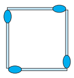
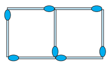
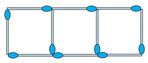
8 Algebraic Expressions and Operations on them

Let's learn. Algebraic Expressions

Look at the arrangements of sticks given below and observe the pattern.

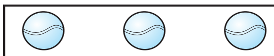
Arrangements of sticks				......	..	......	..	......
Squares	1	2	3	4	..	10	..	n
Number of sticks	4	7	10	13	..	......	..	......
	3 + 1	6 + 1	9 + 1	12 + 1	..	......	..	......
	$3 \times 1 + 1$	$3 \times 2 + 1$	$3 \times 3 + 1$	$3 \times 4 + 1$		$3 \times 10 + 1$		$3 \times n + 1$

On observing the pattern above, we notice that

Number of sticks = 3 × number of squares + 1

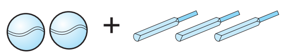
Here, the number of squares changes. It could be any of the numbers 2, 3, 4, ... , 10, ... If we do not know the number of squares, we write a letter in its place. Here, the number of squares is shown by the letter n.

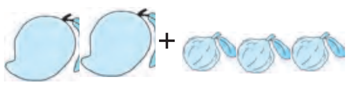
'n' is a variable. $3 \times n + 1$ which is the same as $3n + 1$ is an algebraic expression in the variable n.

 = 3 balls

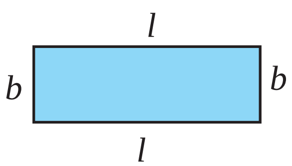
 = 3 triangles

t t t = $3t$'s

 = $\square$ balls + $\square$ bats

 = $\square$ mangoes + $\square$ guavas

$x + x + y + y + y = 2x + 3y$



Perimeter of a rectangle $= 2l + 2b$
$= 2(l + b)$

Now I know!

$3n + 1$, $3t$, $2x + 3y$, $2(l + b)$ are algebraic expressions.

n, t, y, l, b, x are the variables in these expressions.

Let's learn.

In the expression $3x$, 3 is the coefficient of the variable x.

In the expression $-15t$, -15 is the coefficient of the variable t.

An expression in which multiplication is the only operation is called a 'term'.

An algebraic expression may have one term or may be the sum of several terms.

Term	Coefficient	Variables
$11mn$	11	m, n
$-9x^2y^3$	-9	x, y
$\frac{5}{6}p$	$\frac{5}{6}$	p
a	1	a

Example In the algebraic expression $4x^2-2y+\frac{5}{6}xz$, $4x^2$ is the first term and 4 is the coefficient in it.

$-2y$ is the second term with the coefficient 2.

$\frac{5}{6}xz$ is the third term and $\frac{5}{6}$ is the coefficient in it.

Remember :

- The algebraic expression $15-x$ has two terms. The first term 15 is a number. The second term is $-x$. Here, the coefficient of the variable x is (-1).
- Terms which have the same variables with the same powers are called 'like terms'.

Like terms	Unlike terms
(i) $2x$, $5x$, $-\frac{2}{3}x$, (ii) $-5x^2y$, $\frac{6}{7}yx^2$	(i) $7xy$, $9y^2$, $-2xyz$, (ii) $8mn$, $8m^2n^2$, $8m^3n$

Types of Algebraic Expressions

Expressions are named after the number of terms they have. Expressions with one term are called monomials, those with two terms, binomials, with three terms, trinominals and if they have more than three terms, they are called polynomials.

Monomials	Binomials	Trinomials	Polynomials
• $4x$	• $2x-3y$	• $a+b+c$	• $a^3-3a^2b+3ab-b^3$
• $\frac{5}{6}m$	• $2l+2b$	• x^2-5x+6	• $4x^4-7x^2+9-5x^3-16x$
• -7	• $3mn-5m^2n$	• $8a^3-5a^2b+c$	• $5x^5-\frac{1}{2}x+8x^3-5$

Practice Set 32

⊙ Classify the following algebraic expressions as monomials, binomials, trinomials or polynomials.

