13 Congruence of triangles

Let's recall.

Write answers to the following questions referring to the adjacent figure.

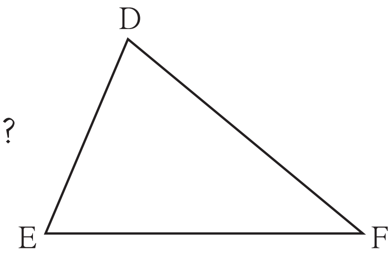
(i) Which is the angle opposite to the side DE?

(ii) Which is the side opposite to ∠ E ?

(iii) Which angle is included by side DE and side DF?

(iv) Which side is included by ∠ E and ∠ F ?

(v) State the angles adjacent to side DE.

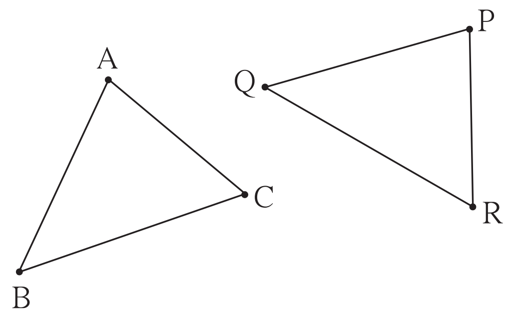


- The figures which exactly coincide with each other are called congruent figures.
- The segments of equal lengths are congruent.
- The angles of equal measures are congruent.

Let's learn.

Congruence of triangles

Activity : Observe the adjacent figures. Copy Δ ABC on a tracing paper. Place it on Δ PQR such that point A coincides with point P, point B with point Q and point C with point R. You will find that both the triangles coincide exactly with each other, that is they are congruent.

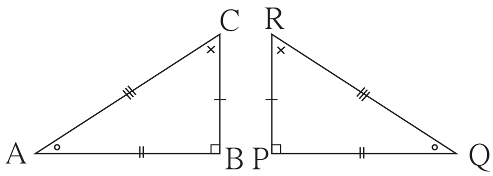


In the activity, one way of placing Δ ABC on Δ PQR is given. But if we place point A on point Q, point B on point R and point C on point P, then two triangles will not coincide with each other. It means, the vertices must be matched in a specific way. The way of matching the vertices is denoted by one-to-one correspondence. Point A corresponds to point P is denoted as A ↔ P. Here, two triangles are congruent in the correspondence A ↔ P, B ↔ Q, C ↔ R. When the two triangles are congruent in this way, we get six congruences, namely ∠A ≅ ∠P, ∠B ≅ ∠Q, ∠C ≅ ∠R, seg AB ≅ seg PQ, seg BC ≅ seg QR, seg CA ≅ seg RP. Therefore, it is said that, Δ ABC and Δ PQR are congruent in the correspondence ABC ↔ PQR and written as Δ ABC ≅ Δ PQR.

Δ ABC ≅ Δ PQR implies the correspomdence A ↔ P, B ↔ Q, C ↔ R and the six congruences mentioned above. Therefore, while writing the congruence of two triangles, we have to take care that the order of vertices observes the one to one correspondence ascertaining congruence.

Let's discuss.

Δ ABC and Δ PQR are congruent. Their congruent parts are indicated by the identical marks.



Anil, Rehana and Surjit had written congruence of the triangles as follows.

Anil : Δ ABC ≅ Δ QPR

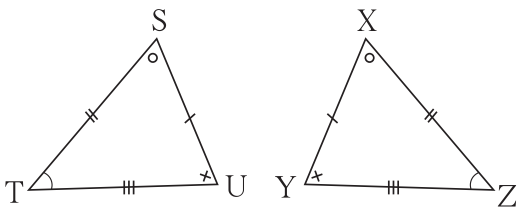
Rehana : Δ BAC ≅ Δ PQR

Surjit : Δ ABC ≅ Δ PQR

Which of the statements is correct and which is wrong ? Discuss.

Solved Example

Ex. (1) In the adjacent figure, parts of triangles indicated by identical marks are congruent (i) Identify the one to one correspondence of vertices in which the two triangles are congruent and write the congruence in two ways.



(ii) State with reason, whether the statement, Δ XYZ ≅ Δ STU is right or wrong.

Solution: By observation, the triangles are congruent in the correspondence STU ↔ XZY hence

(i) Δ STU ≅ Δ XZY is one way; Δ UST ≅ Δ YXZ is another way.

Write the same congruence in some more different ways.

