} \cdot \cos\left[bx + c + n \tan^{-1}\left(\frac{b}{a}\right)\right]$   
 (xii)  $e^{8x} \cdot (10)^n \cos\left[6x + 7 + n \tan^{-1}\left(\frac{3}{4}\right)\right]$

## MISCELLANEOUS EXERCISE 1

(I)

1	2	3	4	5	6	7	8	9	10	11	12
D	C	C	B	A	C	D	C	B	C	A	B

- (II) (1)  $\frac{3}{4}$  (ii) Does not exist (iii)  $-2$  (iv)  $-\frac{1}{2\sqrt{1-x^2}}$  [Hint :  $x = \cos 2\theta$ ]
- (2) (A) 3, (B) 5, (C) 4, (D) 1. (v)  $\frac{3}{1+9x^2} + \frac{5}{1+25x^2}$
- (3) (i)  $-\frac{1}{9}$  (ii)  $-\frac{40}{3}$  (iii)  $-\frac{29}{96}$  (vi)  $\frac{1}{2(1+x^2)}$  [Hint :  $x = \tan \theta$ ]
- (iv)  $-\frac{4}{9}$  (6) (i)  $\frac{\sqrt{1-x^2}}{4(1+x^2)}$
- (4) (i)  $-\frac{x}{\sqrt{1-x^2}}$  [Hint :  $x = \cos 2\theta$ ] (ii)  $-\frac{2x}{\sqrt{1+x^2} \cdot \sin(\log x)}$  (iii) 1
- (ii)  $-\frac{1}{2}$  [Hint :  $x = \cos 2\theta$ ] (iii)  $\frac{3}{2\sqrt{x}(1+x)}$  [Hint :  $\sqrt{x} = \tan \theta$ ]

## 2. APPLICATIONS OF DERIVATIVES

### EXERCISE 2.1

- (1) (i)  $2x - y + 4 = 0, x + 2y - 8 = 0$  (3)  $(2, -2)\left(-\frac{2}{3}, -\frac{14}{27}\right)$  (4)  $y = 0$  and  $y = 4$
- (ii)  $4x - 5y + 12 = 0, 5x + 4y - 26 = 0,$  (5)  $x + 3y - 8 = 0, x + 3y + 8 = 0$
- (iii)  $y = 2, x = \sqrt{3}$  (6)  $a = 2, b = -7$  (7) (4, 11) and  $\left(-4, -\frac{31}{3}\right)$
- (iv)  $\pi x + 2y - 2\pi = 0,$  (8)  $0.8 \pi \text{ cm}^2/\text{sec.}$  (9)  $6 \text{ cm}^3/\text{sec.}$
- $4x - 2\pi y + \pi^2 - 4 = 0$  (10)  $\frac{3\sqrt{6}}{2} \text{ cm}^2/\text{sec.}$  (11)  $8 \text{ cm}^2/\text{sec.}$
- (v)  $2x - y = 0, 4x + 8y - 5\pi = 0$  (12)  $7.2 \text{ cm}^3/\text{sec.}$  (13)  $3 \text{ km/hr}$
- (vi)  $4x + 2y - 3 = 0, 2x - 4y + 1 = 0$  (14) (i)  $\left(\frac{3}{8}\right) \text{ meter/sec.}$  (ii)  $\frac{9}{8} \text{ meter/sec.}$
- (vii)  $17x - 4y - 20 = 0, 8x + 34y - 135 = 0$  (15)  $0.9 \text{ meter/sec.}$  (16)  $\left(\frac{4\pi}{3}\right) \text{ cm}^3/\text{sec.}$
- (2) (4, 1)

### EXERCISE 2.2

- (1) (i) 2.9168 (ii) 3.03704 (iii) 1.9997  
 (iv) 248.32 (v) 64.48
- (2) (i) 0.953 (ii) 0.42423 (iii) 0.4924  
 (iv) 1.02334
- (3) (i) 0.7845 (ii) 0.7859 (iii) 0.7859
- (4) (i) 2.70471 (ii) 8.1279 (iii) 9.09887
- (5) (i) 4.6152 (ii) 2.1983 (iii) 3.006049
- (6) (i) 6.91 (ii) 9.72

### EXERCISE 2.3

- (1) (i) Valid (ii) Valid  
 (iii) Invalid (iv) Valid  
 (v) Invalid (vi) Invalid
- (2)  $b = 1$
- (3) (i)  $\frac{\pi}{4}$  or  $\frac{5\pi}{4}$  (ii)  $c = \pi$  (iii)  $c = \frac{5}{2}$
- (4)  $p = -6, q = 11$  (6)  $c = -2$
- (7) (i)  $e - 1$  (ii)  $2 \pm \frac{2}{\sqrt{3}}$  (iii)  $\frac{1}{7}$   
 (iv)  $\frac{1}{2}$  (v)  $3 + \sqrt{2}$

### EXERCISE 2.4

- (1) (i) Increasing  $\forall x \in R$   
 (ii) Decreasing  $\forall x \in R$   
 (iii) Increasing  $\forall x \in R$
- (2) (i)  $x < -1$  and  $x > 2$  (ii)  $R - \{1\}$   
 (iii)  $x < -2$  and  $x > 6$
- (3) (i)  $-1 < x < 2$  (ii)  $(-5, 5) - \{0\}$   
 (iii)  $x \in (2, 4)$
- (4) (a)  $(-\infty, -4] \cup [12, \infty)$   
 (b)  $-4 \leq x \leq 12$  i.e.  $[-4, 12]$
- (5) (a)  $x < -3$  and  $x > 8$  (b)  $-3 < x < 8$
- (6) (a)  $-1 < x < 1$  (b)  $(-\infty, -1) \cup (1, \infty)$
- (9) (i) Max =  $\frac{36}{25}$ , Min =  $-\frac{16}{27}$   
 (ii) Max =  $-3$ , Min =  $-128$   
 (iii) Max = 20, Min = 16 (iv) Min = 8  
 (v) Min =  $-\frac{1}{e}$  (vi) Max =  $\frac{1}{e}$
- (10) 15, 15 (11) 10, 10 (12) 9 (13) 12.8
- (14)  $l = \sqrt{2}$  and  $b = \frac{1}{\sqrt{2}}$
- (15) Radius = Height =  $a$  (16) 3, 3
- (17) Side of square base = 8 cm, Height = 4 cm
- (18)  $x = 75, P = 4000$  (19) 6, 9
- (22)  $\frac{4\pi r^3}{3\sqrt{3}}$  cm<sup>3</sup>

