



- **Parallelogram**
- **Tests of parallelogram**
- **Rhombus**
- **Rectangles**
- **Square**
- **Trapezium**
- **Mid point theorem**



1.



Pairs of adjacent angles :

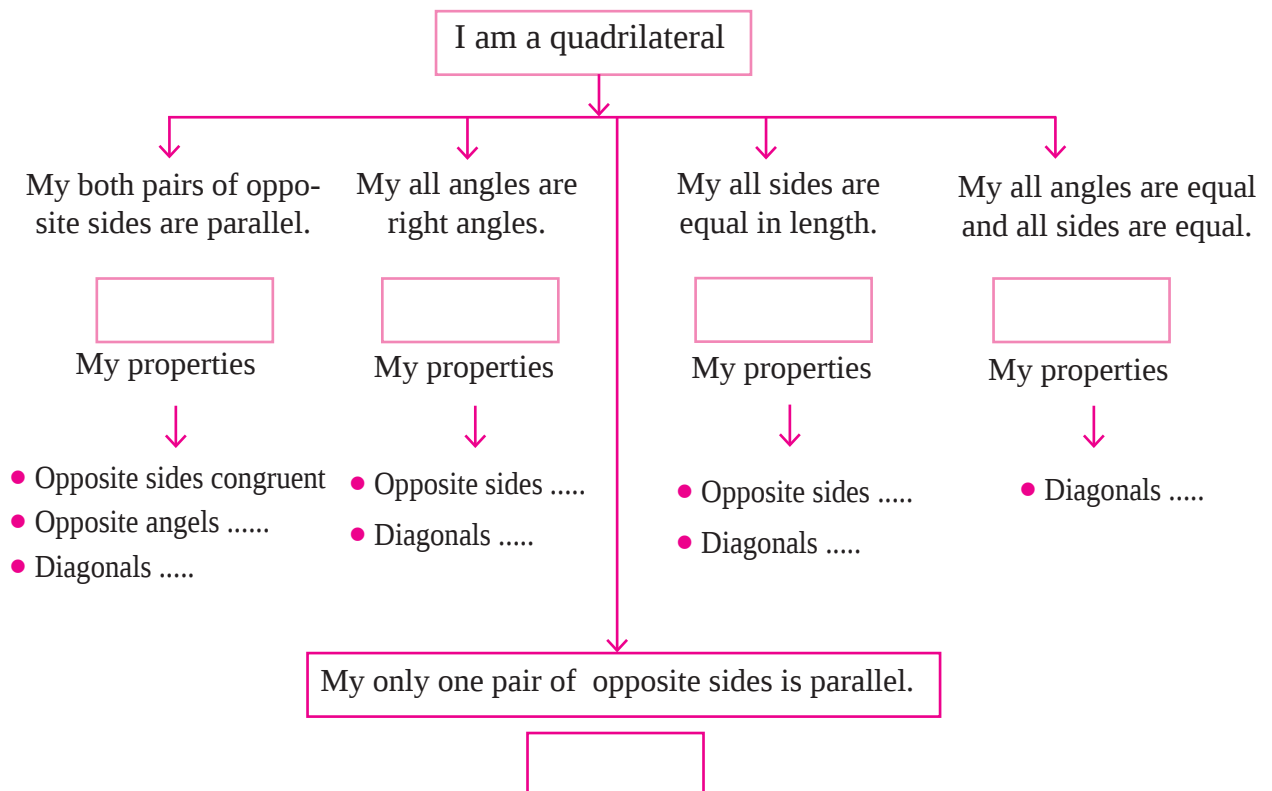
(1) ... , ... (2) ... , ...
(3) ... , ... (4) ... , ...

Pairs of opposite sides (1) , (2) ,

Pairs of opposite angles (1) , (2) ,

Fig. 5.1

Let's recall types of quadrilaterals and their properties .



You know different types of quadrilaterals and their properties. You have learned then through different activities like measuring sides and angles, by paper folding method etc. Now we will study these properties by giving their logical proofs.

A property proved logically is called a proof.

In this chapter you will learn that how a rectangle, a rhombus and a square are parallelograms. Let us start our study from parallelogram.



Let's learn.

Parallelogram

A quadrilateral having both pairs of opposite sides parallel is called a parallelogram.

For proving the theorems or for solving the problems we need to draw figure of a parallelogram frequently. Let us see how to draw a parallelogram.

Suppose we have to draw a parallelogram $\square ABCD$.

Method I :

- Let us draw seg AB and seg BC of any length and making an angle of any measure with each other.
- Now we want seg AD and seg BC parallel to each other. So draw a line parallel to seg BC through the point A.
- Similarly we will draw line parallel to AB through the point C. These lines will intersect in point D.

So constructed quadrilateral ABCD will be a parallelogram.

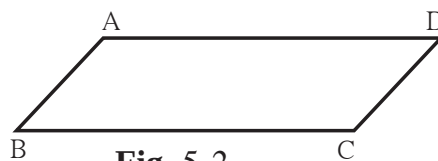


Fig. 5.2

Method II :

- Let us draw seg AB and seg BC of any length and making angle of any measure between them.
- Draw an arc with compass with centre A and radius BC.
- Similarly draw an arc with centre C and radius AB intersecting the arc previously drawn.
- Name the point of intersection of two arcs as D.

Draw seg AD and seg CD.

Quadrilateral so formed is a parallelogram ABCD

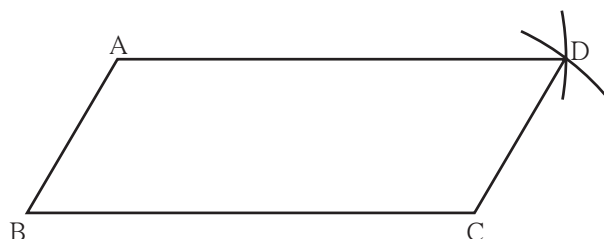


Fig. 5.3

In the second method we have actually drawn $\square ABCD$ in which opposite sides are equal. We will prove that a quadrilateral whose opposite sides are equal, is a parallelogram.

