

12

Equations in one variable



Let's recall.

In the previous standards we have studied equations in one variable.

- The value of the variable which satisfies the given equation is called the solution of the equation.
- Solving an equation means to find the solution of the equation.
- The equation obtained by performing the same operation on its both sides does not change its solution. Using this property, we obtain new but simpler equations and solve the given equation.

The operations done on both sides of an equation are -

- To add the same number to both sides.
- To subtract the same number from both sides.
- To multiply both sides by the same number.
- To divide both sides by the same non zero number.

Fill in the boxes to solve the following equations.

Ex. (1) $x + 4 = 9$

$$\therefore x + 4 - \boxed{} = 9 - \boxed{}$$

$$\therefore x = \boxed{}$$

Ex. (3) $\frac{x}{3} = 4$

$$\therefore \frac{x}{3} \times \boxed{} = 4 \times \boxed{}$$

$$\therefore x = \boxed{}$$

Ex. (2) $x - 2 = 7$

$$\therefore x - 2 + \boxed{} = 7 + \boxed{}$$

$$\therefore x = \boxed{}$$

Ex. (4) $4x = 24$

$$\therefore \frac{4x}{\boxed{}} = \frac{24}{\boxed{}}$$

$$\therefore x = \boxed{}$$



Let's learn.

Solution of equations in one variable

While solving an equation, sometimes we have to perform many operations on it. We will learn how to find solutions of such equations. Study the following examples.

Ex. (1) Solve the following equations.

(i) $2(x - 3) = \frac{3}{5}(x + 4)$

Solution: Multiplying both sides by 5

$$10(x - 3) = 3(x + 4)$$

$$\therefore 10x - 30 = 3x + 12$$

Adding 30 to both sides

$$\therefore 10x - 30 + 30 = 3x + 12 + 30$$

$$10x = 3x + 42$$

Subtracting $3x$ from both sides

$$\therefore 10x - 3x = 3x + 42 - 3x$$

$$\therefore 7x = 42$$

Dividing both sides by 7

$$\frac{7x}{7} = \frac{42}{7}$$

$$\therefore x = 6$$

(ii) $9x - 4 = 6x + 29$

Solution: Adding 4 to both sides

$$9x - 4 + 4 = 6x + 29 + 4$$

$$\therefore 9x = 6x + 33$$

Subtracting $6x$ from both sides

$$\therefore 9x - 6x = 6x + 33 - 6x$$

$$\therefore 3x = 33$$

Dividing both sides by 3

$$\therefore \frac{3x}{3} = \frac{33}{3}$$

$$\therefore x = 11$$

(iii) $\frac{2}{3} + 5a = 4$

Solution: Method I

$$\frac{2}{3} + 5a = 4$$

Multiplying each term by 3

$$3 \times \frac{2}{3} + 3 \times 5a = 4 \times 3$$

$$\therefore 2 + 15a = 12$$

$$\therefore 15a = 12 - 2$$

$$\therefore 15a = 10$$

$$\therefore a = \frac{10}{15}$$

$$\therefore a = \frac{2}{3}$$

Method II

Subtracting $\frac{2}{3}$ from both the sides,

$$\frac{2}{3} + 5a - \frac{2}{3} = 4 - \frac{2}{3}$$

$$\therefore 5a = \frac{12-2}{3}$$

$$\therefore 5a = \frac{10}{3}$$

Dividing both sides by 5

$$\frac{5a}{5} = \frac{10}{3} \times \frac{1}{5}$$

$$\therefore a = \frac{2}{3}$$

If A, B, C, D are nonzero expressions such that $\frac{A}{B} = \frac{C}{D}$ then multiplying both sides by $B \times D$ we get the equation $AD = BC$. Using this we will solve examples.

$$(iv) \quad \frac{(x-7)}{(x-2)} = \frac{5}{4}$$

Solution: $\frac{(x-7)}{(x-2)} = \frac{5}{4}$

$$\therefore 4(x-7) = 5(x-2)$$

$$\therefore 4x - 28 = 5x - 10$$

$$\therefore 4x - 5x = -10 + 28$$

$$\therefore -x = 18 \quad \therefore x = -18$$

$$(v) \quad \frac{8m-1}{2m+3} = 2$$

Solution: $\frac{8m-1}{2m+3} = \frac{2}{1}$

$$1(8m-1) = 2(2m+3)$$

$$\therefore 8m - 1 = 4m + 6$$

$$\therefore 8m - 4m = 6 + 1$$

$$\therefore 4m = 7 \quad \therefore m = \frac{7}{4}$$

Practice Set 12.1

1. Each equation is followed by the values of the variable. Decide whether these values are the solutions of that equation.