### MISCELLANEOUS EXERCISE 2

(I)

1	2	3	4	5	6	7	8	9	10
A	C	B	B	D	C	D	A	D	D



**(II) (2) 4**

(3)  $14x - 13y + 12 = 0, 13x + 14y - 41 = 0$

(4)  $\frac{2}{9\pi}$  ft/sec      (5)  $\left(\frac{16}{3}, 3\right), \left(-\frac{16}{3}, -3\right)$

(6)  $c = 0$       (7)  $c = 2$       (8) 2.025

(9) 1.03565

(10) Decreasing in  $\left(0, \frac{1}{e}\right]$  and

Increasing in  $\left[\frac{1}{e}, \infty\right)$

(11) Increasing in  $[e, \infty)$ , Decreasing in  $(1, e]$

(15)  $l = \frac{60}{\pi+4}, b = \frac{30}{\pi+4}, r = \frac{30}{\pi+4}$

(17) Side =  $\frac{l}{\pi+4}$ , Radius =  $\frac{l}{2(\pi+4)} = \frac{x}{2}$

(18) 24, 45      (21) Max =  $\frac{5}{4}$ , Min = 1

### 3. INDEFINITE INTEGRATION

#### EXERCISE 3.1

(1) (i)  $\frac{x^4}{4} + \frac{x^3}{3} - \frac{x^2}{2} + x + c$     (ii)  $\frac{x^3}{3} - 2x^2 + 4x + c$

(iii)  $3 \tan x - 4 \log x - \frac{2}{\sqrt{x}} - 7x + c$

(iv)  $\frac{x^2}{4} - \frac{5x^2}{2} + 3 \log x - \frac{1}{x^4} + c$

(v)  $\frac{6}{5}x^2\sqrt{x} - 4\sqrt{x} - \frac{10}{\sqrt{x}} + c$

(2) (i)  $\tan x - x + c$       (ii)  $-2 \cos x + c$

(iii)  $\sec x + c$       (iv)  $-\cot x - 2x + c$

(v)  $-\cot x - \tan x + x + c$

(vi)  $\sec x - \tan x + x + c$

(vii)  $\sec x - \tan x + x + c$

(viii)  $\sin x - \cos x + c$       (ix)  $-\sqrt{2} \cos x + c$

(x)  $-\frac{1}{14} \cos 7x - \frac{1}{2} \cos x + c$

(3) (i)  $x - 2 \log(x + 2) + c$

(ii)  $2x + \frac{1}{2} \log(2x + 1) + c$

(iii)  $\frac{5}{3}x - \frac{26}{9} \log(3x - 4) + c$

(iv)  $\frac{2(x+5)^{\frac{3}{2}}}{3} - 14\sqrt{x+5} + c$

(v)  $\frac{1}{12}(4x-1)^{\frac{3}{2}} - \frac{13}{4}\sqrt{4x-1} + c$

(vi)  $-\cos 2x + c$

(vii)  $\frac{2}{5} \left( \sin \frac{5x}{2} - \cos \frac{5x}{2} \right) + c$

(viii)  $\frac{1}{4}(2x + \sin 2x) + c$

(ix)  $-\frac{4}{9} \left[ x^{\frac{3}{2}} + (x+3)^{\frac{3}{2}} \right] + c$

(x)  $\frac{2}{21} \left[ (7x-2)^{\frac{3}{2}} + (7x-5)^{\frac{3}{2}} \right] + c$

(4)  $f(x) = \frac{x^2}{2} + \frac{3}{2x^2} + \frac{7}{2}$

#### EXERCISE 3.2 (A)

1. 1.  $\frac{(\log x)^{n+1}}{n+1} + c$       2.  $\frac{2}{5}(\sin^{-1}x)^{\frac{5}{2}} + c$

3.  $\log(\cosec(x + \log x) - \cot(x + \log x)) + c$

4.  $\frac{-1}{\sqrt{\tan(x^2)}} + c$       5.  $\frac{1}{3}(e^{3x} + 1) + c$





6.  $\sqrt{x^2 - 16x + 63} + \log \left\{ (x-8) + \sqrt{x^2 - 16x + 63} \right\} + c$
7.  $\sqrt{9x-x^2} + \frac{9}{2} \sin^{-1} \left( \frac{2x-9}{9} \right) + c$
8.  $\frac{3}{4\sqrt{2}} \log \left( \frac{2\sqrt{2} \sin x + \sqrt{2} - 2}{2\sqrt{2} \sin x + \sqrt{2} + 2} \right) + c$
9.  $\sqrt{e^{2x}-1} - \log (e^x + \sqrt{e^{2x}-1}) + c$