(i) $7x$ (ii) $5y-7z$ (iii) $3x^3-5x^2-11$ (iv) $1-8a-7a^2-7a^3$

(v) $5m-3$ (vi) a (vii) 4 (viii) $3y^2-7y+5$

Let's learn.

Addition of Algebraic Expressions

✸ Addition of monomials

Example 3 guavas + 4 guavas = (3 + 4) guavas = 7 guavas

Example $3x + 4x = (3 + 4)\,x = 7x$

Like terms are added as we would add up things of the same kind.

Example Add.

(i) $-3x - 8x + 5x = (-3 - 8 + 5)x = -6x$

(ii) $\frac{2}{3}ab - \frac{5}{7}ab = \left(\frac{2}{3} - \frac{5}{7}\right) ab = \frac{-1}{21}\,ab$

(iii) $-2p^2 + 7p^2 = (-2 + 7)p^2 = 5p^2$

> Think about it.
> $3x + 4y$ = How many ?
> 3 guavas + 4 mangoes = 7 guavas ?
> $7m - 2n = 5m$?

✸ Addition of binomial expressions

Horizontal arrangement

Example $(2x + 4y) + (3x + 2y)$

$= 2x + 3x + 4y + 2y$

$= 5x + 6y$

Vertical arrangement

$$\begin{array}{rr} & 2x + 4y \\ + & 3x + 2y \\ \hline & 5x + 6y \end{array}$$

To add like terms, we add their coefficients and write the variable after their sum.

Example Add. $9x^2y^2 - 7xy$; $3x^2y^2 + 4xy$

Horizontal arrangement

$(9x^2y^2 - 7xy) + (3x^2y^2 + 4xy)$

$= 9x^2y^2 - 7xy + 3x^2y^2 + 4xy$

$= (9x^2y^2 + 3x^2y^2) + (-7xy + 4xy)$

$= 12x^2y^2 - 3xy$

Vertical arrangement

$$\begin{array}{rr} & 9x^2y^2 - 7xy \\ + & 3x^2y^2 + 4xy \\ \hline & 12x^2y^2 - 3xy \end{array}$$

Take care!

In $3x + 7y$, the two terms are not like terms. Hence, their sum can only be written as $3x + 7y$ or as $7y + 3x$.

Practice Set 33

⦿ Add.

(i) 9p + 16q ; 13p + 2q

(ii) 2a + 6b + 8c; 16a + 13c + 18b

(iii) $13x^2 - 12y^2$; $6x^2 - 8y^2$

(iv) $17a^2b^2$ + 16c ; 28c $- 28a^2b^2$

(v) $3y^2 - 10y + 16$; $2y - 7$

(vi) $-3y^2 + 10y - 16$; $7y^2 + 8$

Let's learn. ## Subtraction of Algebraic Expressions

We have learnt that to subtract one integer from another is to add its opposite integer to the other.

We shall use the same rule for subtraction of algebraic expressions.

Example $18 - 7$
$= 18 + (-7) = 11$

Example $9x - 4x$
$= [9 + (-4)]x = 5x$

Example Subtract the second expression from the first.
$16x + 23y + 12z$; $9x - 27y + 14z$

Horizontal arrangement

$$
\begin{aligned}
& (16x + 23y + 12z) - (9x - 27y + 14z) \\
= \ & 16x + 23y + 12z - 9x + 27y - 14z \\
= \ & (16x - 9x) + (23y + 27y) + (12z - 14z) \\
= \ & 7x + 50y - 2z
\end{aligned}
$$

Vertical arrangement

$$
\begin{array}{rrrr}
 & 16x & + \ 23y & + \ 12z \\
- & \oplus\ 9x & \ominus\ 27y & \oplus\ 14z \\
 & - & + & - \\
\hline
 & 7x & + \ 50y & - \ 2z
\end{array}
$$

(Change the sign of every term in the expression to be subtracted and then add the two expressions.)

Practice Set 34

⊙ Subtract the second expression from the first.