Activity I Draw five parallelograms by taking various measures of lengths and angles.

For the proving theorems on parallelogram, we use congruent triangles. To understand how they are used, let's do the following activity.

Activity II

- Draw a parallelogram $ABCD$ on a card sheet. Draw diagonal AC . Write the names of vertices inside the triangle as shown in the figure. Then cut it out.

- Fold the quadrilateral on the diagonal AC and see whether $\triangle ADC$ and $\triangle CBA$ match with each other or not.

- Cut $\square ABCD$ along diagonals AC and separate $\triangle ADC$ and $\triangle CBA$. By rotating and flipping $\triangle CBA$, check whether it matches exactly with $\triangle ADC$. What did you find ? Which sides of $\triangle CBA$ match with which sides of $\triangle ADC$? Which angles of $\triangle CBD$ match with which angles of $\triangle ADC$?

Side DC matches with side AB and side AD matches with side CB . Similarly $\angle B$ matches with $\angle D$.

So we can see that opposite sides and angles of a parallelogram are congruent.

We will prove these properties of a parallelogram.

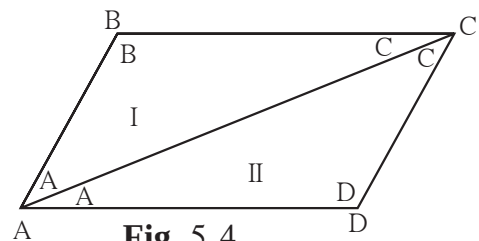


Fig. 5.4

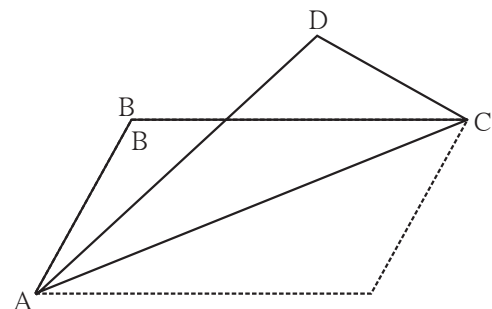


Fig. 5.5

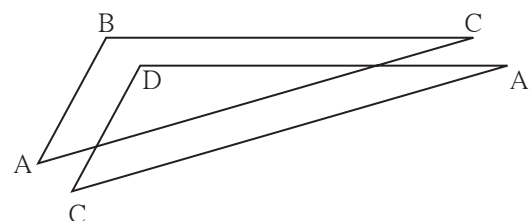


Fig. 5.6

Theorem 1. Opposite sides and opposite angles of a parallelogram are congruent.

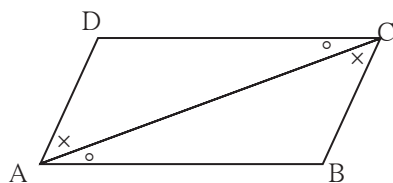


Fig. 5.7

Given : $\square ABCD$ is a parallelogram.

It means side $AB \parallel$ side DC ,

side $AD \parallel$ side BC .

To prove : $\text{seg } AD \cong \text{seg } BC$; $\text{seg } DC \cong \text{seg } AB$
 $\angle ADC \cong \angle CBA$, and $\angle DAB \cong \angle BCD$.

Construction : Draw diagonal AC .

Proof : $\text{seg } DC \parallel \text{seg } AB$ and diagonal AC is a transversal.

$\therefore \angle DCA \cong \angle BAC$ (1)

and $\angle DAC \cong \angle BCA$ (2) }alternate angles

Now , in $\triangle ADC$ and $\triangle CBA$,

$\angle DAC \cong \angle BCA$ from (2)

$\angle DCA \cong \angle BAC$ from (1)

$\text{seg } AC \cong \text{seg } CA$ common side

$\therefore \triangle ADC \cong \triangle CBA$ ASA test

$\therefore \text{side } AD \cong \text{side } CB$ c.s.c.t.

and $\text{side } DC \cong \text{side } AB$ c.s.c.t.,

Also, $\angle ADC \cong \angle CBA$ c.a.c.t.

Similarly we can prove $\angle DAB \cong \angle BCD$.



Use your brain power!

In the above theorem, to prove $\angle DAB \cong \angle BCD$, is any change in the construction needed ? If so, how will you write the proof making the change ?

To know one more property of a parallelogram let us do the following activity.

Activity : Draw a parallelogram $PQRS$. Draw diagonals PR and QS . Denote the intersection of diagonals by letter O . Compare the two parts of each diagonal with a divider. What do you find ?

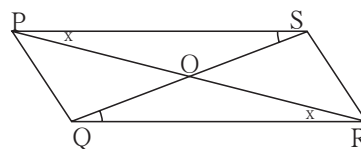


Fig. 5.8



Theorem : Diagonals of a parallelogram bisect each other.

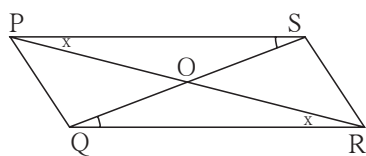


Fig. 5.9

Given : $\square PQRS$ is a parallelogram. Diagonals PR and QS intersect in point O.

To prove : $\text{seg } PO \cong \text{seg } RO$,
 $\text{seg } SO \cong \text{seg } QO$.

Proof : In $\triangle POS$ and $\triangle ROQ$

$\angle OPS \cong \angle ORQ$ alternate angles

side PS $\cong$ side RQ opposite sides of parallelogram

$\angle PSO \cong \angle RQO$ alternate angles

$\therefore \triangle POS \cong \triangle ROQ$ ASA test

$\therefore \text{seg } PO \cong \text{seg } RO$

and $\text{seg } SO \cong \text{seg } QO$ } corresponding sides of congruent triangles



Remember this !

