

1

Sets



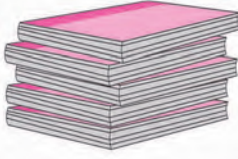
Let's study.

- Sets - Introduction
- Types of sets
- Venn diagrams
- Equal sets, subset
- Universal set
- Intersection and Union of sets
- Number of elements in a set



Let's recall.

Some pictures are given below. It contains the group of things you know.

				1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, ...
Flower bouquet	Bunch of keys	Flock of birds	Pile of note-books	Collection of numbers

We use special word for each of the collection given above. In all the above examples we can clearly list the objects of that collection. We call the collection of such objects as '**Set**'.

Now, observe the collection. 'Happy children in the village', 'Brilliant students of the class'. In both the examples the words 'Happy' and 'Brilliant' are relative terms, because the exact meaning of these words 'to be happy' and 'to be brilliant' differ from person to person. Therefore, these collections are not sets.

See the examples given below and decide whether it is a set or not.

- | | |
|---------------------------------|---------------------------------|
| (1) Days of a week | (2) Months in a year |
| (3) Brave children in the class | (4) First 10 counting numbers |
| (5) Strong forts of Maharashtra | (6) Planets in our solar system |



Let's learn.

Sets

If we can definitely and clearly decide the objects of a given collection then that collection is called a set.

Generally the name of the set is given using capital letters $A, B, C, \dots, Z$

The members or elements of the set are shown by using small letters $a, b, c, \dots$

If a is an element of set A , then we write it as ' $a \in A$ ' and if a is not an element of set A then we write ' $a \notin A$ '.

Now let us observe the set of numbers.

$N = \{1, 2, 3, \dots\}$ is a set of natural numbers.

$W = \{0, 1, 2, 3, \dots\}$ is a set of whole numbers.

$I = \{\dots, -3, -2, -1, 0, 1, 2, \dots\}$ is a set of integers.

Q is a set of rational numbers.

R is a set of real numbers.

Methods of writing sets

There are two methods of writing set.

(1) Listing method or roster method

In this method, we write all the elements of a set in curly bracket. Each of the elements is written only once and separated by commas. The order of an element is not important but it is necessary to write all the elements of the set.

e.g. the set of odd numbers between 1 and 10, can be written as

as, $A = \{3, 5, 7, 9\}$ or $A = \{7, 3, 5, 9\}$

If an element comes more than once then it is customary to write that element only once. e.g. in the word 'remember' the letters 'r, m, e' are repeated more than once. So the set of letters of this word is written as $A = \{r, e, m, b\}$

(2) Rule method or set builder form

In this method, we do not write the list of elements but write the general element using variable followed by a vertical line or colon and write the property of the variable.

e.g. $A = \{x \mid x \in N, 1 < x < 10\}$ and read as 'set A is the set of all ' x ' such that ' x ' is a natural number between 1 and 10'.

e.g. $B = \{ x \mid x \text{ is a prime number between 1 and 10} \}$

set B contains all the prime numbers between 1 and 10. So by using listing method set B can be written as $B = \{2, 3, 5, 7\}$.

Q is the set of rational numbers which can be written in set builder form as

$$Q = \left\{ \frac{p}{q} \mid p, q \in \mathbb{I}, q \neq 0 \right\}$$

and read as 'Q' is set of all numbers in the form $\frac{p}{q}$ such that p and q are integers where q is a non-zero number.'

Illustrations : In the following examples each set is written in both the methods.

Rule method or Set builder form

$A = \{ x \mid x \text{ is a letter of the word 'DIVISION'.} \}$

$B = \{ y \mid y \text{ is a number such that } y^2 = 9 \}$

$C = \{ z \mid z \text{ is a multiple of 5 and is less than 30} \}$

Listing method or Roster method

$A = \{D, I, V, S, O, N\}$

$B = \{ -3, 3 \}$

$C = \{ 5, 10, 15, 20, 25 \}$

Ex. : Fill in the blanks given in the following table.

Listing or Roster Method	Rule Method
$A = \{ 2, 4, 6, 8, 10, 12, 14 \}$	$A = \{ x \mid x \text{ is an even natural number less than 15} \}$
.....	$B = \{ x \mid x \text{ is a perfect square number between 1 to 20} \}$
$C = \{ a, e, i, o, u \}$	
.....	$D = \{ y \mid y \text{ is a colour in the rainbow} \}$
.....	$P = \{ x \mid x \text{ is an integer and } , -3 < x < 3 \}$
$M = \{ 1, 8, 27, 64, 125, \dots \}$	

Practice set 1.1

(1) Write the following sets in roster form.