### EXERCISE 3.3

- I.**
1.  $\frac{x^3}{9} (3 \cdot \log x - 1) + c$
  2.  $-\frac{x^2}{3} \cos 3x + \frac{2}{9} x \sin 3x + \frac{2}{27} \cos 3x + c$
  3.  $\frac{x^2}{2} \tan^{-1} x - \frac{1}{2} (x - \tan^{-1} x) + c$
  4.  $\frac{x^3}{3} \tan^{-1} x - \frac{x^2}{6} + \frac{1}{6} \log (1+x^2) + c$
  5.  $\frac{1}{4} (\tan^{-1} x) (x^4 - 1) - \frac{x}{12} (x^3 - 3x) + c$
  6.  $x [(\log x)^2 - 2 (\log x) + 2] + c$
  7.  $\frac{1}{2} \log (\sec x + \tan x) + \frac{1}{2} \sec x \cdot \tan x + c$
  8.  $\frac{1}{4} \left[ x^2 - x \cdot \sin 2x - \frac{1}{2} \cos 2x \right] + c$
  9.  $\frac{x^4}{4} \log x - \frac{x^4}{16} + c$
  10.  $\frac{e^{2x}}{13} [2 \cos 3x + 3 \sin 3x] + c$
  11.  $\frac{x^2}{2} \sin^{-1} x + \frac{1}{4} x \sqrt{1-x^2} - \frac{1}{4} \sin^{-1} x + c$
  12.  $\frac{x^3}{3} \cos^{-1} x - \frac{1}{3} \sqrt{1-x^2} + \frac{1}{9} (1-x^2)^{\frac{3}{2}} + c$
  13.  $(\log x) [\log (\log x) - 1] + c$
  14.  $-(\sin^{-1} t) \sqrt{1-t^2} + t + c$
- II.**
1.  $\frac{e^{2x}}{13} [2 \sin 3x - 3 \cos 3x] + c$
  2.  $\frac{e^{-x}}{5} [-\cos x + 2 \sin 2x] + c$
  3.  $\frac{x}{2} [\sin (\log x) - \cos (\log x)] + c$
  4.  $\sqrt{5} \left[ \frac{x}{2} \sqrt{x^2 + \frac{3}{5}} + \frac{3}{10} \log \left( x + \sqrt{x^2 + \frac{3}{5}} \right) \right] + c$
  5.  $\frac{x^3}{6} \cdot \sqrt{a^2 - x^6} + \frac{a^2}{2} \sin^{-1} \left( \frac{x^3}{a} \right) + c$
  6.  $\frac{x-5}{2} \sqrt{(x-3)(7-x)} + 2 \sin^{-1} \left( \frac{x-5}{2} \right) + c$
  7.  $\frac{1}{\log 2} \left\{ \frac{2^x}{2} \sqrt{4^x + 4} + 2 \log (2^x + \sqrt{4^x + 4}) \right\} + c$
  8.  $\frac{1}{6} (2x^2 + 3)^{\frac{3}{2}} + \sqrt{2} \left[ \frac{x}{2} \sqrt{x^2 + \frac{3}{2}} + \frac{3}{4} \log \left( x + \sqrt{x^2 + \frac{3}{4}} \right) \right] + c$

$$9. -\frac{1}{3}(5-4x-x^2)^{\frac{3}{2}}-(x+2)\sqrt{5-4x-x^2}-9\sin^{-1}\left(\frac{x+2}{3}\right)+c$$

$$10. \frac{(1+2\tan x)}{4}\sqrt{\tan^2 x+\tan x-7}-\frac{29}{8}\log\left\{\frac{1}{2}+\tan x+\sqrt{\tan^2 x+\tan x-7}\right\}+c$$

$$11. \left(\frac{x+1}{2}\right)\sqrt{x^2+2x+5}+2\log\left\{x+1+\sqrt{x^2+2x+5}\right\}+c$$

$$12. \sqrt{2}\left\{\left(\frac{4x+3}{8}\right)\sqrt{x^2+\frac{3}{2}x+2}+\frac{23}{16\sqrt{2}}\log\left[\left(x+\frac{3}{4}\right)+\sqrt{x^2+\frac{3}{2}x+2}\right]\right\}+c$$

**III.** 1.  $e^x(2+\cot x)+c$       2.  $e^x \cdot \tan \frac{x}{2}+c$       3.  $e^x \cdot \frac{1}{x}+c$       4.  $e^x \cdot \left(\frac{1}{x+1}\right)+c$

5.  $e^x \cdot (\log x)^2+c$       6.  $e^{5x} \cdot \log x+c$       7.  $e^{\sin^{-1} x} \cdot x+c$

8.  $\frac{(1+x)^2}{2}\left(\log(1+x)-\frac{1}{2}\right)+c$       9.  $x \cdot \operatorname{cosec}(\log x)+c$

### EXERCISE 3.4

**I.** 1.  $\frac{1}{4}\log(x-1)-2\log(x+2)+\frac{11}{4}(x+3)+c$

2.  $\frac{1}{6}\tan^{-1}x+\frac{1}{15\sqrt{2}}\log\left(\frac{x-\sqrt{2}}{x+\sqrt{2}}\right)-\frac{\sqrt{3}}{10}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)+c$

3.  $\frac{51}{41}\log(2x+9)+\frac{31}{41}\log(3x-7)+c$       4.  $-\frac{8}{5}\log(x+4)-\frac{2}{5}\log(x-1)+c$

5.  $x-\log(x+3)+\log(x-2)+c$       6.  $x^2+3x+\frac{5}{3}\log(3x+1)+\log(x-1)+c$

7.  $\frac{1}{2}\log\left|\frac{2x+1}{2x-1}\right|+3\log(x+3)+c$       8.  $\frac{1}{5}\log\left(\frac{x^5}{x^5+1}\right)+c$       9.  $\frac{11}{\sqrt{5}}\tan^{-1}\left(\frac{x}{2}\right)-\frac{9}{2}\tan^{-1}\left(\frac{x}{2}\right)+c$

10.  $2\log\left(\frac{x+1}{x-1}\right)+\frac{5}{2\sqrt{2}}\log\left(\frac{x+\sqrt{2}}{x-\sqrt{2}}\right)+c$       11.  $\log\left(\frac{2+x^2}{3+x^2}\right)+c$

12.  $\frac{1}{5\cdot\log 2}\log\left(\frac{2^x-4}{2^x+1}\right)+c$       13.  $\frac{5}{2}\left(\frac{1}{x+1}\right)+\frac{11}{4}\log\left(\frac{x+1}{x+3}\right)+c$

