



Let's study.

- **Parallelogram**
- Tests of parallelogram
- **Rhombus**

- **Rectangles**
- Mid point theorem
- Square
- **Trapezium**



(3) ..., ...

Let's recall.

1.

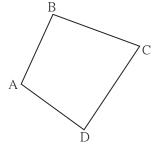


Fig. 5.1

○ ○ ○ ○ ○ ○ ○ ○

Write the following pairs considering □ABCD

Pairs of adjacent sides: (1) ..., ...

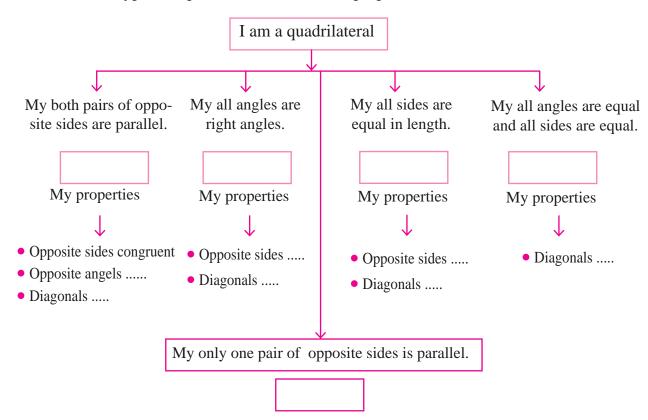
(2) ..., ... (4) ..., ... Pairs of adjacent angles:

(1) ..., ... (2) ..., ... (3) ..., ... (4) ..., ...

Pairs of opposite sides (1), (2),

Pairs of opposite angles (1), (2),

Let's recall types of quadrilaterals and their properties.



You know different types of quadrilaterals and their properties. You have learned then through different activities like measuring sides and angles, by paper folding method etc. Now we will study these properties by giving their logical proofs.

A property proved logically is called a proof.

In this chapter you will learn that how a rectangle, a rhombus and a square are parallelograms. Let us start our study from parallelogram.



Parallelogram

A quadrilateral having both pairs of opposite sides parallel is called a parallelogram.

For proving the theorems or for solving the problems we need to draw figure of a parallelogram frequently. Let us see how to draw a parallelogram.

Suppose we have to draw a parallelogram $\square ABCD$.

Method I:

- Let us draw seg AB and seg BC of any length and making an angle of any measure with each other.
- Now we want seg AD and seg BC parallel to each other. So draw a line parallel to seg BC through the point A.
- Similarly we will draw line parallel to AB through the point C. These lines will intersect in point D.

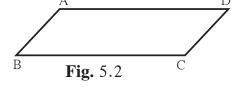


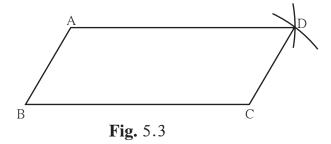
Method II:

- Let us draw seg AB and seg BC of any length and making angle of any measure between them.
- Draw an arc with compass with centre A and radius BC.
- Similarly draw an arc with centre C and radius AB intersecting the arc previously drawn.
- Name the point of intersection of two arcs as D.

Draw seg AD and seg CD.

Quadrilateral so formed is a parallelogram ABCD





In the second method we have actually drawn $\square ABCD$ in which opposite sides are equal. We will prove that a quadrilateral whose opposite sides are equal, is a parallelogram.

Activity I Draw five parallelograms by taking various measures of lengths and angles.

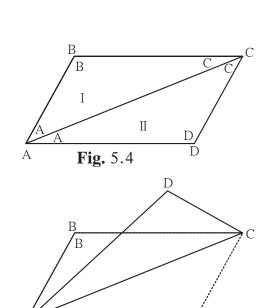
For the proving theorems on parallelogram, we use congruent triangles. To understand how they are used, let's do the following activity.

Activity II

- Draw a parallellogram ABCD on a card sheet. Draw diagonal AC. Write the names of vertices inside the triangle as shown in the figure. Then cut is out.
- Fold the quadrilateral on the diagonal AC and see whether ΔADC and ΔCBA match with each other or not.
- Cut \square ABCD along diagonals AC and separate \triangle ADC and \triangle CBA. By rotating and flipping \triangle CBA, check whether it matchs exactly with \triangle ADC. What did you find ? Which sides of \triangle CBA match with which sides of \triangle ADC? Which angles of \triangle CBD match with which angles of \triangle ADC?