- Adjacent angles of a parallelogram are supplementary.
- Opposite sides of a parallelogram are congruent.
- Opposite angles of a parallelogram are congruent.
- Diagonals of a parallelogram bisect each other.

Solved Examples

Ex (1) $\square PQRS$ is a parallelogram. $PQ = 3.5$, $PS = 5.3$ $\angle Q = 50^\circ$ then find the lengths of remaining sides and measures of remaining angles.

Solution : $\square PQRS$ is a parallelogram.

$\therefore \angle Q + \angle P = 180^\circ$ interior angles are supplementary.

$\therefore 50^\circ + \angle P = 180^\circ$

$\therefore \angle P = 180^\circ - 50^\circ = 130^\circ$

Now , $\angle P = \angle R$ and $\angle Q = \angle S$ opposite angles of a parallelogram.

$\therefore \angle R = 130^\circ$ and $\angle S = 50^\circ$

Similarly, $PS = QR$ and $PQ = SR$ opposite sides of a parallelogram.

$\therefore QR = 5.3$ and $SR = 3.5$

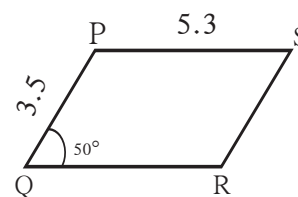


Fig. 5.10

Ex (2) □ABCD is a parallelogram. If $\angle A = (4x + 13)^\circ$ and $\angle D = (5x - 22)^\circ$ then find the measures of $\angle B$ and $\angle C$.

Solution : Adjacent angles of a parallelogram are supplementary.

$\angle A$ and $\angle D$ are adjacent angles.

$$\therefore (4x + 13)^\circ + (5x - 22)^\circ = 180$$

$$\therefore 9x - 9 = 180$$

$$\therefore 9x = 189$$

$$\therefore x = 21$$

$$\therefore \angle A = 4x + 13 = 4 \times 21 + 13 = 84 + 13 = 97^\circ$$

$$\angle D = 5x - 22 = 5 \times 21 - 22 = 105 - 22 = 83^\circ$$

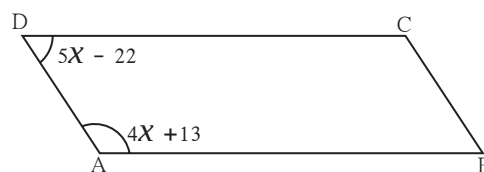


Fig. 5.11

$$\therefore \angle C = 97^\circ$$

$$\therefore \angle B = 83^\circ$$

Practice set 5.1

1. Diagonals of a parallelogram WXYZ intersect each other at point O. If $\angle XYZ = 135^\circ$ then what is the measure of $\angle XWZ$ and $\angle YZW$?
If $l(OY) = 5$ cm then $l(WY) = ?$
2. In a parallelogram ABCD, If $\angle A = (3x + 12)^\circ$, $\angle B = (2x - 32)^\circ$ then find the value of x and then find the measures of $\angle C$ and $\angle D$.
3. Perimeter of a parallelogram is 150 cm. One of its sides is greater than the other side by 25 cm. Find the lengths of all sides.
4. If the ratio of measures of two adjacent angles of a parallelogram is 1 : 2, find the measures of all angles of the parallelogram.
- 5*. Diagonals of a parallelogram intersect each other at point O. If $AO = 5$, $BO = 12$ and $AB = 13$ then show that $\square ABCD$ is a rhombus.

- 6.** In the figure 5.12, $\square PQRS$ and $\square ABCR$ are two parallelograms.

If $\angle P = 110^\circ$ then find the measures of all angles of $\square ABCR$.

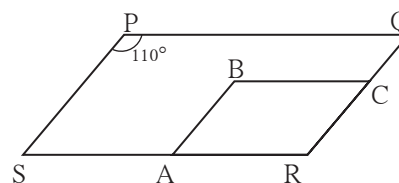


Fig. 5.12

7. In figure 5.13 $\square ABCD$ is a parallelogram. Point E is on the ray AB such that $BE = AB$ then prove that line ED bisects seg BC at point F.

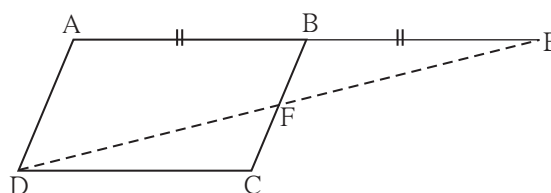


Fig. 5.13



Let's recall.

Tests for parallel lines

1. If a transversal intersects two lines and a pair of corresponding angles is congruent then those lines are parallel.
2. If a transversal intersects two lines and a pair of alternate angles is congruent then those two lines are parallel.
3. If a transversal intersects two lines and a pair of interior angles is supplementary then those two lines are parallel.



Let's learn.

Tests for parallelogram

Suppose, in $\square PQRS$, $PS = QR$ and $PQ = SR$ and we have to prove that $\square PQRS$ is a parallelogram. To prove it, which pairs of sides of $\square PQRS$ should be shown parallel?

Which test can we use to show the sides parallel? Which line will be convenient as a transversal to obtain the angles necessary to apply the test?

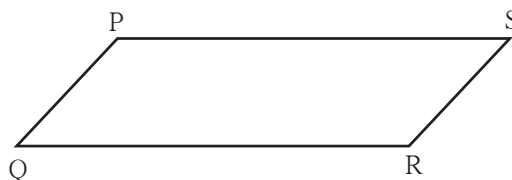


Fig. 5.14

Theorem : If pairs of opposite sides of a quadrilateral are congruent then that quadrilateral is a parallelogram.