- (i) Set of even natural numbers (ii) Set of even prime numbers from 1 to 50
(iii) Set of negative integers (iv) Seven basic sounds of a sargam (sur)

(2) Write the following symbolic statements in words.

- (i) $\frac{4}{3} \in \mathbb{Q}$ (ii) $-2 \notin \mathbb{N}$ (iii) $P = \{ p \mid p \text{ is an odd number} \}$

- (3) Write any two sets by listing method and by rule method.
- (4) Write the following sets using listing method.
- All months in the indian solar year.
 - Letters in the word 'COMPLEMENT'.
 - Set of human sensory organs.
 - Set of prime numbers from 1 to 20.
 - Names of continents of the world.
- (5) Write the following sets using rule method.
- $A = \{ 1, 4, 9, 16, 25, 36, 49, 64, 81, 100 \}$
 - $B = \{ 6, 12, 18, 24, 30, 36, 42, 48 \}$
 - $C = \{ S, M, I, L, E \}$
 - $D = \{ \text{Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday} \}$
 - $X = \{ a, e, t \}$



Let's learn.

Types of sets

Name of set	Definition	Example
Singleton Set	A set consisting of a single element is called a singleton set.	$A = \{2\}$ A is the set of even prime numbers.
Empty Set or Null Set	If there is not a single element in the set which satisfies the given condition then it is called a Null set or an empty set. Null set is represented by $\{ \}$ or a symbol ϕ (phi)	$B = \{x x \text{ is natural number between } 2 \text{ and } 3.\}$ $\therefore B = \{ \}$ or ϕ
Finite Set	If a set is a null set or number of elements are limited and countable then it is called as 'Finite set'.	$C = \{p p \text{ is a number from } 1 \text{ to } 22 \text{ divisible by } 4.\}$ $C = \{4, 8, 12, 16, 20\}$
Infinite Set	If number of elements in a set is unlimited and uncountable then the set is called 'Infinite set'.	$N = \{1, 2, 3, \dots\}$

Ex. Write the following sets using listing method and classify into finite or infinite set.

- (i) $A = \{x \mid x \in \mathbb{N} \text{ and } x \text{ is an odd number}\}$ (ii) $B = \{x \mid x \in \mathbb{N} \text{ and } 3x - 1 = 0\}$
 (iii) $C = \{x \mid x \in \mathbb{N}, \text{ and } x \text{ is divisible by } 7\}$
 (iv) $D = \{(a, b) \mid a, b \in \mathbb{W}, a + b = 9\}$ (v) $E = \{x \mid x \in \mathbb{I}, x^2 = 100\}$
 (vi) $F = \{(a, b) \mid a, b \in \mathbb{Q}, a + b = 11\}$

Solution : (i) $A = \{x \mid x \in \mathbb{N} \text{ and } x \text{ is an odd number.}\}$

$A = \{1, 3, 5, 7, \dots\}$ This is an infinite set.

(ii) $B = \{x \mid x \in \mathbb{N} \text{ and } 3x - 1 = 0\}$

$$3x - 1 = 0 \quad \therefore 3x = 1 \quad x = \frac{1}{3}$$

But $\frac{1}{3} \notin \mathbb{N} \quad \therefore B = \{ \quad \} \quad \therefore B \text{ is finite set.}$

(iii) $C = \{x \mid x \in \mathbb{N} \text{ and } x \text{ is divisible by } 7.\}$

$C = \{7, 14, 21, \dots\}$ This is an infinite set.

(iv) $D = \{(a, b) \mid a, b \in \mathbb{W}, a + b = 9\}$

We have to find the pairs of a and b such that, a and b are whole numbers and $a + b = 9$.

Let us first write the value of a and then the value of b . By keeping this order set D can be written as

$$D = \{(0, 9), (1, 8), (2, 7), (3, 6), (4, 5), (5, 4), (6, 3), (7, 2), (8, 1), (9, 0)\},$$

In this set, number of pairs are finite and could be counted.

$\therefore$ Set D is a finite set.

(v) $E = \{x \mid x \in \mathbb{I}, x^2 = 100\}$

$E = \{-10, 10\}$. $\therefore E$ is a finite set.

(vi) $F = \{(a, b) \mid a, b \in \mathbb{Q}, a + b = 11\}$

$F = \{(6, 5), (3, 8), (3.5, 7.5), (-15, 26), \dots\}$ infinitely many such pairs can be written.