Side DC matches with side AB and side AD matches with side CB. Similarly $\angle B$ matches with $\angle D$.

So we can see that opposite sides and angles of a parallelogram are congruent.

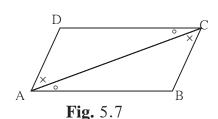


B C A Fig. 5.6

Fig. 5.5

We will prove these properties of a parallelogram.

Theorem 1. Opposite sides and opposite angles of a parallelogram are congruent.



Given $: \Box ABCD$ is a parallelogram.

It means side AB || side DC,

side AD || side BC.

To prove : $\operatorname{seg} AD \cong \operatorname{seg} BC ; \operatorname{seg} DC \cong \operatorname{seg} AB$

 $\angle ADC \cong \angle CBA$, and $\angle DAB \cong \angle BCD$.

Construction: Draw diagonal AC.

Proof: seg DC | seg AB and diagonal AC is a transversal.

$$\therefore$$
 \angle DCA \cong \angle BAC(1)

and
$$\angle DAC \cong \angle BCA$$
(2) }.....alternate angles Now , in $\triangle ADC$ and $\triangle CBA$,

$$\angle DAC \cong \angle BCA$$
 from (2)

$$\angle DCA \cong \angle BAC$$
 from (1)

$$seg AC \cong seg CA$$
 common side

$$\therefore \Delta ADC \cong \Delta CBA$$
 ASA test

∴ side AD
$$\cong$$
 side CB c.s.c.t.

and side DC
$$\cong$$
 side AB c.s.c.t.,

Also,
$$\angle ADC \cong \angle CBA$$
 c.a.c.t.

Similarly we can prove $\angle DAB \cong \angle BCD$.

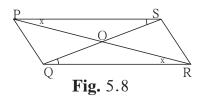


Use your brain power!

In the above theorem, to prove $\angle DAB \cong \angle BCD$, is any change in the construction needed? If so, how will you write the proof making the change?

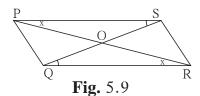
To know one more property of a parallelogram let us do the following activity.

Activity: Draw a parallelogram PORS. Draw diagonals PR and QS. Denote the intersection of diagonals by letter O. Compare the two parts of each diagonal with a divider. What do you find?



60

Theorem: Diagonals of a parallelogram bisect each other.



Given : □PQRS is a parallelogram. Diagonals PR

and QS intersect in point O.

To prove : $seg PO \cong seg RO$,

 $seg SO \cong seg QO.$

Proof : In $\triangle POS$ and $\triangle ROQ$

 $\angle OPS \cong \angle ORQ$ alternate angles

side $PS \cong side RQ$ opposite sides of parallelogram

 $\angle PSO \cong \angle RQO$ alternate angles

 $\therefore \Delta POS \cong \Delta ROQ \dots ASA \text{ test}$



- Adjacent angles of a parallelogram are supplementary.
- Opposite sides of a parallelogram are congruent.
- Opposite angles of a parallelogram are congruent.
- Diagonals of a parallelogram bisect each other.

Solved Examples

Ex (1) \square PQRS is a parallelogram. PQ = 3.5, PS = 5.3 \angle Q = 50° then find the lengths of remaining sides and measures of remaining angles.

Solution : \square PQRS is a parallelogram.

$$\therefore \angle Q + \angle P = 180^{\circ}$$
 interior angles are

$$\therefore 50^{\circ} + \angle P = 180^{\circ}$$

supplementary.

$$\therefore \angle P = 180^{\circ} - 50^{\circ} = 130^{\circ}$$

Now, $\angle P = \angle R$ and $\angle Q = \angle S$ opposite angles of a parallelogram.

$$\therefore$$
 $\angle R = 130^{\circ}$ and $\angle S = 50^{\circ}$

Similarly, PS = QR and PQ = SRopposite sides of a parallelogram.

$$\therefore$$
 QR = 5.3 and SR = 3.5

Fig. 5.10

Ex (2) \square ABCD is a parallelogram. If $\angle A = (4x + 13)^{\circ}$ and $\angle D = (5x - 22)^{\circ}$ then find the measures of $\angle B$ and $\angle C$.

Solution: Adjacent angles of a parallelogram are supplementary.

