4 Constructions of Triangles

23M8J3

Let's study.

To construct a triangle, if following information is given.

- **Base, an angle adjacent to the base and sum of lengths of two remaining sides.**
- **Base, an angle adjacent to the base and difference of lengths of remaining two sides.**
- **Perimeter and angles adjacent to the base.**

Let's recall.

In previous standard we have learnt the following triangle constructions.

* To construct a triangle when its three sides are given.
* To construct a triangle when its base and two adjacent angles are given.
* To construct a triangle when two sides and the included angle are given.
* To construct a right angled triangle when its hypotenuse and one side is given.

Perpendicular bisector Theorem

- Every point on the perpendicular bisector of a segment is equidistant from its end points.
- Every point equidistant from the end points of a segment is on the perpendicular bisector of the segment.

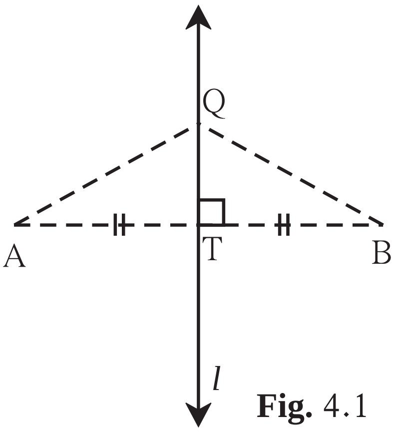


Fig. 4.1

Let's learn.

Constructions of triangles

To construct a triangle, three conditions are required. Out of three sides and three angles of a triangle two parts and some additional information about them is given, then we can construct a triangle using them.

We frequently use the following property in constructions.

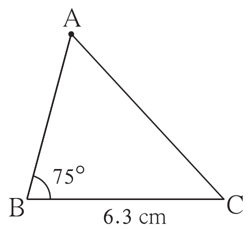
If a point is on two different lines then it is the intersecrtion of the two lines.

Construction I

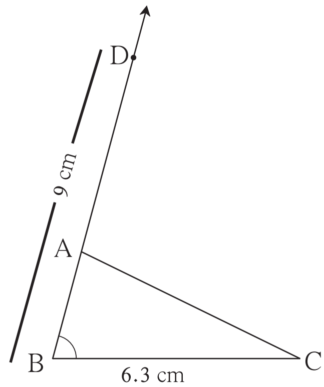
To construct a triangle when its base, an angle adjacent to the base and the sum of the lengths of remaining sides is given.

Ex. Construct Δ ABC in which BC = 6.3 cm, $\angle$B = 75° and AB + AC = 9 cm.

Solution : Let us first draw a rough figure of expected triangle.



Rough figure 4.2



Rough figure 4.3

Explanation : As shown in the rough figure, first we draw seg BC = 6.3 cm of length. On the ray making an angle of 75° with seg BC, mark point D such that

BD = AB + AC = 9 cm

Now we have to locate point A on ray BD.

BA + AD = BA + AC = 9

$\therefore$ AD = AC

$\therefore$ point A is on the perpendicular bisector of seg CD.

$\therefore$ the point of intersection of ray BD and the perpendicular bisector of seg CD is point A.

Steps of construction

(1) Draw seg BC of length 6.3 cm.

(2) Draw ray BP such that m$\angle$ PBC = 75°.

(3) Mark point D on ray BP such that d(B,D) = 9 cm

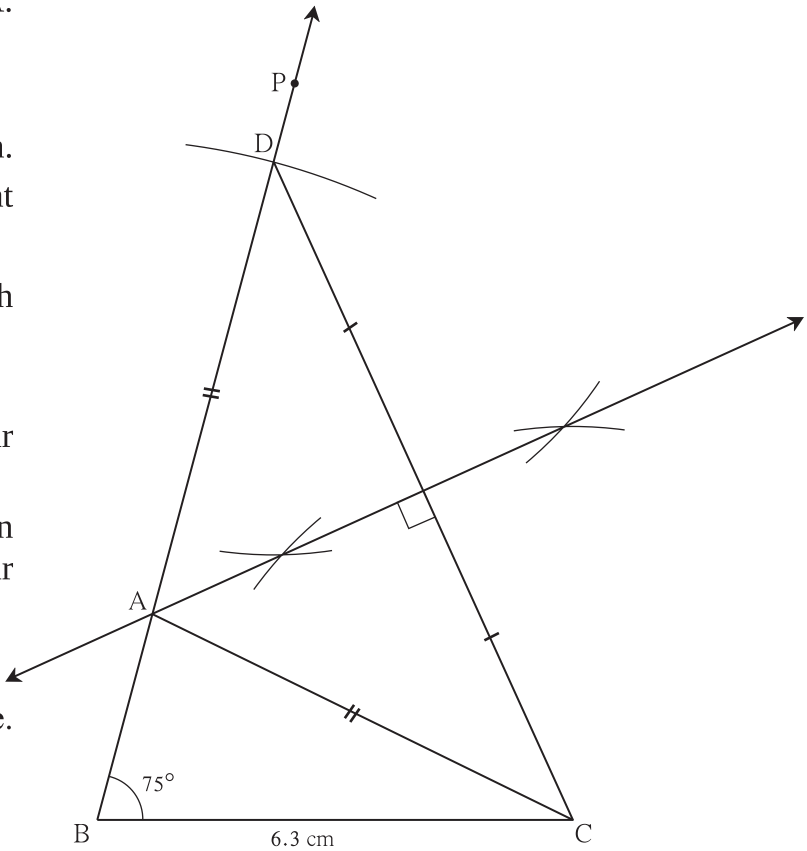
(4) Draw seg DC.