(i) $(4xy - 9z)$; $(3xy - 16z)$ (ii) $(5x + 4y + 7z)$; $(x + 2y + 3z)$

(iii) $(14x^2 + 8xy + 3y^2)$; $(26x^2 - 8xy - 17y^2)$ (iv) $(6x^2 + 7xy + 16y^2)$; $(16x^2 - 17xy)$

(v) $(4x + 16z)$; $(19y - 14z + 16x)$

Let's learn. ## Multiplication of Algebraic Expressions

✱ Multiplying a monomial by a monomial

Example $3x \times 12y$
$= 3 \times 12 \times x \times y$
$= 36xy$

Example $(-12x) \times 3y^2$
$= -12 \times 3 \times x \times y \times y$
$= -36xy^2$

Example $2a^2 \times 3ab^2$
$= 2 \times 3 \times a^2 \times a \times b^2$
$= 6a^3 b^2$

Example $(-3x^2) \times (-4xy)$
$= (-3) \times (-4) \times x^2 \times x \times y$
$= 12x^3y$

When multiplying two monomials, first multiply the coefficients along with the signs. Then multiply the variables.

✸ Multiplying a binomial by a monomial

Example $x(x+y)$
$= x \times x + x \times y$
$= x^2 + xy$

Example $(7x-6y) \times 3z = 7x \times 3z - 6y \times 3z$
$= 7 \times 3 \times x \times z - 6 \times 3 \times y \times z$
$= 21xz - 18yz$

✸ Multiplying a binomial by a binomial

Example

$$\begin{array}{rll} & 3x + 4y & \\ \times & 5x + 7y & \\ \hline & 15x^2 + 20xy & \text{[Multiplying by } 5x] \\ & + 21xy + 28y^2 & \text{[Multiplying by } 7y] \\ \hline & 15x^2 + 41xy + 28y^2 & \text{[adding]} \end{array}$$

$(3x + 4y)\ (5x + 7y)$
$= 3x\,(5x + 7y) + 4y\,(5x + 7y)$
$= 3x \times 5x + 3x \times 7y + 4y \times 5x + 4y \times 7y$
$= 15x^2 + 21xy + 20xy + 28y^2$
$= 15x^2 + 41xy + 28y^2$

Example Find the area of a rectangular field whose length is $(2x + 7)$ m and breadth is $(x + 2)$ m.

Solution: Area of rectangular field = length × breadth $= (2x + 7) \times (x + 2)$
$= 2x\,(x + 2) + 7\,(x + 2)$
$= 2x^2 + 11x + 14$

Area of rectangular field $(2x^2 + 11x + 14)$ m^2

Practice Set 35

1. Multiply.
   (i) $16xy \times 18xy$
   (ii) $23xy^2 \times 4yz^2$
   (iii) $(12a + 17b) \times 4c$
   (iv) $(4x + 5y) \times (9x + 7y)$

2. A rectangle is $(8x + 5)$ cm long and $(5x + 3)$ cm broad. Find its area.

Let's recall.

Equations in One Variable

Solve the following equations.

(1) $x + 7 = 4$ (2) $4p = 12$ (3) $m - 5 = 4$ (4) $\frac{t}{3} = 6$

Let's learn.

Example $2x + 2 = 8$
$\therefore\ 2x + 2 - 2 = 8 - 2$
$\therefore\ 2x = 6$
$\therefore\ x = 3$

Example $3x - 5 = x - 17$
$3x - 5 + 5 - x = x - 17 + 5 - x$
$\therefore\ 2x = -12$
$\therefore\ x = -6$

Example The length of a rectangle is 1 cm more than twice its breadth. If the perimeter of the rectangle is 50 cm, find its length.

Solution:

Let the breadth of the rectangle be x cm.

Then the length of the rectangle will be $(2x+1)$cm.

$2 \times \text{length} + 2 \times \text{breadth} = \text{perimeter of rectangle}$

$$2(2x+1) + 2x = 50$$
$$\therefore \ 4x + 2 + 2x = 50$$
$$\therefore \ 6x + 2 = 50$$
$$\therefore \ 6x = 50 - 2 = 48$$
$$\therefore \ x = 8$$

Breadth of rectangle is 8 cm.