 $\angle A$ and $\angle D$ are adjacent angles.

$$\therefore (4x + 13)^{\circ} + (5x - 22)^{\circ} = 180$$

$$\therefore 9x - 9 = 180$$

$$\therefore 9x = 189$$

$$\therefore x = 21$$

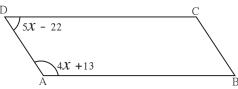


Fig. 5.11

$$\therefore$$
 $\angle A = 4x + 13 = 4 \times 21 + 13 = 84 + 13 = 97°$

$$\therefore \angle C = 97^{\circ}$$

$$\angle D = 5x - 22 = 5 \times 21 - 22 = 105 - 22 = 83^{\circ}$$

$$\therefore \angle B = 83^{\circ}$$

Practice set 5.1

- 1. Diagonals of a parallelogram WXYZ intersect each other at point O. If \angle XYZ = 135° then what is the measure of \angle XWZ and \angle YZW ?

 If l(OY) = 5 cm then l(WY) = ?
- 2. In a parallelogram ABCD, If $\angle A = (3x + 12)^{\circ}$, $\angle B = (2x 32)^{\circ}$ then find the value of x and then find the measures of $\angle C$ and $\angle D$.
- **3.** Perimeter of a parallelogram is 150 cm. One of its sides is greater than the other side by 25 cm. Find the lengths of all sides.
- **4.** If the ratio of measures of two adjacent angles of a parallelogram is 1 : 2, find the measures of all angles of the parallelogram.
- **5*.** Diagonals of a parallelogram intersect each other at point O. If AO = 5, BO = 12 and AB = 13 then show that $\Box ABCD$ is a rhombus.
- 6. In the figure 5.12, □PQRS and □ABCR are two parallelograms.
 If ∠P = 110° then find the measures of all angles of □ABCR.

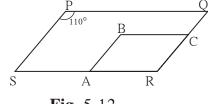


Fig. 5.12

7. In figure 5.13 □ABCD is a parallelogram. Point E is on the ray AB such that BE = AB then prove that line ED bisects seg BC at point F.

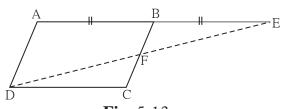
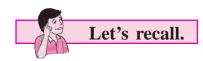


Fig. 5.13



Tests for parallel lines

- 1. If a transversal interesects two lines and a pair of corresponding angles is congruent then those lines are parallel.
- 2. If a transversal intersects two lines and a pair of alternate angles is corgruent then those two lines are parallel.
- 3. If a transversal intersects two lines and a pair of interior angles is supplementary then those two lines are parallel.



Tests for parallelogram

Suppose, in \Box PQRS, PS = QR and PQ = SR and we have to prove that \Box PQRS is a parallelogram. To prove it, which pairs of sides of \Box PQRS should be shown parallel?

Which test can we use to show the sides parallel? Which line will be convenient as a transversal to obtain the angles necessary to apply the test?

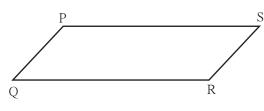


Fig. 5.14

Theorem : If pairs of opposite sides of a quadrilateral are congruent then that quadrilateral is a parallelogram.

Given : In □PQRS

side $PS \cong side QR$

side $PQ \cong side SR$

To prove: \square PQRS is a parallelogram.

Construction: Draw diagonal PR.

Proof : In Δ SPR and Δ QRP

 $\text{side PS} \cong \text{ side QR }\text{given}$

side $SR \cong side QP \dots given$

side PR ≅ side RP common side

 $\therefore \Delta$ SPR $\cong \Delta$ QRP sss test

 $\therefore \angle SPR \cong \angle QRP \dots c.a.c.t.$

Similarly, $\angle PRS \cong \angle RPQ$ c.a.c.t.

 \angle SPR and \angle QRP are alternate angles formed by the transversal PR of seg PS and seg QR.

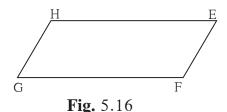
 \therefore side PS || side QR(I) alternate angles test for parallel lines.

Similarly $\angle PRS$ and $\angle RPQ$ are the alternate angles formed by transversal PR of seg PQ and seg SR.

- ∴ side PQ || side SR(II)alternate angle test
- \therefore from (I) and (II) \square PQRS is a parallelogram.

On page 56, two methods to draw a parallelogram are given. In the second method actually we have drawn a quadrilateral of which opposite sides are equal. Did you now understand why such a quadrilateral is a parallelogram?