(5) Construct the perpendicular bisector of seg DC .

(6) Name the point of intersection of ray BP and the perpendicular bisector of CD as A.

(7) Draw seg AC.

Δ ABC is the required triangle.



Fair fig. 4.4

Practice set 4.1

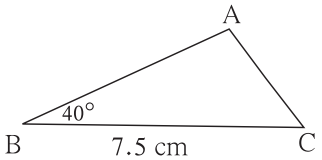
1. Construct Δ PQR, in which QR = 4.2 cm, $m\angle Q = 40^\circ$ and PQ + PR = 8.5 cm
2. Construct Δ XYZ, in which YZ = 6 cm, XY + XZ = 9 cm. $\angle XYZ = 50^\circ$
3. Construct Δ ABC, in which BC = 6.2 cm, $\angle ACB = 50^\circ$, AB + AC = 9.8 cm
4. Construct Δ ABC, in which BC = 3.2 cm, $\angle ACB = 45^\circ$ and perimeter of Δ ABC is 10 cm

Construction II

To construct a triangle when its base, angle adjacent to the base and difference between the remaining sides is given.

Ex (1) Construct Δ ABC, such that BC = 7.5 cm, $\angle ABC = 40^\circ$, AB – AC = 3 cm.

Solution : Let us draw a rough figure.



Rough Figure 4.5

Explanation : AB – AC = 3 cm $\therefore$ AB > AC

Draw seg BC. We can draw the ray BL such that $\angle LBC = 40^\circ$. We have to locate point A on ray BL. Take point D on ray BL such that BD = 3 cm.

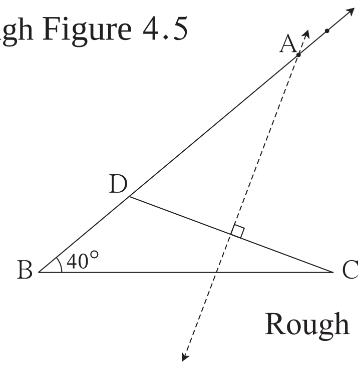
Now , B-D-A and BD = AB – AD = 3.

It is given that AB – AC = 3

$\therefore$ AD = AC

$\therefore$ point A is on the perpendicular bisector of seg DC.

$\therefore$ point A is the intersection of ray BL and the perpendicular bisector of seg DC.



Rough figure 4.6

Steps of construction

(1) Draw seg BC of length7.5 cm.

(2) Draw ray BL such that $\angle LBC = 40^\circ$

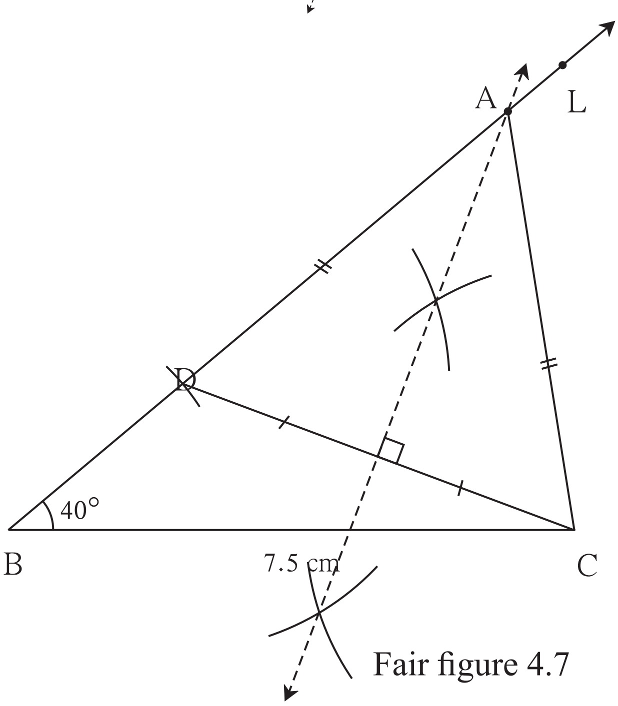
(3) Take point D on ray BL such that BD = 3 cm.