Given : In $\square PQRS$
 side $PS \cong$ side QR
 side $PQ \cong$ side SR

To prove : $\square PQRS$ is a parallelogram.

Construction : Draw diagonal PR .

Proof : In $\triangle SPR$ and $\triangle QRP$
 side $PS \cong$ side QR given
 side $SR \cong$ side QP given
 side $PR \cong$ side RP common side
 $\therefore \triangle SPR \cong \triangle QRP$ sss test
 $\therefore \angle SPR \cong \angle QRP$ c.a.c.t.
 Similarly, $\angle PRS \cong \angle RPQ$ c.a.c.t.

$\angle SPR$ and $\angle QRP$ are alternate angles formed by the transversal PR of seg PS and seg QR .

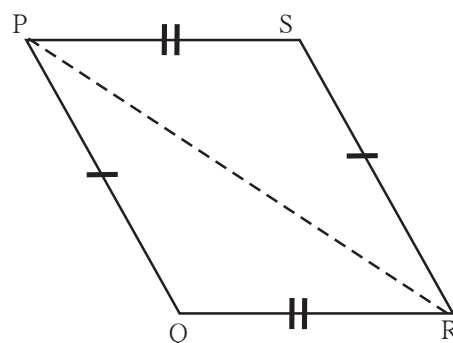


Fig. 5.15

$\therefore$ side PS $\parallel$ side QR(I) alternate angles test for parallel lines.

Similarly $\angle PRS$ and $\angle RPQ$ are the alternate angles formed by transversal PR of seg PQ and seg SR.

$\therefore$ side PQ $\parallel$ side SR(II)alternate angle test

$\therefore$ from (I) and (II) $\square PQRS$ is a parallelogram.

On page 56, two methods to draw a parallelogram are given. In the second method actually we have drawn a quadrilateral of which opposite sides are equal. Did you now understand why such a quadrilateral is a parallelogram ?

Theorem : If both the pairs of opposite angles of a quadrilateral are congruent then it is a parallelogram.

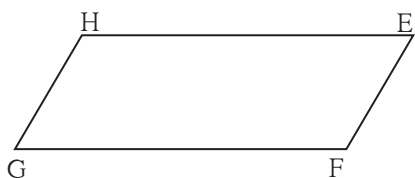


Fig. 5.16

Given : In $\square EFGH$ $\angle E \cong \angle G$
and $\angle \dots \cong \angle \dots$

To prove : $\square EFGH$ is a

Proof : Let $\angle E = \angle G = x$ and $\angle H = \angle F = y$

Sum of all angles of a quadrilateral is

$\therefore \angle E + \angle G + \angle H + \angle F = \dots$

$\therefore x + y + \dots + \dots = \dots$

$\therefore \square x + \square y = \dots$

$\therefore x + y = 180^\circ$

$\therefore \angle G + \angle H = \dots$

$\angle G$ and $\angle H$ are interior angles formed by transversal HG of seg HE and seg GF.

$\therefore$ side HE $\parallel$ side GF (I) interior angle test for parallel lines.

Similarly, $\angle G + \angle F = \dots$

$\therefore$ side $\parallel$ side (II) interior angle test for parallel lines.

$\therefore$ From (I) and (II), $\square EFGH$ is a

Solved examples -

Ex (1) □PQRS is parallelogram. M is the midpoint of side PQ and N is the mid point of side RS. Prove that □PMNS and □MQRN are parallelograms.

Given : □ PQRS is a parallelogram.
M and N are the midpoints of
side PQ and side RS respectively.

To prove : □PMNS is a parallelogram.

□MQRN is a parallelogram.

Proof : side PQ || side SR

$$\therefore \text{side PM} \parallel \text{side SN} \dots\dots (\because \text{P-M-Q; S-N-R}) \dots\dots(\text{I})$$

side PQ $\cong$ side SR.

$$\therefore \frac{1}{2} \text{side PQ} = \frac{1}{2} \text{side SR}$$

$\therefore$ side PM $\cong$ side SN ($\because$ M and N are midpoints.).....(II)

$\therefore$ From (I) and (II), $\square PMNQ$ is a parallelogram,

Similarly, we can prove that $\square\text{MQRN}$ is parallelogram.

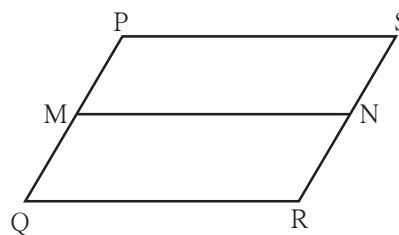


Fig. 5.19

Ex (2) Points D and E are the midpoints of side AB and side AC of $\triangle ABC$ respectively.

Point F is on ray ED such that $ED = DF$. Prove that $\square AFBE$ is a parallelogram.

For this example write ‘given’ and ‘to prove’ and complete the proof given below.

Given : -----

To prove : -----

Proof : seg AB and seg EF are □ of $\square$ AFBE.

$$\text{seg AD} \cong \text{seg DB} \dots\dots$$
$$\text{seg } \boxed{} \cong \text{seg } \boxed{} \dots \text{construction.}$$

$\therefore$ Diagonals of $\square$ AFBE each other

$\therefore \square AFBE$ is a parallelogram ...by □ test.

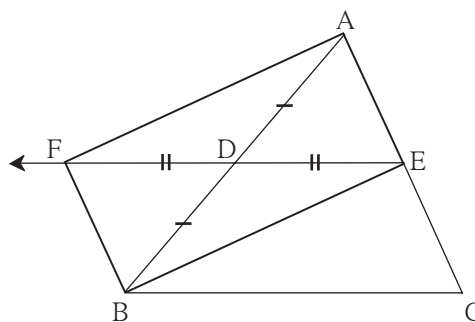


Fig. 5.20

Ex (3) Prove that every rhombus is a parallelogram.