Theorem : If both the pairs of opposite angles of a quadrilateral are congruent then it is a parallelogram.



Given : In $\square EFGH \angle E \cong \angle G$

and \angle \cong \angle

To prove : □EFGH is a

Proof: Let $\angle E = \angle G = x$ and $\angle H = \angle F = y$

Sum of all angles of a quadrilateral is

$$\therefore$$
 $\angle E + \angle G + \angle H + \angle F = \dots$

$$\therefore$$
 $x + y + \dots + \dots = \dots$

$$\therefore \Box x + \Box y = \dots$$

$$\therefore x + y = 180^{\circ}$$

$$\therefore \angle G + \angle H = \dots$$

 $\angle G$ and $\angle H$ are interior angles formed by transversal HG of seg HE and seg GF.

∴ side HE || side GF (I) interior angle test for parallel lines.

Similarly, $\angle G + \angle F = \dots$

∴ side || side (II) interior angle test for parallel lines.

 \therefore From (I) and (II), \square EFGH is a

Theorem: If the diagonals of a quadrilateral bisect each other then it is a parallelogram.

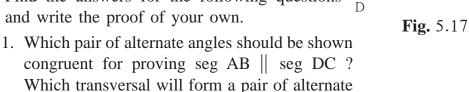
Given: Diagonals of \square ABCD bisect each other in the point E.

It means $seg AE \cong seg CE$ and $seg BE \cong seg DE$

To prove : □ABCD is a parallelogram.

angles?

Proof: Find the answers for the following questions and write the proof of your own.



- 2. Which triangles will contain the alternate angles formed by the transversal?
- 3. Which test will enable us to say that the two triangles congruent?
- 4. Similarly, can you prove that seg AD | seg BC?

The three theorems above are useful to prove that a given quadrilateral is a parallelogram. Hence they are called as tests of a parallelogram.

One more theorem which is useful as a test for parallelogram is given below.

Theorem : A quadrilateral is a parallelogram if a pair of its opposite sides is parallel and congruent.

Given : In □ABCD

 $seg CB \cong seg DA$ and $seg CB \parallel seg DA$

To prove : \square ABCD is a parallelogram.

Construction: Draw diagonal BD.

Write the complete proof which is given in short.

Fig. 5.18

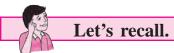
 Δ CBD $\cong \Delta$ ADBSAS test

 \therefore \angle CDB \cong \angle ABD c.a.c.t.

∴ seg CD || seg BA alternate angle test for parallel lines

Remember this!

- A quadrilateral is a parallelogram if its pairs of opposite angles are congruent.
- A quadrilateral is a parallelogram if its pairs of opposite sides are congruent.
- A quadrilateral is a parallelogram if its diagonals bisect each other.
- A quadrilateral is a parallelogram if a pair of its opposite sides is parallel and congruent.
 These theorems are called tests for parallelogram.



Lines in a note book are parallel. Using these lines how can we draw a parallelogram?

Solved examples -

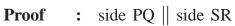
Ex (1) \Box PQRS is parallelogram. M is the midpoint of side PQ and N is the mid point of side RS. Prove that \Box PMNS and \Box MQRN are parallelograms.

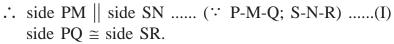
Given: □ PQRS is a parallelogram.

M and N are the midpoints of side PQ and side RS respectively.

To prove: \Box PMNS is a parallelogram.

□MQRN is a parallelogram.





$$\therefore \frac{1}{2} \text{ side PQ} = \frac{1}{2} \text{ side SR}$$

 \therefore side PM \cong side SN (\because M and N are midpoints.).....(II)

 \therefore From (I) and (II), \Box PMNQ is a parallelogram,

Similarly, we can prove that \square MQRN is parallelogram.

Ex (2) Points D and E are the midpoints of side AB and side AC of Δ ABC respectively. Point F is on ray ED such that ED = DF. Prove that \Box AFBE is a parallelogram. For this example write 'given' and 'to prove' and complete the proof given below.

Given : -----To prove : ------

Proof: seg AB and seg EF are _____ of □AFBE.

seg AD ≅ seg DB......

∴ Diagonals of □AFBE each other

∴ □AFBE is a parallelogram ...by

Ex (3) Prove that every rhombus is a parallelogram.

Given : \Box ABCD is a rhombus.

To prove: □ABCD is parallelogram.