(1) $x - 4 = 3$, $x = -1, 7, -7$

(2) $9m = 81$, $m = 3, 9, -3$

(3) $2a + 4 = 0$, $a = 2, -2, 1$

(4) $3 - y = 4$, $y = -1, 1, 2$

2. Solve the following equations.

(1) $17p - 2 = 49$

(2) $2m + 7 = 9$

(3) $3x + 12 = 2x - 4$

(4) $5(x - 3) = 3(x + 2)$

(5) $\frac{9x}{8} + 1 = 10$

(6) $\frac{y}{7} + \frac{y-4}{3} = 2$

(7) $13x - 5 = \frac{3}{2}$

(8) $3(y + 8) = 10(y - 4) + 8$

(9) $\frac{x-9}{x-5} = \frac{5}{7}$

(10) $\frac{y-4}{3} + 3y = 4$

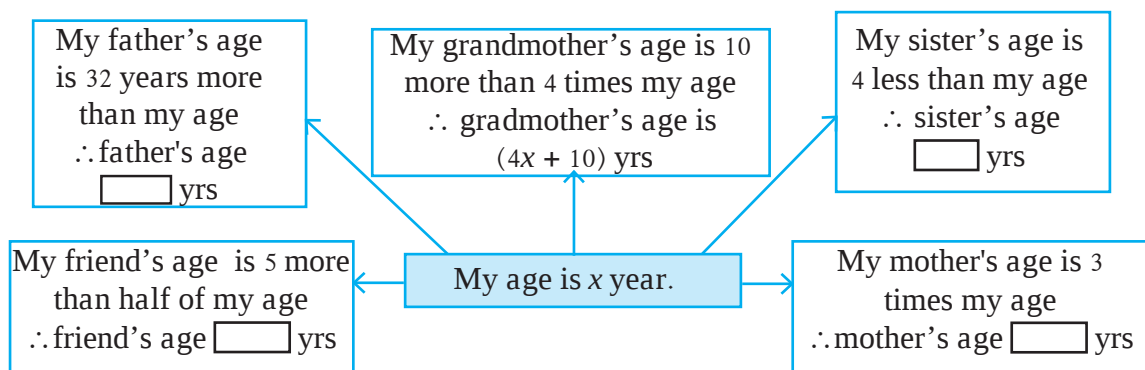
(11) $\frac{b+(b+1)+(b+2)}{4} = 21$



Let's learn.

Word Problems

Let's learn how the information given in the word problem can be converted into an algebraic expression using variable.



From the above information find my age if my friend's age is 12 years.

$$\text{My age} = x \text{ years} \quad \therefore \text{friend's age} = \frac{x}{2} + 5$$

$$\frac{x}{2} + 5 = 12 \quad \dots\dots (\text{Given})$$

$$\therefore x + 10 = 24 \quad \dots\dots (\text{Multiplying each term by 2})$$

$$\therefore x = 24 - 10$$

$$\therefore x = 14,$$

$\therefore$ My age is 14 years.

Find the ages of other persons from the above information.

Activity : Write correct numbers in the boxes given.

length is 3 times the breadth

I am a rectangle.
My perimeter is
40 cm.

breadth
 x

Perimeter of the rectangle = 40

$$2(\square x + \square x) = 40$$

$$2 \times \square x = 40$$

$$\square x = 40$$

$$x = \square$$

$\therefore$ Breadth of rectangle = $\square$ cm and Length of rectangle = $\square$ cm

Solved Examples

Ex. (1) Joseph's weight is two times the weight of his younger brother. Find Joseph's weight if sum of their weights is 63 kg

Solution: Let the weight of Joseph's younger brother be x kg

$\therefore$ Joseph's weight is two times the weight of his younger brother = $2x$

$\therefore$ from the given information $x + 2x = 63$

$$\therefore 3x = 63 \quad \therefore x = 21$$

$$\therefore \text{Joseph's weight} = 2x = 2 \times 21 = 42 \text{ kg}$$

Ex. (2) Numerator of a fraction is 5 more than its denominator. If 4 is added to numerator and denominator, the fraction obtained is $\frac{6}{5}$. Find the fraction.

Solution: Let the demonator of the fraction be x .

$\therefore$ Numerator of the fraction is 5 more than denominator means $x + 5$.