(4) Construct the perpendicular bisector of seg CD.

(5) Name the point of intersection of ray BL and the perpendicular bisector of seg CD as A.

(6) Draw seg AC.

Δ ABC is required triangle.



Fair figure 4.7

Ex. 2   Construct Δ ABC, in which side BC = 7 cm, $\angle$B = 40° and AC – AB = 3 cm.

Solution : Let us draw a rough figure.

seg BC = 7 cm. AC > AB.

We can draw ray BT such that

$\angle$ TBC = 40°

Point A is on ray BT. Take point D on opposite ray of ray BT such that

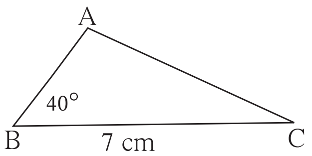
BD = 3 cm.

Now AD = AB + BD = AB + 3 = AC

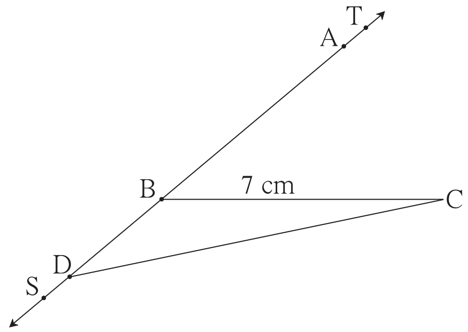
($\because$ AC – AB = 3 cm.)

$\therefore$ AD = AC

$\therefore$ point A is on the perpendicular bisector of seg CD.



Rough figure 4.8



Rough figure 4.9

Steps of construction

(1) Draw BC of length 7 cm.

(2) Draw ray BT such that

$\angle$ TBC = 40°

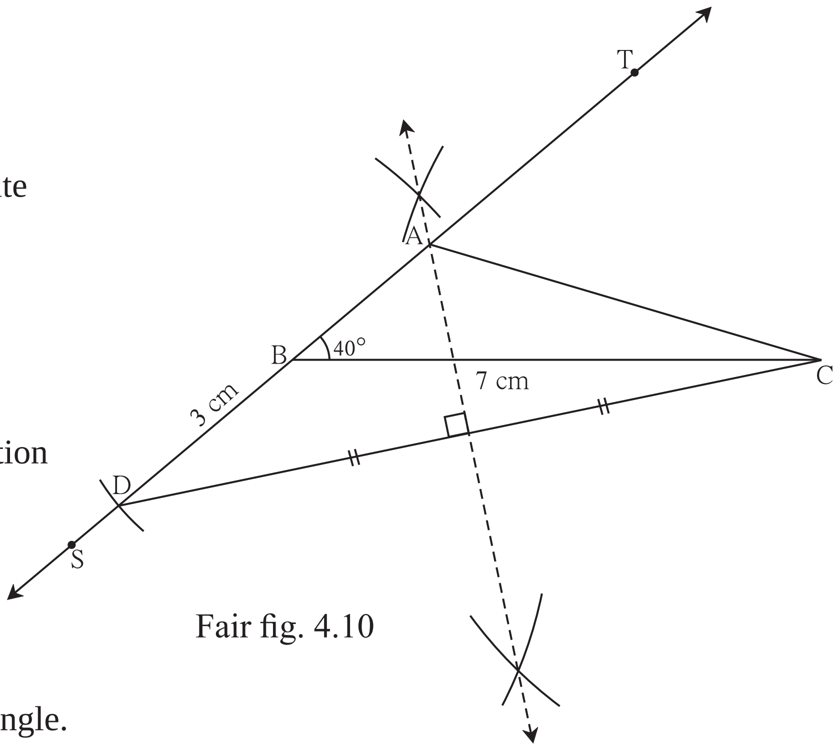
(3) Take point D on the opposite ray BS of ray BT such that BD = 3 cm.

(4) Construct perpendicular bisector of seg DC.

(5) Name the point of intersection of ray BT and the perpendicular bisector of DC as A.

(6) Draw seg AC.

Δ ABC is the required triangle.