Proof : $seg AB \cong seg BC \cong seg CD \cong seg DA (given)$

∴ side $AB \cong side CD$ and side $BC \cong side AD$

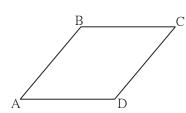


Fig. 5.20

Fig. 5.19

Fig. 5.21

В

test.

Practice set 5.2

1. In figure 5.22, □ABCD is a parallelogram, P and Q are midpoints of side AB and DC respectively, then prove □APCQ is a parallelogram.

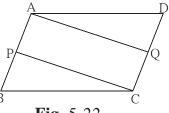
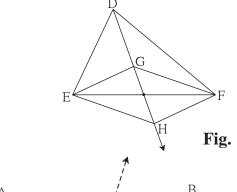
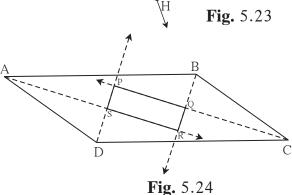


Fig. 5.22

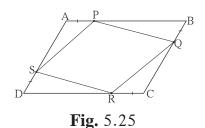
- **2.** Using opposite angles test for parallelogram, prove that every rectangle is a parallelogram.
- 3. In figure 5.23, G is the point of concurrence of medians of Δ DEF. Take point H on ray DG such that D-G-H and DG = GH, then prove that \Box GEHF is a parallelogram.



4. Prove that quadrilateral formed by the intersection of angle bisectors of all angles of a parallelogram is a rectangle. (Figure 5.24)



5. In figure 5.25, if points P, Q, R, S are on the sides of parallelogram such that AP = BQ = CR = DS then prove that □PQRS is a parallelogram.





Properties of rectangle, rhombus and square

Rectangle, rhombus and square are also parallelograms. So the properties that opposite sides are equal, opposite angles are equal and diagonals bisect each other hold good in these types of quadrilaterals also. But there are some more properties of these quadrilaterals.

Proofs of these properties are given in brief. Considering the steps in the given proofs, write the proofs in detail.

Theorem: Diagonals of a rectangle are congruent.

Given : \Box ABCD is a rectangle.

To prove: Diagonal AC ≅ diagonal BD

Proof : Complete the proof by giving suitable reasons.

$$\Delta$$
 ADC \cong Δ DAB SAS test

 \therefore diagonal AC \cong diagonal BD.... c.s.c.t.

Theorem: Diagonals of a square are congruent.

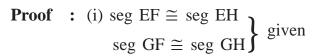
Write 'Given', 'To prove' and 'proof' of the theorem.

Theorem: Diagonals of a rhombus are perpendicular bisectors of each other.

Given : □EFGH is a rhombus

To prove : (i) Diagonal EG is the perpendicular bisector of diagonal HF.

(ii) Diagonal HF is the perpendicular bisector of diagonal EG.



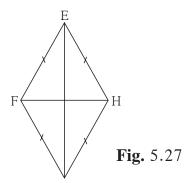


Fig. 5.26

В

Every point which is equidistant from end points of a segment is on the perpendicular bisector of the segment.

.. point E and point G are on the perpendicular bisector of seg HF.

One and only one line passes through two distinct points.

- \therefore line EG is the perpendicular bisector of diagonal HF.
- :. diagonal EG is the perpendicular bisector of diagonal HF.
- (ii) Similarly, we can prove that diagonal HF is the perpendicular bisector of EG.

Write the proofs of the following statements.

- Diagonals of a square are perpendicular bisectors of each other.
- Diagonals of a rhombus bisect its opposite angles.
- Diagonals of a square bisect its opposite angles.

Rei

Remember this!

- Diagonals of a rectangle are congruent.
- Diagonals of a square are congruent.
- Diagonals of a rhombus are perpendicular bisectors of each other.
- Diagonals of a rhombus bisect the pairs of opposite angles.
- Diagonals of a square are perpendicular bisectors of each other.
- Diagonals of a square bisect opposite angles.