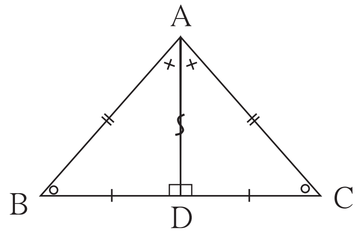
(ii) If the congruence is written as Δ XYZ ≅ Δ STU, it will mean side ST ≅ side XY, which is wrong.

∴ statement Δ XYZ ≅ Δ STU is wrong.

(The writing Δ XYZ ≅ Δ STU includes some more mistakes. Students should find them out. Note that, to show that an answer is wrong it is sufficient to point out one mistake.)

Ex. (2) In the given figure, the identical marks show the congruent parts in the pair of triangles. State the correspondence between the vertices of the triangles in which the two triangles are congruent.

Solution : In Δ ABD and Δ ACD, side AD is common. Every segment is congruent to itself. Therefore,



Correspondence: A ↔ A, B ↔ C, D ↔ D. Δ ABD ≅ Δ ACD

Note: It is a convension to indicate a common side by the symbol 'ʂ'

Let's learn.

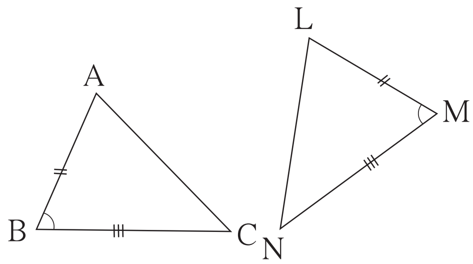
To show that a pair of triangles is congruent, it is not necessary to show that all six corresponding parts of the two triangles are congruent. If three specific parts of one triangle are respectively congruent with the three corresponding parts of the other, then the remaining three corresponding parts are also congruent with each other. It means, the specific three parts ascertain the test of congruence.

We have learnt to construct triangles. The three specific parts of a triangle which define a unique triangle decide a test of congruence. Let us verify this.

(1) Two sides and the included angle : SAS Test

Draw Δ ABC and Δ LMN such that two pairs of their sides and the angles included by them are congruent.

Draw Δ ABC and Δ LMN, l(AB) = l(LM), l(BC) = l(MN), m∠ABC = m∠LMN

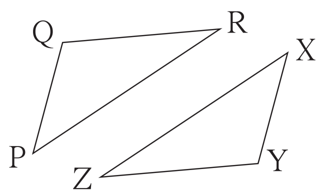


Copy Δ ABC on a tracing paper. Place the paper on Δ LMN in such a way that point A coincides with point L, side AB overlaps side LM, ∠B overlaps ∠M and side BC overlaps side MN. You will notice that Δ ABC ≅ Δ LMN.

(2) Three corresponding sides: SSS test

Draw ΔPQR and ΔXYZ such that l(PQ) = l(XY), l(QR) = l(YZ), l(RP) = l(ZX).

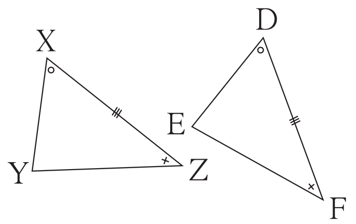
Copy ΔPQR on a tracing paper. Place it on ΔXYZ observing the correspondence P $\leftrightarrow$ X, Q $\leftrightarrow$ Y, R $\leftrightarrow$ Z. You will notice that Δ PQR $\cong$ ΔXYZ.



(3) Two angles and their included side: ASA test

Draw ΔXYZ and ΔDEF such that,
l(XZ) = l(DF), $\angle$X $\cong$ $\angle$D and $\angle$Z $\cong$ $\angle$F.

Copy ΔXYZ on a tracing paper and place it over ΔDEF. Note that Δ XYZ $\cong$ ΔDEF in the correspondence X $\leftrightarrow$ D, Y $\leftrightarrow$ E, Z $\leftrightarrow$ F.

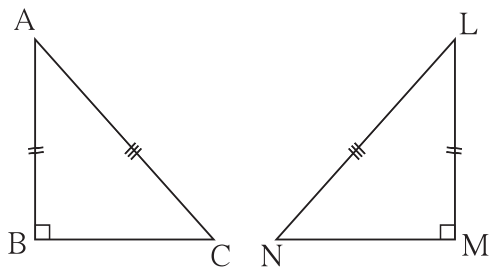


(4) AAS (or SAA) test :

The sum of the measures of angles in a triangle is 180°. Therefore, if two corresponding pairs of angles in two triangles are congruent, then the remaining pair of angles is also congruent. Hence, if two angles and a side adjacent to one of them are congruent with corresponding parts of the other triangle then the condition for ASA test is fulfilled. So the triangles are congruent.