14.  $6\cdot\log x-\log(x+1)-\frac{9}{x+1}+c$       15.  $\frac{1}{8}\log\left(\frac{x^6(x^3+3)}{(3x^3+1)^3}\right)+c$

16.  $\frac{1}{3}\log(x-1)-\frac{1}{6}\log(x^2+x+1)-\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+c$

17.  $3 \cdot \log(\sin x - 2) - \frac{4}{\sin x - 2} + c$
18.  $\frac{1}{2} \log(\cos x + 1) + \frac{1}{6} \log(\cos x - 1) - \frac{2}{3} \log(2 \cos x + 1) + c$
19.  $\frac{1}{8} \log\left(\frac{\cos x - 1}{\cos x + 1}\right) + \frac{1}{4 \cdot (\cos x + 1)} + c$
20.  $\frac{1}{6} \log\left[\frac{(1 + 2 \sin x)^4}{(1 - \sin x)(1 + \sin x)^3}\right] + c$
21.  $\frac{1}{10} \log(1 - \cos x) - \frac{1}{2} \log(1 + \cos x) + \frac{2}{5} \log(3 + 2 \cos x) + c$
22.  $\frac{1}{2} \log\left[\frac{e^x + 1}{(e^{2x} + 9)^{\frac{1}{2}}}\right] + \frac{1}{6} \tan^{-1}\left(\frac{e^x}{3}\right) + c$
23.  $\frac{5}{26} \log\left[\frac{(3 \log x + 2)^2}{\sqrt{(\log x)^2 + 1}}\right] + \frac{11}{26} \tan^{-1}(\log x) + c$

### MISCELLANEOUS EXERCISE 3

(I)

1	2	3	4	5	6	7	8	9	10
B	A	B	A	D	B	A	A	C	B
11	12	13	14	15	16	17	18	19	20
A	A	D	C	A	D	A	D	C	A

- (II) (1)  $\frac{2}{7} x^{\frac{7}{2}} - \frac{8}{5} x^{\frac{5}{2}} - \frac{8}{3} x^{\frac{3}{2}} + c$
- (2)  $\frac{x^7}{7} - \frac{x^6}{6} + \frac{x^5}{5} - \frac{x^4}{4} + \frac{x^3}{3} - \frac{x^2}{2} + x - \log(x + 1) + c$
- (3)  $\frac{1}{15} (6x + 5)^{\frac{5}{2}} + c$
- (4)  $\frac{t^2}{2} - 2t + 3 \cdot \log(t + 1) + \frac{1}{t + 1} + c$
- (5)  $3 \tan x - 2 \sec x + c$
- (6)  $\tan \theta - \cot \theta - 3 \theta + c$
- (7)  $\frac{1}{48} (2 \sin 6x + 3 \sin 4x + 6 \sin 2x + 12x) + c$
- (8)  $\frac{1}{2} \sin 2x - \frac{1}{3} \sin 3x + c$
- (9)  $\frac{\pi}{4} x - \frac{1}{4} x^2 + c$
- (III) (1)  $\frac{1}{4} (1 + \log x)^4 + c$
- (2)  $(\tan^{-1} x)x - \frac{1}{2} \log(1 + x^2) - (1 - x) \tan^{-1}(1 - x) + \frac{1}{2} \log(x^2 - 2x + 2) + c$
- (3)  $-\cot(\log x) + c$
- (4)  $\frac{2}{3} \sec x^{\frac{3}{2}} + c$

$$(5) \quad x \log(1 + \cos x) + c$$

$$(6) \quad \frac{1}{3} \sin^{-1}(x^3) + c$$

$$(7) \quad \frac{1}{4} \log(3 - 2 \cot x) + c$$

$$(8) \quad x \left( \log(\log x) - \frac{1}{\log x} \right) + c$$

$$(9) \quad \frac{2}{\sqrt{13}} \tan^{-1} \left( \frac{2 \tan \left( \frac{x}{2} \right) - 3}{\sqrt{13}} \right) + c$$

$$(10) \quad \frac{1}{4} \left( 2 \sec^{-1} x + \frac{2\sqrt{x^2 - 1}}{x^2} \right) + c$$

$$(11) \quad -\frac{3}{2} \sqrt{-2x^2 + x + 3} + \frac{7}{4\sqrt{2}} \sin^{-1} \left( \frac{2x - 1}{\sqrt{7}} \right) + c$$

$$(12) \quad x \cdot \log(x^2 + 1) - 2 [x - \tan^{-1} x] + c$$

$$(13) \quad \frac{1}{4} e^{2x} \cdot [\sin 2x - \cos 2x] + c$$

$$(14) \quad \frac{1}{18} \log(3x - 1) + \frac{1}{2} \log(x - 1) - \frac{4}{9} \log(3x - 2) + c$$

$$(15) \quad \frac{1}{6} \log \left\{ \frac{(\cos x - 1)(\cos x + 1)^3}{(2 \cos x + 1)^4} \right\} + c$$

$$(16) \quad \left( \frac{\tan x - 1}{2} \right) \sqrt{7 + 2 \tan x - \tan^2 x} + 4 \sin^{-1} \left( \frac{\tan x - 1}{2\sqrt{2}} \right) + c$$

$$(17) \quad \frac{1}{4} \log \left\{ \frac{(x - 1)^3(x + 3)}{(x + 1)^4} \right\} + c$$

$$(18) \quad \frac{1}{5} \log \left( \frac{x^5}{x^5 + 1} \right) + c$$

$$(19) \quad 2 \sqrt{\tan x} + c$$

$$(20) \quad \frac{1}{3 \cot^3 x} + \frac{2}{\cot x} - \cot x + c$$

## 4. DEFINITE INTEGRATION

### EXERCISE 4.1

I. (1) 4

(2)  $\frac{64}{3}$

(3)  $e^2 - 1$

(4) 6

(5) 20

### EXERCISE 4.2

I. (1)  $\frac{64}{3}$

(2)  $\log \left( \frac{25}{24} \right)$

(3)  $- \left( 1 + \frac{\pi}{4} \right)$

(4) 2

(5)  $\frac{1}{18} [13\sqrt{13} + 7\sqrt{7} - 3\sqrt{3} - 27]$  (6)  $1 - \frac{3\pi}{4}$  (7)  $\frac{4}{7\sqrt{2}}$  (8) 1 (9)  $\frac{3\pi}{16}$