$\therefore F$ is an infinite set.



Remember this !

$\mathbb{N}, \mathbb{W}, \mathbb{I}, \mathbb{Q}, \mathbb{R}$ all these sets are infinite sets.



Let's learn.

Equal sets

Two sets A and B are said to be equal, if every element of set A is in set B and every element of set B is in set A.

'Set A and set B are equal sets', symbolically it is written as $A = B$.

Ex (1) $A = \{x \mid x \text{ is a letter of the word 'listen'}\}$ $\therefore A = \{l, i, s, t, e, n\}$

$B = \{y \mid y \text{ is a letter of the word 'silent'}\}$ $\therefore B = \{s, i, l, e, n, t\}$

Though the elements of set A and B are not in the same order but all the elements are identical.

$\therefore A = B$

Ex (2) $A = \{x \mid x = 2n, n \in \mathbb{N}, 0 < x \leq 10\}$, $A = \{2, 4, 6, 8, 10\}$

$B = \{y \mid y \text{ is an even number}, 1 \leq y \leq 10\}$, $B = \{2, 4, 6, 8, 10\}$

$\therefore A$ and B are equal sets.

Now think of the following sets.

$C = \{1, 3, 5, 7\}$ $D = \{2, 3, 5, 7\}$

Are C and D equal sets? Obviously 'No'

Because $1 \in C, 1 \notin D, 2 \in D, 2 \notin C$

$\therefore C$ and D are not equal sets. It is written as $C \neq D$

Ex (3) If $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4\}$ then $A \neq B$ verify it.

Ex (4) $A = \{x \mid x \text{ is prime number and } 10 < x < 20\}$ and $B = \{11, 13, 17, 19\}$

Here $A = B$. Verify,

Practice set 1.2

(1) Decide which of the following are equal sets and which are not? Justify your answer.

$$A = \{x \mid 3x - 1 = 2\}$$

$$B = \{x \mid x \text{ is a natural number but } x \text{ is neither prime nor composite.}\}$$

$$C = \{x \mid x \in \mathbb{N}, x < 2\}$$

(2) Decide whether set A and B are equal sets. Give reason for your answer.

$$A = \text{Even prime numbers} \quad B = \{x \mid 7x - 1 = 13\}$$

(3) Which of the following are empty sets? why?

(i) $A = \{a \mid a \text{ is a natural number smaller than zero.}\}$

(ii) $B = \{x \mid x^2 = 0\}$ (iii) $C = \{x \mid 5x - 2 = 0, x \in \mathbb{N}\}$

(4) Write with reasons, which of the following sets are finite or infinite.

- (i) $A = \{x \mid x < 10, x \text{ is a natural number}\}$ (v) Set of apparatus in laboratory
 (ii) $B = \{y \mid y < -1, y \text{ is an integer}\}$ (vi) Set of whole numbers
 (iii) $C = \text{Set of students of class 9 from your school.}$ (vii) Set of rational number
 (iv) Set of people from your village.



Let's learn.

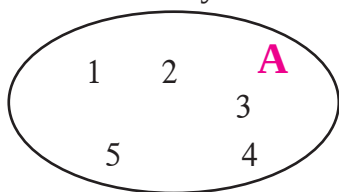
Venn diagrams

British logician John Venn was the first to use closed figures to represent sets. Such representations are called 'Venn diagrams'. Venn diagrams are very useful, in order to understand and illustrate the relationship among sets and to solve the examples based on the sets.

Let us understand the use of Venn diagrams from the following example.

e.g. $A = \{1, 2, 3, 4, 5\}$

Set A is shown by Venn diagram.



1834–1923

John Venn is the first Mathematician who gave the Mathematical form to 'logic' and 'probability'. His famous book is 'Logic of chance'.

$B = \{x \mid -10 \leq x \leq 0, x \text{ is an integer}\}$

Venn diagram given alongside represents the set B.

0	-1	-2	-3
-4	-5	-6	-7
-8	-9	-10	

Subset

If A and B are two given sets and every element of set B is also an element of set A then B is a subset of A which is symbolically written as $B \subseteq A$. It is read as 'B is a subset of A' or 'B subset A'.