(5) Hypotenuse side test for right angled triangles : (Hypotenuse-side test)

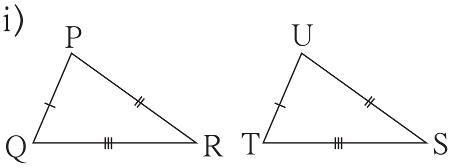
There is a unique right angled triangle of a given side and hypotenuse. Draw two right angled triangles such that a side and the hypotenuse of one is congruent with the corresponding parts of the other. Verify that they are congruent by the method given above.



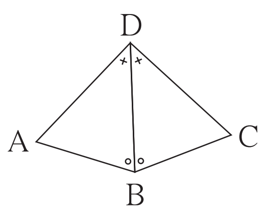
Solved Examples

Ex. (1) In the given figures parts of triangles bearing identical marks are congruent. State the test and the one to one correspondence of vertices by which the triangles in each pair are congruent.

(i)



(ii)

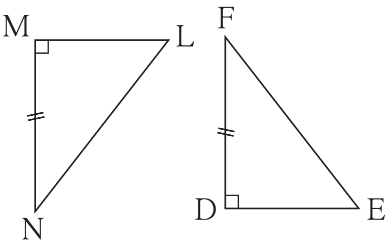


Solution: (i) By SSS test, in the correspondence PQR ↔ UTS

(ii) By ASA test, in the correspondence DBA ↔ DBC

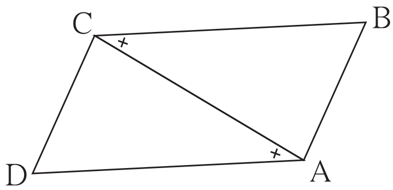
Ex. (2) In each pair of triangles in the following figures, parts indicated by indentical marks are congruent. A test of congruence of triangles is given below each figure. State the additional information which is necessary to show that the triangles are congruent by the given test. With this additional information, state the one to one correspondence in which the triangles will be congruent.

(i)



Hypotenuse - side test

(ii)



ASA test

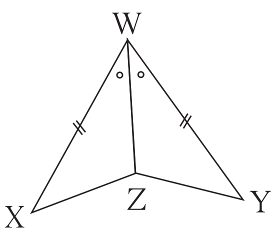
Solution: (i) The given traiangles are right angled. One of the sides of a trangle is congruent with a side of the other triangle. So the additional information necessary is hypotenuse LN ≅ hypotenuse EF. With this information the triangles will be congruent in the correspondence LMN ↔ EDF.

(ii) The side CA of the two triangles is common. So the additional necessary information is ∠ DCA ≅ ∠ BAC. With this information, the triangles will be congruent in the correspondence DCA ↔ BAC.

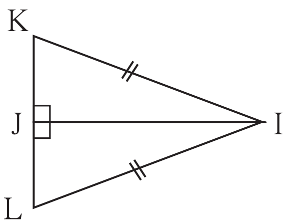
Practice Set 13.1

1. In each pair of triangles in the following figures, parts bearing identical marks are congruent. State the test and correspondence of vertices by which triangles in each pair are congruent .

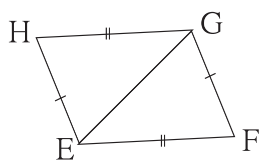
(i)



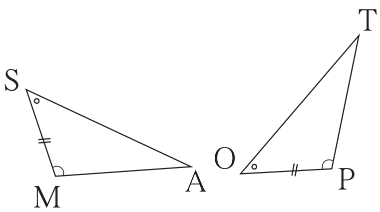
(ii)



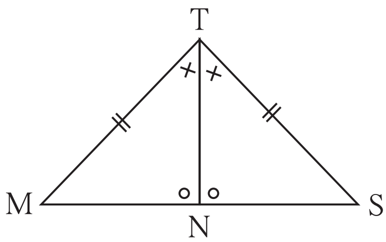
(iii)



(iv)



(v)



Now I know.

(1) S-A-S test : If two sides and the included angle of a triangle are congruent with two corresponding sides and the included angle of the other triangle then the triangles are congruent with each other.

(2) S-S-S test : If three sides of a triangle are congruent with three corresponding sides of the other triangle, then the two triangles are congruent.

(3) A-S-A test : If two angles of a triangle and a side included by them are congruent with two corresponding angles and the side included by them of the other triangle, then the triangles are congruent with each other.