Practice set 5.3

- 1. Diagonals of a rectangle ABCD intersect at point O. If AC = 8 cm then find the length of BO and if $\angle CAD = 35^{\circ}$ then find the measure of $\angle ACB$.
- 2. In a rhombus PQRS if PQ = 7.5 then find the length of QR. If $\angle QPS = 75^{\circ}$ then find the measure of $\angle PQR$ and $\angle SRQ$.
- 3. Diagonals of a square IJKL intersects at point M, Find the measures of \angle IMJ, \angle JIK and \angle LJK.
- **4.** Diagonals of a rhombus are 20 cm and 21 cm respectively, then find the side of rhombus and its perimeter.
- 5. State with reasons whether the following statements are 'true' or 'false'.
 - (i) Every parallelogram is a rhombus.
 - (ii) Every rhombus is a rectangle.
 - (iii) Every rectangle is a parallelogram.
 - (iv) Every squre is a rectangle.
 - (v) Every square is a rhombus.
 - (vi) Every parallelogram is a rectangle.



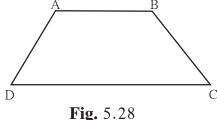
Trapezium

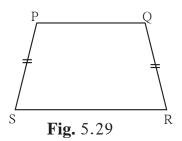
When only one pair of opposite sides of a quadrilateral is parallel then the quadrilateral is called a trapezium.

In the adjacent figure only side AB and side DC of \Box ABCD are parallel to each other. So this is a trapezium. \angle A and \angle D is a pair of adjacent angles and so is the pair of \angle B and \angle C. Therefore by property of parallel lines both the pairs are supplementary.

If non-parallel sides of a trapezium are congruent then that quadrilateral is called as an **Isoceles trapezium**.

 $\Diamond \Diamond \Diamond \Diamond \Diamond \Diamond \Diamond \Diamond \Diamond \Diamond$





The segment joining the midpoints of non parallel sides of a trapezium is called the median of the trapezium.

Solved examples

Ex (1) Measures of angles of \square ABCD are in the ratio 4:5:7:8. Show that \square ABCD is a trapezium.

Solution : Let measures of $\angle A$, $\angle B$, $\angle C$ and $\angle D$ are $(4x)^{\circ}$, $(5x)^{\circ}$, $(7x)^{\circ}$, and $(8x)^{\circ}$ respectively.

Sum of all angles of a quadrialteral is 360°.

$$\therefore 4x + 5x + 7x + 8x = 360$$

$$\therefore 24x = 360$$
 $\therefore x = 15$

$$\angle A = 4 \times 15 = 60^{\circ}, \ \angle B = 5 \times 15 = 75^{\circ}, \ \angle C = 7 \times 15 = 105^{\circ},$$

and
$$\angle D = 8 \times 15 = 120^{\circ}$$

Now,
$$\angle B + \angle C = 75^{\circ} + 105^{\circ} = 180^{\circ}$$

But
$$\angle B + \angle A = 75^{\circ} + 60^{\circ} = 135^{\circ} \neq 180^{\circ}$$

- : side BC and side AD are not parallel(II)
- ∴ □ABCD is a trapezium.[from (I) and (II)]

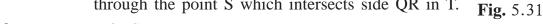
Ex (2) In \square PQRS, side PS \parallel side QR and side PQ \cong side SR,

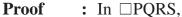
side QR > side PS then prove that $\angle PQR \cong \angle SRQ$

: In \square PQRS, side PS \parallel side QR, Given side $PQ \cong \text{side SR}$ and side QR > side PS.

To prove : $\angle PQR \cong \angle SRQ$

Construction: Draw the segment parallel to side PQ through the point S which intersects side QR in T.





$$\therefore \angle PQT \cong \angle STR \dots corresponding angles (I)$$

and seg PQ
$$\cong$$
 seg STopposite sides of parallelogram

But seg PQ
$$\cong$$
 seg SRgiven

$$\therefore$$
 seg ST \cong seg SR

$$\therefore \angle STR \cong \angle SRT$$
.....isosceles triangle theorem (II)

$$\therefore \angle PQT \cong \angle SRT \dots [from (I) and (II)]$$

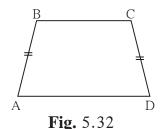
Hence, it is proved that base angles of an isosceles trapezium are congruent.

Fig. 5.30

Practice set 5.4

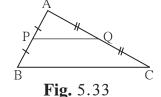
- 1. In \Box IJKL, side IJ || side KL \angle I = 108° \angle K = 53° then find the measures of \angle J and \angle L.
- 2. In \Box ABCD, side BC || side AD, side AB \cong side DC If \angle A = 72° then find the measures of \angle B, and \angle D.
- 3. In \Box ABCD, side BC < side AD (Figure 5.32) side BC \parallel side AD and if side BA \cong side CD then prove that \angle ABC \cong \angle DCB.