(10)  $\frac{1}{3} \left( \tan^{-1} \frac{4}{3} + \tan^{-1} \frac{2}{3} \right)$  (11)  $\pi$  (12)  $\frac{\pi}{6}$  (13) 1 (14)  $\frac{\pi}{4} - \frac{1}{2}$  (15) 1

- II. (1)  $\frac{\pi}{4} - \frac{1}{2} \log 2$  (2)  $\frac{1}{2} \log 2$  (3)  $\frac{\pi}{4}$   
 (4) 0 (5)  $\frac{2}{3} \tan^{-1}\left(\frac{1}{3}\right)$  (6)  $\frac{1}{4} \log\left(\frac{2\sqrt{2}+1}{2\sqrt{2}-1}\right)$   
 (7)  $\log\left(\frac{4}{3}\right)$  (8)  $\frac{1}{ab} \left[ \tan^{-1}\left(\frac{ae}{b}\right) - \tan^{-1}\left(\frac{a}{be}\right) \right]$   
 (9)  $\frac{\pi}{4}$  (10)  $\frac{4}{3}$  (11)  $\frac{\pi}{2} - 1$   
 (12)  $\frac{8}{3}$  (13)  $\frac{\pi}{2} - 1$   
 (14)  $e^{\frac{\pi}{4}} \left[ \frac{\pi}{4} + 1 \right] - \left[ \frac{\pi}{2} + 1 \right]$  (15)  $\sin(\log 3)$
- III. (1)  $\frac{\pi}{4}$  (2) 0 (3) 0 (4) 0 (5)  $\frac{16}{77}(3)^{\frac{7}{2}}$   
 (6) 0 (7) 0 (8)  $\frac{\pi^2}{6\sqrt{3}}$  (9) 0 (10) 0  
 (11)  $4 \log\left(\frac{1+\sqrt{5}}{2}\right)$  (12) 0 (13)  $\frac{16}{105}$  (14)  $\frac{\pi}{3}$  (15)  $\frac{\pi}{2} \log\left(\frac{1}{2}\right)$

### MISCELLANEOUS EXERCISE 4

(I)

1	2	3	4	5	6	7	8	9	10
A	A	C	C	D	C	A	D	B	A

- (II) (1)  $\frac{1}{10}(3 - \log 3)$  (2)  $2 - \sqrt{2}$  (3)  $6 - 4 \log 2$  (4)  $\frac{1}{8}$  (5)  $\frac{1}{21}$   
 (6)  $\pi - 2$  (7)  $\frac{1}{3} \log 2$  (8)  $\frac{\pi}{5}$  (9) 0 (10)  $\frac{\pi}{2}$   
 (III) (1)  $\frac{\pi^2}{16}$  (2)  $\frac{2}{\sqrt{35}} \tan^{-1} \sqrt{\frac{7}{5}}$  (3)  $\frac{1}{\sqrt{5}a} \log\left(\frac{7+3\sqrt{5}}{2}\right)$  (4)  $\frac{\pi}{20}$   
 (5)  $\frac{\pi}{2} - \log 2$  (6)  $\frac{1}{2} \left( \frac{\pi}{4} - \log \sqrt{2} \right)$  (7)  $-\frac{\pi}{2} \log 2$  (8)  $\frac{\pi^3}{6}$   
 (9)  $\log\left(\frac{5+3\sqrt{3}}{1+\sqrt{3}}\right)$  (10)  $\frac{17}{2}$
- (IV) (1)  $\frac{1}{2}$  when  $a = 0$ ;  $\frac{9}{2}$  when  $a = 4$  (2)  $k = \frac{1}{2}$



## 5. APPLICATION OF DEFINITE INTEGRAL

### EXERCISE 5.1

- (1) (i) 25      (ii) 16      (iii) 20  
 (iv) 1      (v)  $2 \log 4$       (vi)  $\frac{32}{3}$   
 (vii)  $\frac{128}{3}$  sq. units

- (2) (i)  $\frac{128}{3}$       (ii)  $\frac{16}{3}$   
 (3) (i)  $\frac{1}{12}$       (ii)  $\frac{8}{3}$       (iii)  $\frac{32}{3}$   
 (iv)  $8 \frac{a^2}{3}$       (v)  $\frac{1}{6}$

### MISCELLANEOUS EXERCISE 5

(I)

1	2	3	4	5	6	7	8	9	10
A	A	C	B	A	D	B	D	A	B

  

11	12	13	14	15	16	17	18	19	20
A	D	B	B	C	C	A	D	A	C

(II)

1. (i) 10      (ii) 2      (iii)  $\frac{1}{2}$       5.  $\frac{\pi}{3}$       6.  $\frac{1}{6}$       7.  $\frac{\pi}{4} - \frac{1}{2}$   
 2.  $9\pi$       3.  $20\pi$       8.  $\frac{56}{3}$       9.  $36\frac{3}{4}$       10.  $\frac{7}{3}$   
 4. (i)  $\frac{16}{3}$       (ii)  $\frac{8}{3}$       (iii)  $\frac{1}{3}$