Length of the rectangle $= 2x + 1 = 2 \times 8 + 1$

$\therefore$ Length of rectangle = 17 cm.

Example The sum of two consecutive natural numbers is 69. Find the numbers.

Solution:

Let one natural number be x.

The next natural number is $x + 1$

$$(x) + (x+1) = 69$$
$$\therefore \ x + x + 1 = 69$$
$$\therefore \ 2x + 1 = 69$$
$$2x = 69 - 1$$
$$\therefore 2x = 68$$
$$\therefore \ x = 34$$

1st natural number = 34

2nd natural number = 34 + 1

= 35

Remember :

From the solved examples above, we see that if a term is 'transposed' from one side to the other of the '=' sign in an equation, that term's sign must be changed.

Practice Set 36

1. Simplify $(3x-11y) - (17x+13y)$ and choose the right answer.

   (i) $7x-12y$ (ii) $-14x-54y$ (iii) $-3(5x+4y)$ (iv) $-2(7x+12y)$

2. The product of $(23x^2y^3z)$ and $(-15x^3yz^2)$ is ....

   (i) $-345x^5y^4z^3$ (ii) $345x^2y^3z^5$ (iii) $145x^3y^2z$ (iv) $170x^3y^2z^3$

3. Solve the following equations.

   (i) $4x + \frac{1}{2} = \frac{9}{2}$ (ii) $10 = 2y + 5$ (iii) $5m - 4 = 1$

   (iv) $6x - 1 = 3x + 8$ (v) $2(x-4) = 4x + 2$ (vi) $5(x+1) = 74$

4. Rakesh's age is less than Sania's age by 5 years. The sum of their ages is 27 years. How old are they?

5. When planting a forest, the number of jambhul trees planted was greater than the number of ashoka trees by 60. If there are altogether 200 trees of these two types, how many jambhul trees were planted?

6. Shubhangi has twice as many 20-rupee notes as she has 50-rupee notes. Altogether, she has 2700 rupees. How many 50-rupee notes does she have?

7*. Virat made twice as many runs as Rohit. The total of their scores is 2 less than a double century. How many runs did each of them make?

Miscellaneous Problems : Set 1

1. Solve the following.

   (i) $(-16) \times (-5)$ (ii) $(72) \div (-12)$ (iii) $(-24) \times (2)$

   (iv) $125 \div 5$ (v) $(-104) \div (-13)$ (vi) $25 \times (-4)$

2. Find the prime factors of the following numbers and find their LCM and HCF.

   (i) 75, 135 (ii) 114, 76 (iii) 153, 187 (iv) 32, 24, 48

3*. Simplify.

   (i) $\frac{322}{391}$ (ii) $\frac{247}{209}$ (iii) $\frac{117}{156}$

4. Find the square root of the following numbers.

   (i) 784 (ii) 225 (iii) 1296 (iv) 2025 (v) 256

5. There are four polling booths for a certain election. The numbers of men and women who cast their vote at each booth is given in the table below. Draw a joint bar graph for this data.

Polling Booths	Navodaya Vidyalaya	Vidyaniketan School	City High School	Eklavya School
Women	500	520	680	800
Men	440	640	760	600

6. Simplify the expressions.

   (i) $45 \div 5 + 20 \times 4 - 12$ (ii) $(38 - 8) \times 2 \div 5 + 13$

   (iii) $\frac{5}{3} + \frac{4}{7} \div \frac{32}{21}$ (iv) $3 \times \{ 4 [85 + 5 - (15 \div 3)] + 2 \}$

7. Solve.

   (i) $\frac{5}{12} + \frac{7}{16}$ (ii) $3\frac{2}{5} - 2\frac{1}{4}$ (iii) $\frac{12}{5} \times \frac{(-10)}{3}$ (iv*) $4\frac{3}{8} \div \frac{25}{18}$