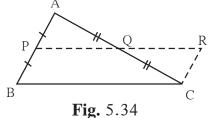


Theorem of midpoints of two sides of a triangle

- **Statement:** The segment joining midpoints of any two sides of a triangle is parallel to the third side and half of it.
- **Given** : In Δ ABC, point P is the midpoint of seg AB and point Q is the midpoint of seg AC.
- **To prove** : seg PQ ∥ seg BC
 - and $PQ = \frac{1}{2} BC$
- **Construction :** Produce seg PQ upto R such that PQ = QR Draw seg RC.



- **Proof** : In Δ AQP and Δ CQR
 - seg PQ ≅ seg QR construction
 - $seg AQ \cong seg QC \dots given$
 - $\angle AQP \cong \angle CQR$ vertically opposite angles.



- -8'

- $\therefore \Delta AQP \cong \Delta CQR \dots SAS \text{ test}$
- $\angle PAQ \cong \angle RCQ \dots$ (1) c.a.c.t.
- \therefore seg AP \cong seg CR(2) c.s.c.t.
- From (1) line AB | line CR.....alternate angle test
- from (2) seg AP \cong seg CR
- Now, $seg AP \cong seg PB \cong seg CR$ and $seg PB \parallel seg CR$
- ∴ □PBCR is a parallelogram.
- :. seg PQ | seg BC and PR = BC opposite sides are congruent

$$PQ = \frac{1}{2} PR$$
 (construction)

$$\therefore$$
 PQ = $\frac{1}{2}$ BC \therefore PR = BC

Converse of midpoint theroem

Theorem: If a line drawn through the midpoint of one side of a triangle and parallel to the other side then it bisects the third side.

For this theorem 'Given', To prove', 'construction' is given below. Try to write the proof.

Given: Point D is the midpoint of side AB of Δ ABC. Line l passing through the point D and parallel to side BC intersects side AC in point E.

To prove : AE = EC

Construction : Draw a line parallel to seg AB passing through the point C. Name the point of intersection where this line and line *l* will intersect as F.

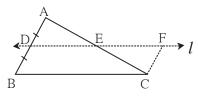
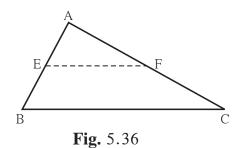


Fig. 5.35

- **Proof** : Use the construction and line $l \parallel \text{seg BC}$ which is given. Prove Δ ADE $\cong \Delta$ CFE and complete the proof.
- Ex (1) Points E and F are mid points of seg AB and seg AC of Δ ABC respectively. If EF = 5.6 then find the length of BC.
- **Solution :** In Δ ABC, point E and F are midpoints of side AB and side AC respectively.

$$EF = \frac{1}{2} BC$$
midpoint theorem

$$5.6 = \frac{1}{2}$$
 BC \therefore BC = $5.6 \times 2 = 11.2$



- Ex (2) Prove that the quadrilateral formed by joining the midpoints of sides of a quadrilateral in order is a parallelogram.
- Given: □ABCD is a quadrilateral.
 P, Q, R, S are midpoints of the sides AB, BC, CD and AD respectively.

To prove : \square PQRS is a parallelogram.

Construction: Draw diagonal BD

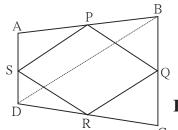


Fig. 5.37

Proof : In \triangle ABD, the midpoint of side AD is S and the midpoint of side AB is P.

 \therefore by midpoint theorem, PS || DB and PS = $\frac{1}{2}$ BD(1)

In Δ DBC point Q and R are midpoints of side BC and side DC respectively.

- \therefore QR || BD and QR = $\frac{1}{2}$ BDby midpoint theorem (2)
- ∴ PS \parallel QR and PS = QR from (1) and (2)
- \therefore \Box PQRS is a parallelogram.

Practice set 5.5

1. In figure 5.38, points X, Y, Z are the midpoints of side AB, side BC and side AC of Δ ABC respectively. AB = 5 cm, AC = 9 cm and BC = 11 cm. Find the length of XY, YZ, XZ.

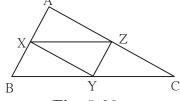


Fig. 5.38

In figure 5.39, $\square PQRS$ □MNRL are and rectangles. If point M is the midpoint of side PR then prove that, (i) SL = LR, (ii) $LN = \frac{1}{2}SQ$.