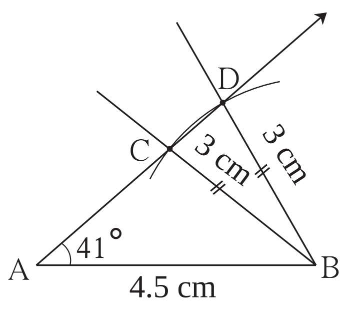
(4) A-A-S test : If two angles of a triangle and a side not included by them are congruent with corresponding angles and a corresponding side not included by them of the other triangle then the triangles are congruent with each other.

(5) **Hypotenuse - side test** : If the hypotenuse and a side of a right angled triangle are congruent with the hypotenuse and the corresponding side of the other right angled triangle, then the two triangles are congruent with each other.

For more information

If two sides and an angle not included by them are congruent with corresponding parts of the other triangle, will the two triangles be congruent ?

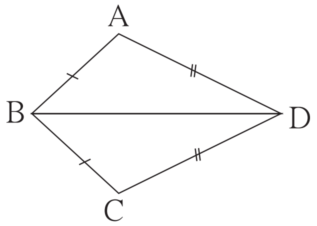
See the adjoining figure. In Δ ABC and Δ ABD, side AB is common, side BC ≅ side BD ∠ A is common, but it is not included by those sides. That is, three parts of a triangle are congruent with three corresponding parts of the other, but the two triangles are not congruent.



Therefore, if two sides and an angle not included by them are congruent with corresponding parts of the other, then the two triangles are not necessarily congruent.

Solved Example

Ex. (1) In the adjacent figure, congruent sides of □ ABCD are shown by identical marks. State if there are any pairs of congruent angles in the figure.



Solution: In Δ ABD and Δ CBD,

side AB ≅ side CB ..... (given)

side DA ≅ side DC ..... (given)

side BD is common

∴ Δ ABD ≅ Δ CBD .... (S-S-S test)

∴ ∠ ABD ≅ ∠ CBD

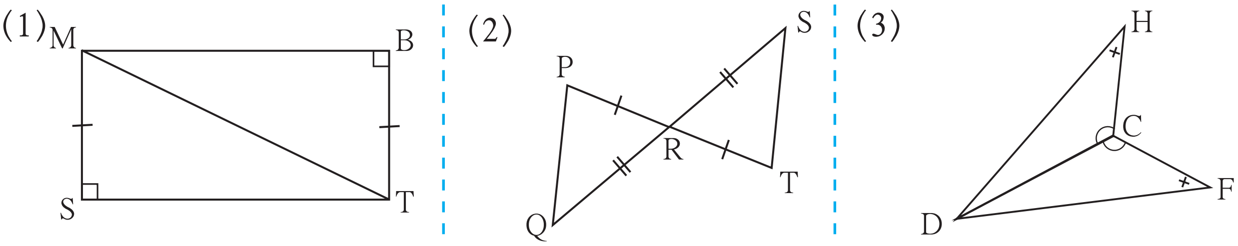
∠ ADB ≅ ∠ CDB

∠ BAD ≅ ∠ BCD

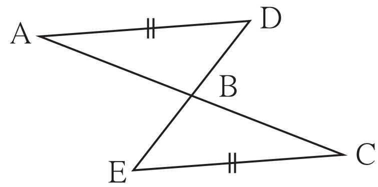
} ....(corresponding angles of congruent triangles)

Practice Set 13.2

1. In each pair of triangles given below, parts shown by identical marks are congruent. State the test and the one to one correspondence of vertices by which triangles in each pair are congruent and remaining congruent parts.



2*. In the adjacent figure, seg AD ≅ seg EC Which additional information is needed to show that Δ ABD and Δ EBC will be congruent by A-A-S test ?



Answers

Practice Set 13.1 **1.** (i) SAS, XWZ ↔ YWZ (ii) Hypotenuse - side test, KJI ↔ LJI (iii) SSS, HEG ↔ FGE (iv) ASA, SMA ↔ OPT (v) ASA or SAA, MTN ↔ STN

Practice Set 13.2

1. (1) Δ MST ≅ Δ TBM - Hypotenuse - side, side ST ≅ side MB, ∠ SMT ≅ ∠ BTM, ∠ STM ≅ ∠ BMT (2) Δ PRQ ≅ Δ TRS - SAS, side PQ ≅ side TS, ∠ RPQ ≅ ∠ RTS, ∠ PQR ≅ ∠ TSR (3) Δ DCH ≅ Δ DCF - SAA, ∠ DHC ≅ ∠ DFC, side HC ≅ side FC

2. ∠ ADB ≅ ∠ CEB and ∠ ABD ≅ ∠ CBE ; or ∠ DAB ≅ ∠ ECB and ∠ ABD ≅ ∠ CBE