Given : $\square ABCD$ is a rhombus.

To prove : □ABCD is parallelogram.

Proof : $\text{seg AB} \cong \text{seg BC} \cong \text{seg CD} \cong \text{seg DA}$ (given)

$$\therefore \text{side AB} \cong \text{side CD} \quad \text{and} \quad \text{side BC} \cong \text{side AD}$$

$\therefore$ $\square ABCD$ is a parallelogram..... opposite side test for parallelogram

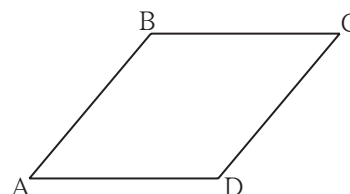


Fig. 5.21

Practice set 5.2

1. In figure 5.22, $\square ABCD$ is a parallelogram, P and Q are midpoints of side AB and DC respectively, then prove $\square APCQ$ is a parallelogram.

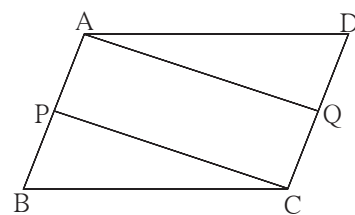


Fig. 5.22

2. Using opposite angles test for parallelogram, prove that every rectangle is a parallelogram.

3. In figure 5.23, G is the point of concurrence of medians of $\triangle DEF$. Take point H on ray DG such that $D-G-H$ and $DG = GH$, then prove that $\square GEHF$ is a parallelogram.

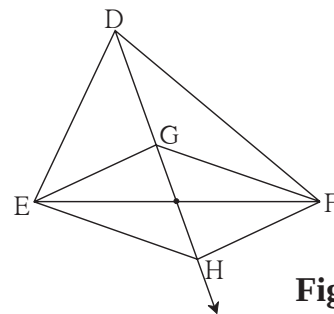


Fig. 5.23

4. Prove that quadrilateral formed by the intersection of angle bisectors of all angles of a parallelogram is a rectangle. (Figure 5.24)

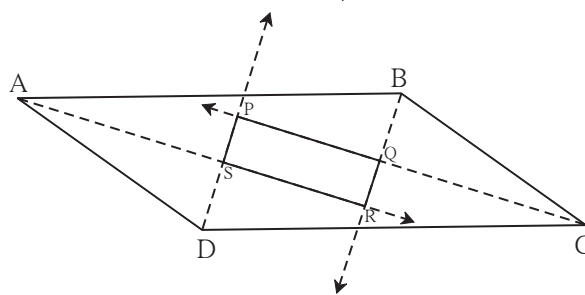


Fig. 5.24

5. In figure 5.25, if points P, Q, R, S are on the sides of parallelogram such that $AP = BQ = CR = DS$ then prove that $\square PQRS$ is a parallelogram.

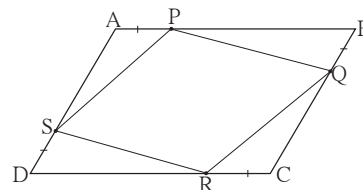


Fig. 5.25



Let's learn.

Properties of rectangle, rhombus and square

Rectangle, rhombus and square are also parallelograms. So the properties that opposite sides are equal, opposite angles are equal and diagonals bisect each other hold good in these types of quadrilaterals also. But there are some more properties of these quadrilaterals.

Proofs of these properties are given in brief. Considering the steps in the given proofs, write the proofs in detail.

Theorem : Diagonals of a rectangle are congruent.

Given : $\square ABCD$ is a rectangle.

To prove : Diagonal $AC \cong$ diagonal BD

Proof : Complete the proof by giving suitable reasons.

$\triangle ADC \cong \triangle DAB$ SAS test

$\therefore$ diagonal $AC \cong$ diagonal BD c.s.c.t.

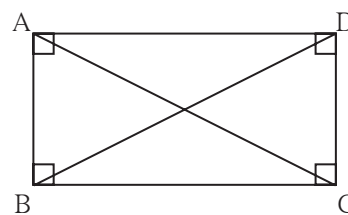


Fig. 5.26

Theorem : Diagonals of a square are congruent.

Write 'Given', 'To prove' and 'proof' of the theorem.

Theorem : Diagonals of a rhombus are perpendicular bisectors of each other.

Given : $\square EFGH$ is a rhombus

To prove : (i) Diagonal EG is the perpendicular bisector of diagonal HF .

(ii) Diagonal HF is the perpendicular bisector of diagonal EG .

Proof : (i) $\left. \begin{array}{l} \text{seg } EF \cong \text{seg } EH \\ \text{seg } GF \cong \text{seg } GH \end{array} \right\} \text{ given}$

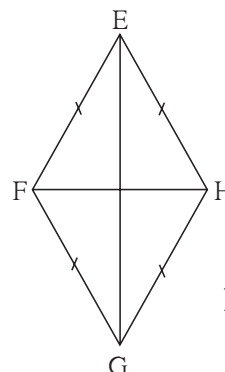


Fig. 5.27

Every point which is equidistant from end points of a segment is on the perpendicular bisector of the segment.

$\therefore$ point E and point G are on the perpendicular bisector of seg HF .

One and only one line passes through two distinct points.

$\therefore$ line EG is the perpendicular bisector of diagonal HF .

$\therefore$ diagonal EG is the perpendicular bisector of diagonal HF .

(ii) Similarly, we can prove that diagonal HF is the perpendicular bisector of EG .

Write the proofs of the following statements.

- Diagonals of a square are perpendicular bisectors of each other.
- Diagonals of a rhombus bisect its opposite angles.
- Diagonals of a square bisect its opposite angles.



Remember this !