## 6. DIFFERENTIAL EQUATIONS

### EXERCISE 6.1

- (1) (i) 2, 1      (ii) 2, 3      (iii) 1, 2      (iv) 3, 1      (v) 2, 1      (vi) 3, 2      (vii) 2, not defined      (viii) 2, 2  
 (ix) 3, 3      (x) 2, 1      (xi)  $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$       (xii)  $8 \left( \frac{dy}{dx} \right)^3 - 27y = 0$   
 (xiii)  $\frac{d^2y}{dx^2} - 25y = 0$       (xiv)  $2 \frac{d^2y}{dx^2} + \left( \frac{dy}{dx} \right)^3 = 0$   
 (xv)  $(x^2 + xy) \frac{dy}{dx} + y = 0$       (xvi)  $\frac{d^2y}{dx^2} - 7 \frac{dy}{dx} + 10y = 0$

### EXERCISE 6.2

- (1) (i)  $2x^3 + 3xy^2 \frac{dy}{dx} - y^3 = 0$       (ix)  $xy \frac{d^2y}{dx^2} + x \left( \frac{dy}{dx} \right)^2 - 2y \frac{dy}{dx} = 0$   
 (ii)  $xy \frac{d^2y}{dx^2} + x \left( \frac{dy}{dx} \right)^2 - y \frac{dy}{dx} = 0$       (x)  $\frac{d^2y}{dx^2} + 4 \frac{dy}{dx} + 5y = 0$

$$(2) \frac{d^2y}{dx^2} = 0 \quad (3) 2a \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = 0$$

$$(4) x + 4y \frac{dy}{dx} = 0 \quad (5) 3 \frac{dy}{dx} + 2 = 0$$

$$(6) 81 \left(\frac{d^2y}{dx^2}\right)^2 = \left[\left(\frac{dy}{dx}\right)^2 + 1\right]^3$$

$$(7) y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$$

### EXERCISE 6.3

- (2) (i)  $\tan^{-1} y = \tan^{-1} x + c$   
(ii)  $2e^{-3y} + 3e^{2x} = c$       (iii)  $x = cy$   
(iv)  $\tan x \cdot \tan y = c$       (v)  $\sin y \cdot \cos x = c$   
(vi)  $y = -kx + c$   
(vii)  $2(x^2 + y^2) + 2(x \sin 2x + y \sin 2y) + \cos 2y + \cos 2x + c = 0$

- (viii)  $2y^2 \tan^{-1} x + 1 = cy^2$   
(ix)  $4e^x + 3e^{-2y} = c$   
(x)  $3e^x + 3e^{-y} + x^3 = c$

$$(3) (i) (1 + e^x)^3 \tan y = 0$$

$$(ii) (1 + x^2)(1 - y^2) = 5$$

$$(iii) y = ex \log x \quad (iv) (\sin x)(e^y + 1) = \sqrt{2}$$

$$(v) 2(2 + e^y) = 3(x + 1)$$

$$(vi) \cos\left(\frac{y-2}{x}\right) = a$$

$$(4) (i) \tan\left(\frac{x+y}{2}\right) = x + c$$

$$(ii) c + 2y = a \log\left(\frac{x-y-a}{x-y+a}\right)$$

$$(iii) \sin(x^2 + y^2) + 2x = c$$

$$(iv) x = \tan(x - 2y) + c$$

$$(v) (2x - y) - \log(x - y + 2) + 1 = 0$$

### EXERCISE 6.4

$$(1) \cos\left(\frac{y}{x}\right) dy = \log(x) + c$$

$$(2) x^2 - y^2 = cx \quad (3) x + 2ye^{\frac{x}{y}} = c$$

$$(4) xy^2 = c^2(x + 2y) \quad (5) x^2 + y^2 = cx$$

$$(6) y = c(x + y)^3 + x$$

$$(7) x \left[1 - \cos\left(\frac{y}{x}\right)\right] = \sin\left(\frac{y}{x}\right)$$

$$(8) x + ye^{\frac{x}{y}} = c \quad (9) \log(y) + \frac{y}{x} = c$$

$$(10) x^2y = 4 \quad (11) x^2 + y^2 = x^4$$

$$(12) \tan^{-1}\left(\frac{y}{x}\right) = \log(x) + c$$

$$(13) (3x + y)^3 (x + y)^2 = c$$

$$(14) c = \log(x) + \frac{x}{x+y} \quad (15) x^2 - y^2 = cx$$

### EXERCISE 6.5

$$1. (i) \frac{x^5}{5} - \frac{3x^2}{2} - xy = c$$

$$(ii) ye^{\tan x} = e^{\tan x} (\tan x - 1) + c$$

$$(iii) x = y(c + y^2)$$

$$(iv) y(\sec x + \tan x) = \sec x + \tan x - x + c$$

$$(v) x^2y = \frac{x^4 \log x}{4} - \frac{x^4}{16} + c$$

$$(vi) x + y + 1 = ce^y$$

$$(vii) 2y = (x + a)^5 + 2c(x + a)^3$$

$$(viii) r \sin^2 \theta + \frac{\sin^4 \theta}{2} = c$$

$$(ix) \frac{y^3}{3} = xy + c$$

$$(x) y = \sqrt{1 - x^2} + c(1 - x^2)$$

$$(xi) y = \frac{1}{2} e^{\tan^{-1} x} + c e^{-\tan^{-1} x}$$

2.  $3(x + 3y) = 2(1 - e^{3x})$

4.  $y = 4 - x - 2e^x$

3.  $4x^2 + 9y^2 = 36$

5.  $1 + y = 2e^{\frac{x^2}{2}}$

### EXERCISE 6.6

1. 8 times of original.