Fair fig. 4.10

Practice set 4.2

1. Construct Δ XYZ, such that YZ = 7.4 cm, $\angle$XYZ = 45° and XY – XZ = 2.7 cm.

2. Construct Δ PQR, such that QR = 6.5 cm, $\angle$PQR = 40° and PQ – PR = 2.5 cm.

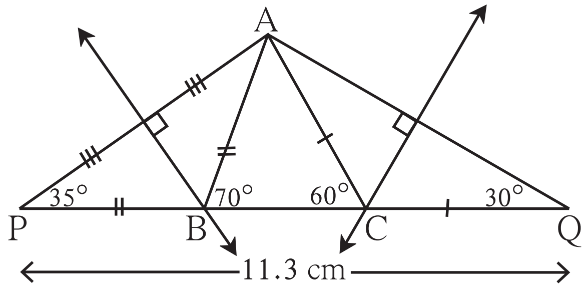
3. Construct Δ ABC, such that BC = 6 cm, $\angle$ABC = 100° and AC – AB = 2.5 cm.

Construction III

To construct a triangle, if its perimeter, base and the angles which include the base are given.

Ex. Construct Δ ABC such that AB + BC + CA = 11.3 cm, $\angle B = 70^\circ$, $\angle C = 60^\circ$.

Solution : Let us draw a rough figure.



Rough Fig. 4.11

Explanation : As shown in the figure, points P and Q are taken on line BC such that,

PB = AB, CQ = AC

$\therefore$ PQ = PB + BC + CQ = AB + BC +AC = 11.3 cm.

Now in ΔPBA, PB = BA

$\therefore$ $\angle APB = \angle PAB$ and $\angle APB + \angle PAB$ = extieror angleABC = 70°

......theorem of remote interior angles

$\therefore$ $\angle APB = \angle PAB = 35^\circ$ Similarly, $\angle CQA = \angle CAQ = 30^\circ$

Now we can draw Δ PAQ, as its two angles and the included side is known.

Since BA = BP , point B lies on the perpendicular bisector of seg AP.

Similarly, CA = CQ, therefore point C lies on the perpendicular bisector of seg AQ

$\therefore$ by constructing the perpendicular bisectors of seg AP and AQ we can get points B and C, where the perpendicular bisectors intersect line PQ.

Steps of construction

(1) Draw seg PQ of 11.3 cm length.

(2) Draw a ray making angle of 35° at point P.

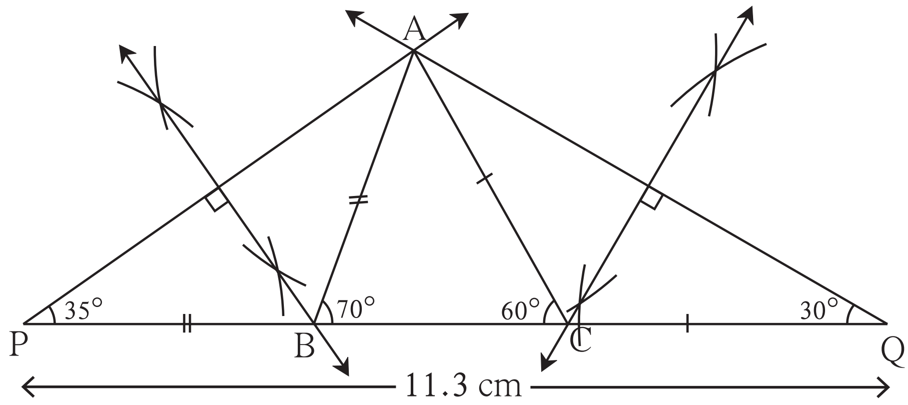
(3) Draw another ray making an angle of 30° at point Q.

(4) Name the point of intersection of the two rays as A.

(5) Draw the perpendicular bisector of seg AP and seg AQ. Name the points as B and C respectively where the perpendicular bisectors intersect line PQ.

(6) Draw seg AB and seg AC.

Δ ABC is the required triangle.



Final Fig. 4.12

Practice set 4.3

1. Construct Δ PQR, in which ∠Q = 70°, ∠R = 80° and PQ + QR + PR = 9.5 cm.
2. Construct Δ XYZ, in which ∠Y = 58°, ∠X = 46° and perimeter of triangle is 10.5 cm.
3. Construct Δ LMN, in which ∠M = 60°, ∠N = 80° and LM + MN + NL = 11 cm.

Problem set 4

1. Construct Δ XYZ, such that XY + XZ = 10.3 cm, YZ = 4.9 cm, ∠XYZ = 45°.
2. Construct Δ ABC, in which ∠B = 70°, ∠C = 60°, AB + BC + AC = 11.2 cm.
3. The perimeter of a triangle is 14.4 cm and the ratio of lengths of its side is 2 : 3 : 4. Construct the triangle.
4. Construct Δ PQR, in which PQ – PR = 2.4 cm, QR = 6.4 cm and ∠PQR = 55°.

ICT Tools or Links

Do constructions of above types on the software Geogebra and enjoy the constructions. The third type of construction given above is shown on Geogebra by a different method. Study that method also.