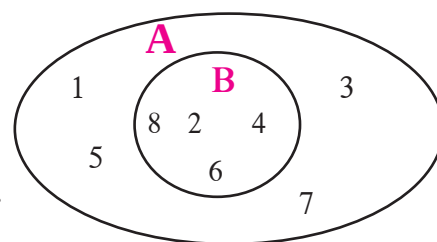
Ex (1) $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$B = \{2, 4, 6, 8\}$

Every element of set B is also an element of set A.

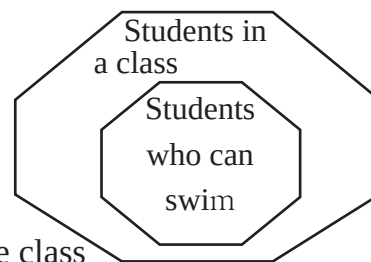
$\therefore B \subseteq A$.

This can be represented by Venn diagram as shown above.



Activity : Set of students in a class and set of students in the same class who can swim, are shown by the Venn diagram.

Observe the diagram and draw Venn diagrams for the following subsets.



- (1) (i) set of students in a class
(ii) set of students who can ride bicycles in the same class
- (2) A set of fruits is given as follows.
{guava, orange, mango, jack fruit, chickoo, jamun, custard apple, papaya, plum}
Show these subsets. (i) fruit with one seed (ii) fruit with more than one seed.

Let's see some more subsets.

Ex (2) N = set of natural numbers.

I = set of integers.

Here $N \subseteq I$ because all natural numbers are integers.

Ex (3) $P = \{x \mid x \text{ is square root of } 25\}$ $S = \{y \mid y \in I, -5 \leq y \leq 5\}$

Let's write set P as $P = \{-5, 5\}$

Let's write set S as $S = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$

Here every element of set P is also an element of set S .

$\therefore P \subseteq S$



Remember this !

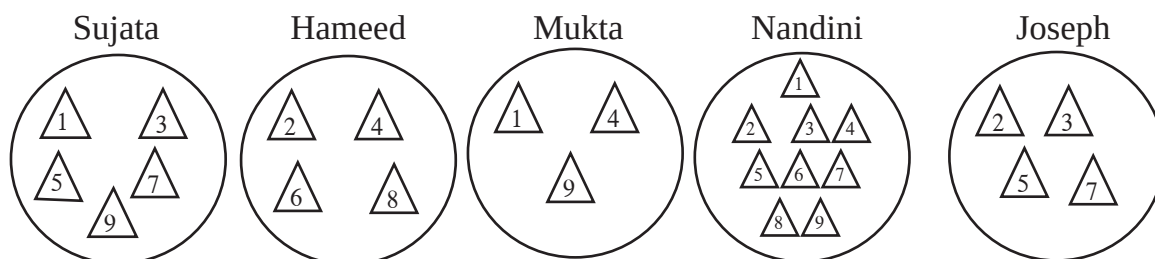
- (i) Every set is a subset of itself. i.e. $A \subseteq A$
- (ii) Empty set is a subset of every set i.e. $\phi \subseteq A$
- (iii) If $A = B$ then $A \subseteq B$ and $B \subseteq A$
- (iv) If $A \subseteq B$ and $B \subseteq A$ then $A = B$

Ex. If $A = \{1, 3, 4, 7, 8\}$ then write all possible subsets of A .

i.e. $P = \{1, 3\}$, $T = \{4, 7, 8\}$, $V = \{1, 4, 8\}$, $S = \{1, 4, 7, 8\}$

In this way many subsets can be written. Write five more subsets of set A .

Activity : Every student should take 9 triangular sheets of paper and one plate. Numbers from 1 to 9 should be written on each triangle. Everyone should keep some numbered triangles in the plate. Now the triangles in each plate form a subset of the set of numbers from 1 to 9.



Look at the plates of Sujata, Hameed, Mukta, Nandini, Joseph with the numbered triangles. Guess the thinking behind selecting these numbers. Hence write the subsets in set builder form.



Let's discuss.

Ex.. Some sets are given below.

$$A = \{ \dots, -4, -2, 0, 2, 4, 6, \dots \}$$

$$B = \{ 1, 2, 3, \dots \}$$

$$C = \{ \dots, -12, -6, 0, 6, 12, 18, \dots \}$$

$$D = \{ \dots, -8, -4, 0, 4, 8, \dots \}$$

$$I = \{ \dots, -3, -2, -1, 0, 1, 2, 3, 4, \dots \}$$

Discuss and decide which of the following statements are true.

(i) A is a subset of sets B, C and D.

(ii) B is a subset of all the sets which are given above.



Let's learn.