2. 95.4 years

3.  $36.36^\circ\text{C}$

4. 5656

5.  $\frac{\log 3}{k}$

6.  $\frac{27}{5}$  gms

7.  $(3000) \left(\frac{4}{9}\right)^{\frac{t}{40}}$

8. 1 hour

10.  $r = 3 - t$

11. 27,182

12.  $\left(10 - \frac{p}{10}\right)^2 \%$

### MISCELLANEOUS EXERCISE 6

(I)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
D	A	C	B	A	D	C	B	C	D	B	A	B	B	B

(II) (1) (i) 2, 1 (ii) 3, 10 (iii) 2, 3 (iv) 1.4 (v) 4, not defined

(3) (i)  $xy \frac{d^2y}{dx^2} + x \left(\frac{dy}{dx}\right)^2 - 2y \frac{dy}{dx} = 0$  (ii)  $\frac{d^2y}{dx^2} + y = 0$  (iii)  $(y - a) \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$

(iv)  $2x^2y \frac{d^2y}{dx^2} + 2x^2 \left(\frac{dy}{dx}\right)^2 + y = 0$  (v)  $\frac{d^2y}{dx^2} - 9y = 0$

(4) (i)  $2xy \frac{dy}{dx} + x^2 - y^2 = 0$  (ii)  $2b \frac{d^2y}{dx^2} - 1 = 0$  (iii)  $x + 4y \frac{dy}{dx} = 0$  (iv)  $2 \frac{dy}{dx} - 3 = 0$

(5) (i)  $2e^{-3y} + 3e^{2x} + 6c = 0$  (ii)  $\log(y) = \frac{x^3}{3} + x + c$  (iii)  $y = \frac{x}{2} \log(x^2) + 2 + cx$

(iv)  $y = 1 + x \log x + cx$  (v)  $y = x^2 + c \cdot \text{cosec } x$  (vi)  $x \log y = (\log y)^2 + c$

(vii)  $4xe^{2y} + 5e^{-y} = c$

(6) (i)  $ex \log x - y = 0$  (ii)  $x = 2y^2$  (iii)  $y \text{ cosec}^2 x + 2 = 4 \sin 2x$

(iv)  $\log \sqrt{x^2 + y^2} + \tan^{-1} \left(\frac{y}{x}\right) = \frac{\pi}{4}$  (v)  $x + 2ye^{\frac{x}{y}} = 2$

(8)  $x^2 + y^2 = 4x + 5$  (9)  $r = (63t + 27)^{\frac{1}{3}}$  (10)  $\frac{20}{9}$  years

## 7. PROBABILITY DISTRIBUTIONS

### EXERCISE 7.1

1.  $\{-6, -4, -2, 0, 2, 4, 6\}$

2.  $\{0, 1, 2\}$

3. (i) p.m.f      (ii) Not p.m.f  
 (iii) p.m.f      (iv) Not p.m.f  
 (v) Not p.m.f      (vi) p.m.f

5.

$X$	0	1	2
$P(X)$	$\frac{2}{3}$	$\frac{2}{9}$	$\frac{1}{9}$

6.

$X$	0	1	2	3	4
$P(X)$	$\left(\frac{4}{5}\right)^4$	$\left(\frac{4}{5}\right)^3 \frac{1}{5}$	$\left(\frac{4}{5}\right)^2 \left(\frac{1}{5}\right)^2$	$\frac{4}{5} \left(\frac{1}{5}\right)^3$	$\left(\frac{1}{5}\right)^4$

7.

$X$	0	1	2
$P(X)$	$\frac{9}{16}$	$\frac{3}{8}$	$\frac{1}{16}$

4. (i)

$X$	0	1	2
$P(X)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

(ii)

$X$	0	1	2	3
$P(X)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

(iii)

$X$	0	1	2	3	4
$P(X)$	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

8. (i)  $\frac{1}{10}$       (ii)  $\frac{3}{10}$       (iii)  $\frac{1}{5}$

9.  $-0.05, 2.2475$     10.  $\frac{7}{3}, \frac{524}{54}$     11.  $1.5$

12.  $\frac{1}{3}$       13.  $4.67$       14.  $2.41$

15.  $17.53, 4.9, 2.21$       16.  $0.7, 0.21$

### EXERCISE 7.2

1. (i) p.d.f.      (ii) Not a p.d.f  
 (iii) Not a p.d.f

2. (a)  $\frac{2.25}{16}$ ,      (b)  $\frac{3}{16}$ ,      (c)  $\frac{3}{4}$

3. (i) p.d.f.      (ii)  $\frac{1}{9}$       (iii)  $\frac{1}{9}$

4. (i)  $\frac{1}{2}, \frac{35}{64}$       (ii)  $6, \frac{11}{32}, \frac{1}{2}$

8. (i)  $\frac{x^2}{16}$       (ii)  $\frac{1}{64}, 0.18, 1$

5. (i)  $\frac{1}{4}$       (ii)  $\frac{1}{2}$       (iii)  $\frac{7}{16}$

9.  $\frac{2}{9}, 0, \frac{8}{9}, \frac{7}{9}$

6. (i)  $\frac{2}{5}$       (ii)  $\frac{1}{5}$

10.  $\frac{1}{\log 3}, \frac{4}{\log 3}, \frac{4(\log 3 - 1)}{(\log 3)^2}$

7. (i)  $\frac{1}{2}$       (ii)  $\frac{11}{16}$       (iii)  $0.6328$

### MISCELLANEOUS EXERCISE 7

(I)

1	2	3	4	5	6	7	8	9	10
B	C	A	B	C	B	B	A	D	B

(II) Solve the following :

(1) (i) Discrete  $\{1, 2, 3, \dots, 100000\}$       (ii) Continuous.      (iii) Continuous.