8. Construct ΔABC such that $m\angle A = 55°$, $m\angle B = 60°$, and $l(AB) = 5.9$ cm.

9. Construct ΔXYZ such that, $l(XY) = 3.7$ cm, $l(YZ) = 7.7$ cm, $l(XZ) = 6.3$ cm.

10. Construct ΔPQR such that, $m\angle P = 80°$, $m\angle Q = 70°$, $l(QR) = 5.7$ cm.

11. Construct ΔEFG from the given measures. $l(FG) = 5$ cm, $m\angle EFG = 90°$, $l(EG) = 7$ cm.

12. In ΔLMN, $l(LM) = 6.2$ cm, $m\angle LMN = 60°$, $l(MN) = 4$ cm. Construct ΔLMN.

13. Find the measures of the complementary angles of the following angles.

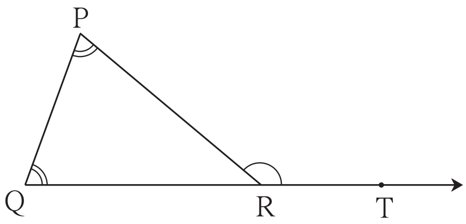
    (i) $35°$ (ii) $a°$ (iii) $22°$ (iv) $(40 - x)°$

14. Find the measures of the supplements of the following angles.

    (i) $111°$ (ii) $47°$ (iii) $180°$ (iv) $(90 - x)°$

15. Construct the following figures.

    (i) A pair of adjacent angles (ii) Two supplementary angles which are not adjacent angles. (iii) A pair of adjacent complementary angles.

16. 

In ΔPQR, the measures of $\angle P$ and $\angle Q$ are equal and $m\angle PRQ = 70°$. Find the measures of the following angles.
(i) $m\angle PRT$ (ii) $m\angle P$ (iii) $m\angle Q$

17. Simplify.
(i) $5^4 \times 5^3$ (ii) $\left(\frac{2}{3}\right)^6 \div \left(\frac{2}{3}\right)^9$ (iii) $\left(\frac{7}{2}\right)^8 \times \left(\frac{7}{2}\right)^{-6}$ (iv) $\left(\frac{4}{5}\right)^2 \div \left(\frac{5}{4}\right)$

18. Find the value.
(i) $17^{16} \div 17^{16}$ (ii) 10^{-3} (iii) $(2^3)^2$ (iv) $4^6 \times 4^{-4}$

19. Solve.
(i) $(6a-5b-8c) + (15b+2a-5c)$ (ii) $(3x+2y)(7x-8y)$
(iii) $(7m-5n) - (-4n-11m)$ (iv) $(11m-12n+3p) - (9m+7n-8p)$

20. Solve the following equations.
(i) $4(x + 12) = 8$ (ii) $3y + 4 = 5y - 6$

Multiple Choice Questions

Choose the right answer from the options given after every question.

1. The three angle bisectors of a triangle are concurrent. Their point of concurrence is called the ....
(i) circumcentre (ii) apex (iii) incentre (iv) point of intersection.

2. $\left[\left(\frac{3}{7}\right)^{-3}\right]^4 =$ ....
(i) $\left(\frac{3}{7}\right)^{-7}$ (ii) $\left(\frac{3}{7}\right)^{-10}$ (iii) $\left(\frac{7}{3}\right)^{12}$ (iv) $\left(\frac{3}{7}\right)^{20}$

3. The simplest form of $5 \div \left(\frac{3}{2}\right) - \frac{1}{3}$ is ....
(i) 3 (ii) 5 (iii) 0 (iv) $\frac{1}{3}$

4. The solution of the equation $3x - \frac{1}{2} = \frac{5}{2} + x$ is ....
(i) $\frac{5}{3}$ (ii) $\frac{7}{2}$ (iii) 4 (iv) $\frac{3}{2}$

5*. Which of the following expressions has the value 37?
(i) $10 \times 3 + (5 + 2)$ (ii) $10 \times 4 + (5 - 3)$
(iii) $8 \times 4 + 3$ (iv) $(9 \times 3) + 2$