Universal set

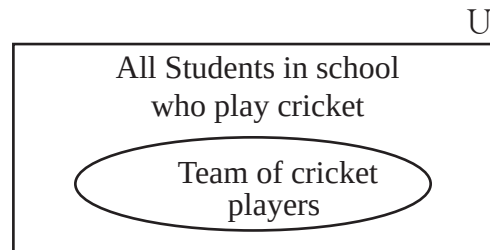
Think of a bigger set which will accommodate all the given sets under consideration which in general is known as Universal set. So that the sets under consideration are the subsets of this Universal set.

Ex (1) Suppose we want to study the students in class 9 who frequently remained absent. Then we have to think of all the students of class 9 who are in the school. So all the students in a school or the students of all the divisions of class 9 in the school is the Universal set.

Let us see the another example.

Ex (2) A cricket team of 15 students is to be selected from a school. Here all the students from school who play cricket is the Universal set. A team of 15 cricket players is a subset of that Universal set.

Generally, the universal set is denoted by 'U' and in Venn diagram it is represented by a rectangle.



Complement of a set

Suppose U is an universal set. If $B \subseteq U$, then the set of all elements in U, which are not in set B is called the complement of B. It is denoted by B' or B^c .

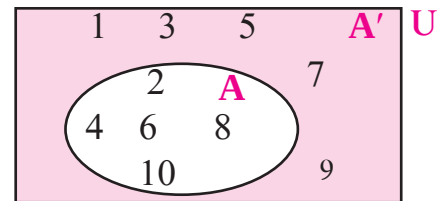
B' is defined as follows.

$$B' = \{x \mid x \in U, \text{ and } x \notin B\}$$

Ex (1) $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$$A = \{2, 4, 6, 8, 10\}$$

$$\therefore A' = \{1, 3, 5, 7, 9\}$$



Ex (2) Suppose $U = \{1, 3, 9, 11, 13, 18, 19\}$

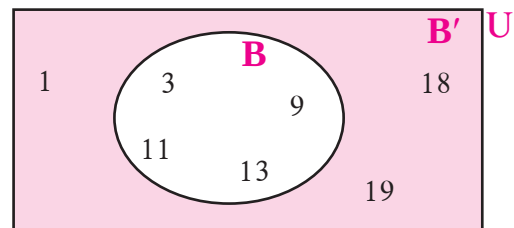
$$B = \{3, 9, 11, 13\}$$

$$\therefore B' = \{1, 18, 19\}$$

Find $(B')'$ and draw the inference.

$(B')'$ is the set of elements which are not in B' but in U.

is $(B')' = B$?



Understand this concept with the help of Venn diagram.

Complement of a complement is the given set itself.



Remember this !

Properties of complement of a set.

- (i) No elements are common in A and A' .
- (ii) $A \subseteq U$ and $A' \subseteq U$
- (iii) Complement of set U is empty set. $U' = \phi$
- (iv) Complement of empty set is U. $\phi' = U$

Practice set 1.3

- (1) If $A = \{a, b, c, d, e\}$, $B = \{c, d, e, f\}$, $C = \{b, d\}$, $D = \{a, e\}$
then which of the following statements are true and which are false ?
(i) $C \subseteq B$ (ii) $A \subseteq D$ (iii) $D \subseteq B$ (iv) $D \subseteq A$ (v) $B \subseteq A$ (vi) $C \subseteq A$
- (2) Take the set of natural numbers from 1 to 20 as universal set and show set X and Y using Venn diagram.
(i) $X = \{x \mid x \in \mathbb{N}, \text{ and } 7 < x < 15\}$
(ii) $Y = \{y \mid y \in \mathbb{N}, y \text{ is prime number from 1 to 20}\}$
- (3) $U = \{1, 2, 3, 7, 8, 9, 10, 11, 12\}$
 $P = \{1, 3, 7, 10\}$
then (i) show the sets U, P and P' by Venn diagram. (ii) Verify $(P')' = P$
- (4) $A = \{1, 3, 2, 7\}$ then write any three subsets of A.
- (5) (i) Write the subset relation between the sets.
P is the set of all residents in Pune.
M is the set of all residents in Madhya Pradesh.
I is the set of all residents in Indore.
B is the set of all residents in India.
H is the set of all residents in Maharashtra.
(ii) Which set can be the universal set for above sets ?
- (6*) Which set of numbers could be the universal set for the sets given below?
(i) $A = \text{set of multiples of 5}$, $B = \text{set of multiples of 7}$.
 $C = \text{set of multiples of 12}$
(ii) $P = \text{set of integers which are multiples of 4}$.
 $T = \text{set of all even square numbers}$.
- (7) Let all the students of a class is an Universal set. Let set A be the students who secure 50% or more marks in Maths. Then write the complement of set A.