(iv) Discrete  $\{0, 1, 2, 3, 4, 5\}$       (v) Continuous

(2) (i)  $\frac{1}{21}$       (ii)  $\frac{10}{21}, \frac{1}{7}, \frac{6}{7}$       (3) (i)  $0.5$       (ii)  $0.7$       (iii)  $0.55$       (iv)  $0.45$

(5)  $\frac{1}{4}$

$x$	1	2	3	4	5
$P(X)$	$\frac{1}{20}$	$\frac{3}{20}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{20}$
$F(x)$	$\frac{1}{20}$	$\frac{1}{5}$	$\frac{9}{20}$	$\frac{19}{20}$	1

(6)

$X$	0	1	2	3	4
$P(X)$	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

$$\frac{{}^4C_x}{2^4}$$

(7) (i)

$X$	0	1	2
$P(X)$	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

(ii)

$X$	0	1	2
$P(X)$	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

$$(8) \quad (\text{i}) \frac{1}{10} \quad (\text{ii}) \frac{17}{100} \quad (\text{iii}) \frac{3}{10}$$

(9)

$X$	-3	-2	-1	0	1	2	3	4
$F(X)$	0.1	0.3	0.5	0.65	0.75	0.85	0.9	1
$P(x)$	0.1	0.2	0.2	0.15	0.10	0.10	0.05	0.10

$$(\text{i}) \quad 0.55 \quad (\text{ii}) \quad 0.25$$

$$(10) \quad (\text{i}) \frac{11}{5}, \frac{14}{25}, \frac{\sqrt{14}}{5} \quad (\text{ii}) \frac{1}{5}, \frac{14}{25}, \frac{\sqrt{14}}{5} \quad (\text{iii}) \frac{n+1}{2}, \frac{n^2-1}{12}, \sqrt{\frac{n^2-1}{12}} \quad (\text{iv}) \frac{5}{2}, \frac{5}{4}, \frac{\sqrt{5}}{2}$$

$$(11) \quad \text{₹ } 5.5, 8.25 \quad (12) \quad 0, 1 \quad (13) \quad (\text{i}) \frac{1}{2} \quad (\text{ii}) \frac{11}{16} \quad (\text{iii}) \frac{81}{128}$$

$$(15) \quad k = \frac{1}{\theta}, \frac{1}{e} \quad (16) \quad k = \frac{1}{4}, F(x) = \frac{\sqrt{x}}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}$$

## 8. BINOMIAL DISTRIBUTION

### EXERCISE 8.1

$$1. \quad (\text{i}) \frac{3}{2^5} \quad (\text{ii}) \frac{7}{2^6} \quad (\text{iii}) \frac{63}{64}$$

$$8. \quad (\text{i}) \quad 1 - \left( \frac{99}{100} \right)^{50} \quad (\text{ii}) \quad 50 \left( \frac{99^{49}}{100^{50}} \right)$$

$$2. \quad \frac{25}{216} \quad 3. \quad 29 \left( \frac{19^9}{20^{10}} \right)$$

$$(\text{iii}) \quad 1 - 149 \left( \frac{99^{49}}{100^{50}} \right)$$

$$4. \quad (\text{i}) \quad \frac{1}{1024} \quad (\text{ii}) \quad \frac{45}{1024}$$

$$9. \quad (\text{i}) \frac{1}{20^3} \quad (\text{ii}) \quad 3 \left( \frac{19}{20^3} \right) \quad (\text{iii}) \quad 3 \left( \frac{19^2}{20^3} \right) \quad (\text{iv}) \left( \frac{19}{20} \right)^3$$

$$5. \quad (\text{i}) \quad (0.95)^5 \quad (\text{ii}) \quad (1.2)(0.95)^4 \\ (\text{iii}) \quad 1 - (1.2)(0.95)^4 \quad (\text{iv}) \quad 1 - (0.95)^5$$

$$10. \quad \frac{7}{3} \left( \frac{5}{6} \right)^5 \quad 11. \quad 22 \left( \frac{9^3}{10^{11}} \right)$$

$$6. \quad \left( \frac{9}{10} \right)^4 \quad 7. \quad \frac{11}{243}$$

$$12. \quad (\text{i}) \quad 4, 2.4 \quad (\text{ii}) \quad 10, 2.4 \quad (\text{iii}) \quad \frac{2}{5}; \sqrt{6} \quad (\text{iv}) \quad \frac{8}{5}$$

### MISCELLANEOUS EXERCISE 8

(I)

1	2	3	4	5	6	7
B	D	D	C	B	C	B

**(II) Solve the following :**

(1) (i)  $2 \times (0.8)^9$     (ii)  $1 - (0.8)^{10}$   
 (iii)  $1 - (8.2)(0.2)^9$

(2) (i)  $p = \frac{1}{2}$ ,  $Var(X) = 2.5$   
 (ii)  $n = 10, p = \frac{1}{2}$

(3) (i)  $\frac{63}{256}$     (ii)  $\frac{105}{512}$

(4)  $45 \left( \frac{2^{26}}{2^{10}} \right)$

(5) (i)  $0.65 \times (0.95)^{16}$   
 (ii)  $(2.0325) \times (0.95)^{14}$   
 (iii)  $1 - (1.6) \times (0.95)^{16}$

(6) 0.2114    (7)  $1.4 \times (0.9)^4$

(8)  $6.97 \times (0.97)^{19}$

(9) 0.3456    (10)  $\frac{30.44}{5^8}$

(11) (i)  $(0.998)^8$     (ii)  $1.014 \times (0.998)^7$

(iii)  $1 - 1.014 \times (0.998)^7$

(12)  $775.44 \times (0.003)^{38}$

(13) (i)  $0.9^{10}$     (ii)  $0.9^9$

(iii)  $0.45 \times (0.9)^8$

(iv)  $1 - 2.16 \times (0.9)^8$

(14) (i)  $\frac{1}{5^4}, \frac{16}{5^4}, \frac{96}{5^4}, \frac{256}{5^4}, \frac{256}{5^4}$

(ii) (a)  $\frac{608}{5^4}$     (b)  $1 - \frac{33}{5^8}$

(15) (i)  $35 \times 8 \times \frac{81}{5^7}$     (ii)  $1 - \frac{12393}{5^7}$

(16) (i)  $\frac{\log 0.5}{\log 0.99}$

(17)  $\frac{1}{5}$





4!



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