- Diagonals of a rectangle are congruent.
- Diagonals of a square are congruent.
- Diagonals of a rhombus are perpendicular bisectors of each other.
- Diagonals of a rhombus bisect the pairs of opposite angles.
- Diagonals of a square are perpendicular bisectors of each other.
- Diagonals of a square bisect opposite angles.



Practice set 5.3

1. Diagonals of a rectangle ABCD intersect at point O. If $AC = 8$ cm then find the length of BO and if $\angle CAD = 35^\circ$ then find the measure of $\angle ACB$.
2. In a rhombus PQRS if $PQ = 7.5$ then find the length of QR.
If $\angle QPS = 75^\circ$ then find the measure of $\angle PQR$ and $\angle SRQ$.
3. Diagonals of a square IJKL intersect at point M, Find the measures of $\angle IMJ$, $\angle JIK$ and $\angle LJK$.
4. Diagonals of a rhombus are 20 cm and 21 cm respectively, then find the side of rhombus and its perimeter.
5. State with reasons whether the following statements are 'true' or 'false'.
 - (i) Every parallelogram is a rhombus.
 - (ii) Every rhombus is a rectangle.
 - (iii) Every rectangle is a parallelogram.
 - (iv) Every square is a rectangle.
 - (v) Every square is a rhombus.
 - (vi) Every parallelogram is a rectangle.



Let's learn.

Trapezium

When only one pair of opposite sides of a quadrilateral is parallel then the quadrilateral is called a trapezium.

In the adjacent figure only side AB and side DC of $\square ABCD$ are parallel to each other. So this is a trapezium. $\angle A$ and $\angle D$ is a pair of adjacent angles and so is the pair of $\angle B$ and $\angle C$. Therefore by property of parallel lines both the pairs are supplementary.

If non-parallel sides of a trapezium are congruent then that quadrilateral is called as an **Isoceles trapezium**.

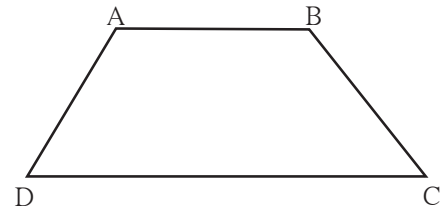


Fig. 5.28

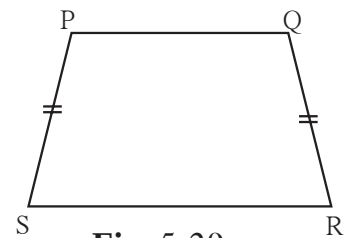


Fig. 5.29

The segment joining the midpoints of non parallel sides of a trapezium is called the median of the trapezium.

Solved examples

Ex (1) Measures of angles of $\square ABCD$ are in the ratio 4 : 5 : 7 : 8. Show that $\square ABCD$ is a trapezium.

Solution : Let measures of $\angle A$, $\angle B$, $\angle C$ and $\angle D$ are $(4x)^\circ$, $(5x)^\circ$, $(7x)^\circ$, and $(8x)^\circ$ respectively.

Sum of all angles of a quadrilateral is 360° .

$$\therefore 4x + 5x + 7x + 8x = 360$$

$$\therefore 24x = 360 \quad \therefore x = 15$$

$$\angle A = 4 \times 15 = 60^\circ, \quad \angle B = 5 \times 15 = 75^\circ, \quad \angle C = 7 \times 15 = 105^\circ,$$

$$\text{and } \angle D = 8 \times 15 = 120^\circ$$

$$\text{Now, } \angle B + \angle C = 75^\circ + 105^\circ = 180^\circ$$

$$\therefore \text{side } CD \parallel \text{side } BA \dots\dots (I)$$

$$\text{But } \angle B + \angle A = 75^\circ + 60^\circ = 135^\circ \neq 180^\circ$$

$$\therefore \text{side } BC \text{ and side } AD \text{ are not parallel } \dots\dots (II)$$

$$\therefore \square ABCD \text{ is a trapezium. } \dots\dots [\text{from (I) and (II)}]$$

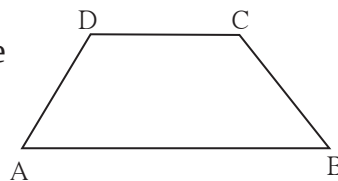


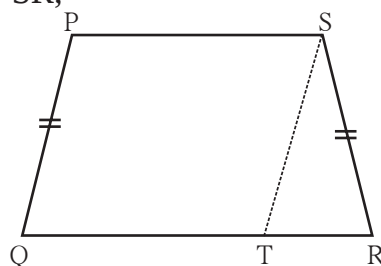
Fig. 5.30

Ex (2) In $\square PQRS$, side $PS \parallel$ side QR and side $PQ \cong$ side SR , side $QR >$ side PS then prove that $\angle PQR \cong \angle SRQ$

Given : In $\square PQRS$, side $PS \parallel$ side QR ,
side $PQ \cong$ side SR and side $QR >$ side PS .

To prove : $\angle PQR \cong \angle SRQ$

Construction : Draw the segment parallel to side PQ through the point S which intersects side QR in T . **Fig. 5.31**



Proof : In $\square PQRS$,

seg $PS \parallel$ seg QT given

seg $PQ \parallel$ seg ST construction

$\therefore \square PQTS$ is a parallelogram

$\therefore \angle PQT \cong \angle STR$ corresponding angles (I)

and seg $PQ \cong$ seg ST opposite sides of parallelogram

But seg $PQ \cong$ seg SR given

$\therefore$ seg $ST \cong$ seg SR

$\therefore \angle STR \cong \angle SRT$isosceles triangle theorem (II)

$\therefore \angle PQT \cong \angle SRT$ [from (I) and (II)]

$\therefore \angle PQR \cong \angle SRQ$

Hence, it is proved that base angles of an isosceles trapezium are congruent.