$\therefore$ The fraction is $\frac{x+5}{x}$.

If 4 is added to both numerator and denominator, fraction obtained is $\frac{6}{5}$

$$\therefore \frac{x+5+4}{x+4} = \frac{6}{5}$$

$$\therefore \frac{x+9}{x+4} = \frac{6}{5}$$

$$\therefore 5(x+9) = 6(x+4)$$

$$\therefore 5x + 45 = 6x + 24$$

$$\therefore 45 - 24 = 6x - 5x$$

$$\therefore 21 = x$$

$\therefore$ denominator of fraction = 21,
and numerator = $21 + 5 = 26$

$$\therefore \text{the fraction is} = \frac{26}{21}$$

Ex. (3) Ratna has ₹200 more than three times the amount Rafik has. If ₹ 300 from the amount with Ratna are given to Rafik, amount with Ratna will be $\frac{7}{4}$ times the amount with Rafik. Find the initial amount with Rafik.

To find the initial amount, complete the following activity.

Solution: The amount with Ratna is ₹ 200 more than three times the amount with Rafik.

Let the initial amount with Rafik be ₹ x $\therefore$ Ratna has ₹

$\therefore$ ₹ 300 from Ratna are given to Rafik.

$\therefore$ Remaining amount with Ratna is .

$\therefore$ Now Rafik has ₹ $(x + 300)$

Now the amount with Ratna is $\frac{7}{4}$ times the amount with Rafik

$$\therefore \frac{\text{Amount with Ratna}}{\text{Amount with Rafik}} = \frac{\text{}}{\text{}}$$

$$\therefore \frac{3x-100}{x+300} = \frac{\text{}}{\text{}}$$

$$\therefore 4 \text{ } = 7 \text{ }$$

$$\therefore 12x - 400 = 7x + 2100$$

$$\therefore 12x - 7x = \text{}$$

$$\therefore 5x = \text{}$$

$$\therefore x = \text{}$$

$\therefore$ Rafik initially has ₹ with him.

Practice Set 12.2

- Mother is 25 year older than her son. Find son's age if after 8 years ratio of son's age to mother's age will be $\frac{4}{9}$.
- The denominator of a fraction is greater than its numerator by 12. If the numerator is decreased by 2 and the denominator is increased by 7, the new fraction is equivalent with $\frac{1}{2}$. Find the fraction.

3. The ratio of weights of copper and zinc in brass is 13:7. Find the weight of zinc in a brass utensil weighing 700 gm.
- 4*. Find three consecutive whole numbers whose sum is more than 45 but less than 54.
5. In a two digit number, digit at the ten's place is twice the digit at units's place. If the number obtained by interchanging the digits is added to the original number, the sum is 66. Find the number.
- 6*. Some tickets of ₹ 200 and some of ₹ 100, of a drama in a theatre were sold. The number of tickets of ₹ 200 sold was 20 more than the number of tickets of ₹ 100. The total amount received by the theatre by sale of tickets was ₹ 37000. Find the number of ₹ 100 tickets sold.
7. Of the three consecutive natural numbers, five times the smallest number is 9 more than four times the greatest number, find the numbers.
8. Raju sold a bicycle to Amit at 8% profit. Amit repaired it spending ₹ 54. Then he sold the bicycle to Nikhil for ₹ 1134 with no loss and no profit. Find the cost price of the bicycle for which Raju purchased it.
9. A Cricket player scored 180 runs in the first match and 257 runs in the second match. Find the number of runs he should score in the third match so that the average of runs in the three matches be 230.
10. Sudhir's present age is 5 more than three times the age of Viru. Anil's age is half the age of Sudhir. If the ratio of the sum of Sudhir's and Viru's age to three times Anil's age is 5:6, then find Viru's age.



Answers

Practice Set 12.1 1. Values which are solutions. (1) $x = 7$ (2) $m = 9$ (3) $a = -2$
 (4) $y = -1$ 2. (1) $p = 3$ (2) $m = 1$ (3) $x = -16$ (4) $x = \frac{21}{2}$ (5) $x = 8$ (6) $y = 7$
 (7) $x = \frac{1}{2}$ (8) $y = 8$ (9) $x = 19$ (10) $y = \frac{8}{5}$ (11) $b = 27$

Practice Set 12.2 1. 12 years 2. $\frac{23}{35}$ 3. 245 gm
 4. 15, 16, 17 or 16, 17, 18 5. 42 6. 110
 7. 17, 18, 19 8. ₹ 1000 9. 253 10. 5 years