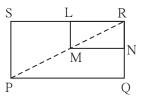


Fig. 5.39

In figure 5.40, \triangle ABC is an equilateral traingle. Points F,D and E are midpoints of side AB, side BC, side AC respectively. Show that Δ FED is an equilateral traingle.

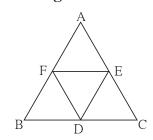


Fig. 5.40

In figure 5.41, seg PD is a median of Δ PQR. Point T is the mid point of seg PD. Produced QT intersects PR at M. Show that $\frac{PM}{PR} = \frac{1}{3}$. [Hint: draw DN || QM.]

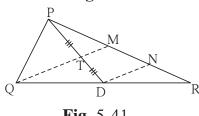
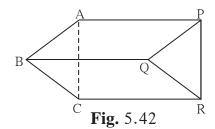


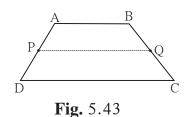
Fig. 5.41

- Choose the correct alternative answer and fill in the blanks. 1.
 - (i) If all pairs of adjacent sides of a quadrilateral are congruent then it is called (A) rectangle (B) parallelogram (C) trapezium, (D) rhombus

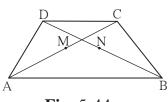
- (ii) If the diagonal of a square is $12\sqrt{2}$ cm then the perimeter of square is (A) 24 cm (B) $24\sqrt{2}$ cm (C) 48 cm (D) $48\sqrt{2}$ cm
- (iii) If opposite angles of a rhombus are $(2x)^{\circ}$ and $(3x 40)^{\circ}$ then value of x is ... (A) 100 ° (B) 80 ° (C) 160 ° (D) 40 °
- 2. Adjacent sides of a rectangle are 7 cm and 24 cm. Find the length of its diagonal.
- 3. If diagonal of a square is 13 cm then find its side.
- **4.** Ratio of two adjacent sides of a parallelogram is 3:4, and its perimeter is 112 cm. Find the length of its each side.
- **5.** Diagonals PR and QS of a rhombus PQRS are 20 cm and 48 cm respectively. Find the length of side PQ.
- **6.** Diagonals of a rectangle PQRS are intersecting in point M. If $\angle QMR = 50^{\circ}$ then find the measure of $\angle MPS$.
- 7. In the adjacent Figure 5.42, if seg AB || seg PQ, seg AB ≅ seg PQ, seg AC || seg PR, seg AC ≅ seg PR then prove that, seg BC || seg QR and seg BC ≅ seg QR.



8*. In the Figure 5.43, \square ABCD is a trapezium. AB \parallel DC. Points P and Q are midpoints of seg AD and seg BC respectively. Then prove that, PQ \parallel AB and PQ = $\frac{1}{2}$ (AB + DC).



9. In the adjacent figure 5.44, □ABCD is a trapezium. AB || DC. Points M and N are midpoints of diagonal AC and DB respectively then prove that MN || AB.



Activity

To verify the different properties of quadrilaterals

Material: A piece of plywood measuring about 15 cm× 10 cm, 15 thin screws, twine, scissors.

Note: On the plywood sheet, fix five screws in a horizontal row keeping a distance of 2cm between any two adjacent screws. Similarly make two more rows of screws exactly below the first one. Take care that the vertical distance between any two adjacent screws is also 2cm.

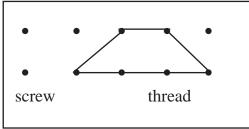


Fig. 5.45

With the help of the screws, make different types of quadrilaterals of twine. Verify the properties of sides and angles of the quadrilaterals.

Additional information

You know the property that the point of concurrence of medians of a triangle divides the medians in the ratio 2:1. Proof of this property is given below.

Given : seg AD and seg BE are the medians of Δ ABC which intersect at point G.

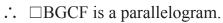
To prove: AG : GD = 2 : 1

Construction: Take point F on ray AD such that

G-D-F and GD = DF

Proof: Diagonals of □BGCF bisect each other

.... given and construction



Now point E is the midpoint of side AC of Δ AFC given seg EB \parallel seg FC

Line passing through midpoint of one side and parallel to the other side bisects the third side.

: point G is the midpoint of side AF.

$$\therefore$$
 AG = GF

But GF = 2GD construction

$$\therefore$$
 AG = 2 GD

$$\therefore \frac{AG}{GD} = \frac{2}{1} \text{ i.e. } AG : GD = 2:1$$

Fig. 5.46