Practice set 5.4

1. In $\square IJKL$, side $IJ \parallel$ side KL $\angle I = 108^\circ$ $\angle K = 53^\circ$ then find the measures of $\angle J$ and $\angle L$.
2. In $\square ABCD$, side $BC \parallel$ side AD , side $AB \cong$ side DC If $\angle A = 72^\circ$ then find the measures of $\angle B$, and $\angle D$.
3. In $\square ABCD$, side $BC <$ side AD (Figure 5.32)
side $BC \parallel$ side AD and if
side $BA \cong$ side CD
then prove that $\angle ABC \cong \angle DCB$.

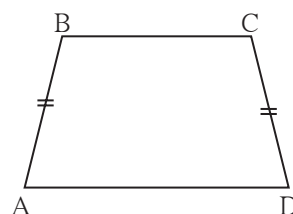


Fig. 5.32



Let's learn.

Theorem of midpoints of two sides of a triangle

Statement : The segment joining midpoints of any two sides of a triangle is parallel to the third side and half of it.

Given : In $\triangle ABC$, point P is the midpoint of seg AB and point Q is the midpoint of seg AC.

To prove : seg PQ $\parallel$ seg BC

$$\text{and } PQ = \frac{1}{2} BC$$

Construction : Produce seg PQ upto R such that PQ = QR
Draw seg RC.

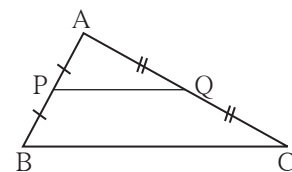


Fig. 5.33

Proof : In $\triangle AQP$ and $\triangle CQR$

seg PQ $\cong$ seg QR construction

seg AQ $\cong$ seg QC given

$\angle AQP \cong \angle CQR$ vertically opposite angles.

$\therefore \triangle AQP \cong \triangle CQR$ SAS test

$\angle PAQ \cong \angle RCQ$ (1) c.a.c.t.

$\therefore$ seg AP $\cong$ seg CR(2) c.s.c.t.

From (1) line AB $\parallel$ line CR.....alternate angle test

from (2) seg AP $\cong$ seg CR

Now, seg AP $\cong$ seg PB $\cong$ seg CR and seg PB $\parallel$ seg CR

$\therefore \square PBCR$ is a parallelogram.

$\therefore$ seg PQ $\parallel$ seg BC and PR = BC opposite sides are congruent

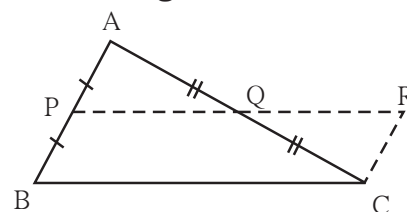


Fig. 5.34

$$PQ = \frac{1}{2} PR \dots\dots (\text{construction})$$

$$\therefore PQ = \frac{1}{2} BC \quad \because PR = BC$$

Converse of midpoint theorem

Theorem : If a line drawn through the midpoint of one side of a triangle and parallel to the other side then it bisects the third side.

For this theorem 'Given', 'To prove', 'construction' is given below. Try to write the proof.

Given : Point D is the midpoint of side AB of $\triangle ABC$. Line l passing through the point D and parallel to side BC intersects side AC in point E.

To prove : $AE = EC$

Construction : Draw a line parallel to seg AB passing through the point C. Name the point of intersection where this line and line l will intersect as F.

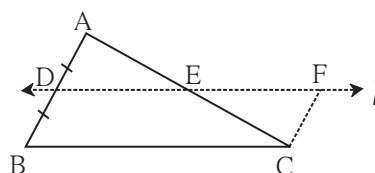


Fig. 5.35

Proof : Use the construction and line $l \parallel \text{seg } BC$ which is given. Prove $\triangle ADE \cong \triangle CFE$ and complete the proof.

Ex (1) Points E and F are mid points of seg AB and seg AC of $\triangle ABC$ respectively. If $EF = 5.6$ then find the length of BC.

Solution : In $\triangle ABC$, point E and F are midpoints of side AB and side AC respectively.

$$EF = \frac{1}{2} BC \dots\dots\text{midpoint theorem}$$

$$5.6 = \frac{1}{2} BC \quad \therefore BC = 5.6 \times 2 = 11.2$$

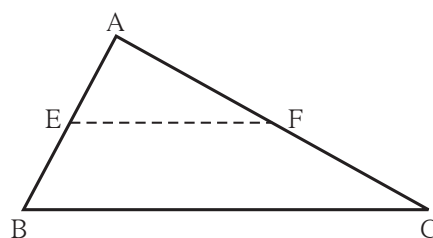


Fig. 5.36

Ex (2) Prove that the quadrilateral formed by joining the midpoints of sides of a quadrilateral in order is a parallelogram.

Given : $\square ABCD$ is a quadrilateral.
P, Q, R, S are midpoints of the sides AB, BC, CD and AD respectively.

To prove : $\square PQRS$ is a parallelogram.

Construction : Draw diagonal BD

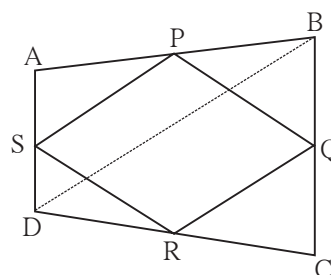


Fig. 5.37

Proof : In $\triangle ABD$, the midpoint of side AD is S and the midpoint of side AB is P.
 $\therefore$ by midpoint theorem, $PS \parallel DB$ and $PS = \frac{1}{2} BD$ (1)
 In $\triangle DBC$ point Q and R are midpoints of side BC and side DC respectively.
 $\therefore QR \parallel BD$ and $QR = \frac{1}{2} BD$ by midpoint theorem (2)
 $\therefore PS \parallel QR$ and $PS = QR$ from (1) and (2)
 $\therefore \square PQRS$ is a parallelogram.

Practice set 5.5

1. In figure 5.38, points X, Y, Z are the midpoints of side AB, side BC and side AC of $\triangle ABC$ respectively. $AB = 5$ cm, $AC = 9$ cm and $BC = 11$ cm. Find the length of XY, YZ, XZ.

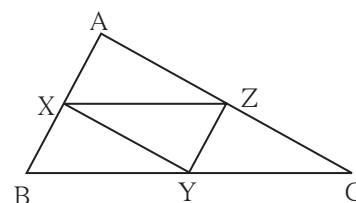


Fig. 5.38

2. In figure 5.39, $\square PQRS$ and $\square MNRL$ are rectangles. If point M is the midpoint of side PR then prove that, (i) $SL = LR$, (ii) $LN = \frac{1}{2} SQ$.

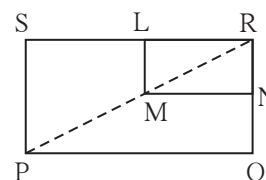


Fig. 5.39

3. In figure 5.40, $\triangle ABC$ is an equilateral triangle. Points F, D and E are midpoints of side AB, side BC, side AC respectively. Show that $\triangle FED$ is an equilateral triangle.

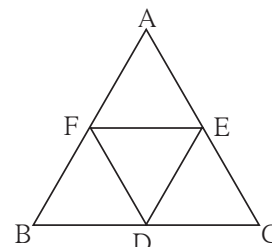


Fig. 5.40

4. In figure 5.41, seg PD is a median of $\triangle PQR$. Point T is the mid point of seg PD. Produced QT intersects PR at M. Show that $\frac{PM}{PR} = \frac{1}{3}$.
 [Hint : draw $DN \parallel QM$.]

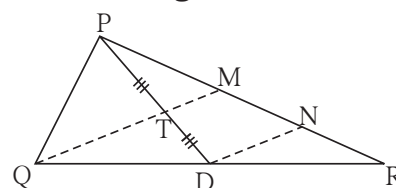


Fig. 5.41

Problem set 5

1. Choose the correct alternative answer and fill in the blanks.
 (i) If all pairs of adjacent sides of a quadrilateral are congruent then it is called
 (A) rectangle (B) parallelogram (C) trapezium, (D) rhombus

- Adjacent sides of a rectangle are 7 cm and 24 cm. Find the length of its diagonal.
 - If diagonal of a square is 13 cm then find its side.
 - Ratio of two adjacent sides of a parallelogram is 3:4, and its perimeter is 112 cm. Find the length of its each side.
 - Diagonals PR and QS of a rhombus PQRS are 20 cm and 48 cm respectively. Find the length of side PQ.
 - Diagonals of a rectangle PQRS are intersecting in point M. If $\angle QMR = 50^\circ$ then find the measure of $\angle MPS$.
 - In the adjacent Figure 5.42, if
 $\text{seg } AB \parallel \text{seg } PQ$, $\text{seg } AB \cong \text{seg } PQ$,
 $\text{seg } AC \parallel \text{seg } PR$, $\text{seg } AC \cong \text{seg } PR$
 then prove that,
 $\text{seg } BC \parallel \text{seg } QR$ and $\text{seg } BC \cong \text{seg } QR$.
-
- Fig. 5.42



- Fig. 5.43**

- Fig. 5.44**

Activity

To verify the different properties of quadrilaterals

Material : A piece of plywood measuring about 15 cm× 10 cm, 15 thin screws, twine, scissors.

Note : On the plywood sheet, fix five screws in a horizontal row keeping a distance of 2cm between any two adjacent screws. Similarly make two more rows of screws exactly below the first one. Take care that the vertical distance between any two adjacent screws is also 2cm.

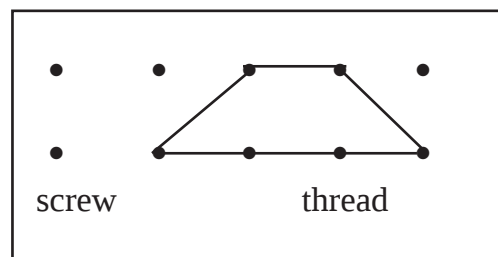


Fig. 5.45

With the help of the screws, make different types of quadrilaterals of twine. Verify the properties of sides and angles of the quadrilaterals.

Additional information

You know the property that the point of concurrence of medians of a triangle divides the medians in the ratio 2 : 1. Proof of this property is given below.

Given : seg AD and seg BE are the medians of $\triangle ABC$ which intersect at point G.

To prove: $AG : GD = 2 : 1$

Construction : Take point F on ray AD such that
G-D-F and $GD = DF$

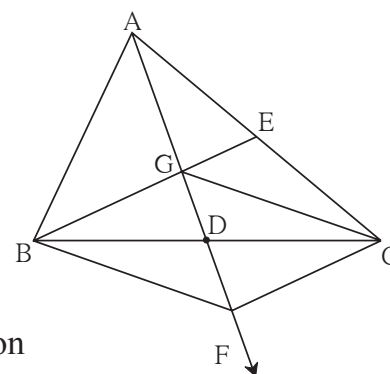


Fig. 5.46

Proof : Diagonals of $\square$ BGCF bisect each other

.... given and construction

$\therefore \square \text{BGCF}$ is a parallelogram.

$$\therefore \text{seg BE} \parallel \text{set FC}$$

Now point E is the midpoint of side AC of $\triangle AFC$ given

seg EB || seg FC

Line passing through midpoint of one side and parallel to the other side bisects the third side.

$\therefore$ point G is the midpoint of side AF.

$$\therefore AG = GF$$

But GF = 2GD construction

$$\therefore AG = 2 GD$$
$$\therefore \frac{AG}{GD} = \frac{2}{1} \text{ i.e. } AG : GD = 2 : 1$$