Let's learn.

Operations on sets

Intersection of two sets

Suppose A and B are two sets. The set of all common elements of A and B is called the intersection of set A and B. It is denoted as $A \cap B$ and read as A intersection B.

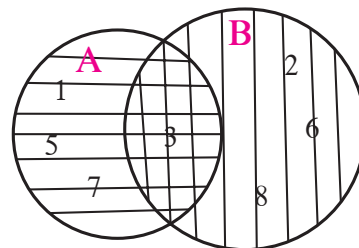
$$\therefore A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

Ex (1) $A = \{ 1, 3, 5, 7 \}$ $B = \{ 2, 3, 6, 8 \}$

Let us draw Venn diagram.

The element 3 is common in set A and B.

$$\therefore A \cap B = \{3\}$$



Ex (2) $A = \{1, 3, 9, 11, 13\}$ $B = \{1, 9, 11\}$

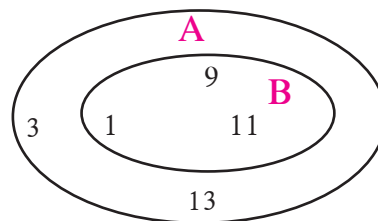
The elements 1, 9, 11 are common in set A and B.

$$\therefore A \cap B = \{1, 9, 11\} \quad \text{But } B = \{1, 9, 11\}$$

$$\therefore A \cap B = B$$

Here set B is the subset of A.

$\therefore$ If $B \subseteq A$ then $A \cap B = B$, similarly, if $B \cap A = B$, then $B \subseteq A$



Remember this !

Properties of Intersection of sets

- | | |
|--|---|
| (1) $A \cap B = B \cap A$ | (2) If $A \subseteq B$ then $A \cap B = A$ |
| (3) If $A \cap B = B$ then $B \subseteq A$ | (4) $A \cap B \subseteq A$ and $A \cap B \subseteq B$ |
| (5) $A \cap A' = \phi$ | (6) $A \cap A = A$ (7) $A \cap \phi = \phi$ |

Activity : Take different examples of sets and verify the above mentioned properties.



Let's learn.

Disjoint sets

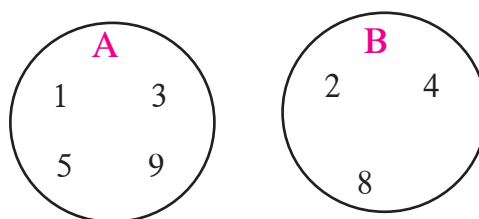
Let, $A = \{ 1, 3, 5, 9 \}$

and $B = \{2, 4, 8\}$ are given.

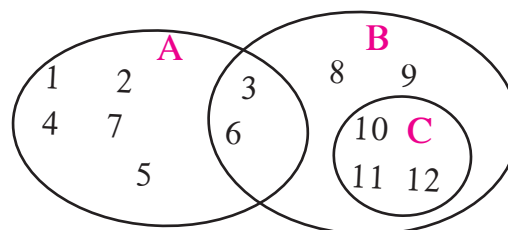
Confirm that not a single element is common in set

A and B. These sets are completely different from each other.

So the set A and B are disjoint sets. Observe its Venn diagram.



Activity I : Observe the set A, B, C given by Venn diagrams and write which of these are disjoint sets.

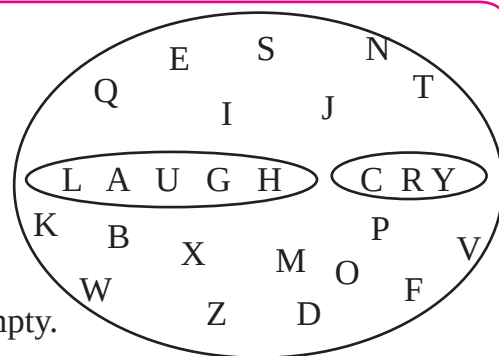


Activity II : Let the set of English alphabets be the Universal set.

The letters of the word 'LAUGH' is one set and the letter of the word 'CRY' is another set.

We can say that these are two disjoint sets.

Observe that intersection of these two sets is empty.



Union of two sets

Let A and B be two given sets. Then the set of all elements of set A and B is called the Union of two sets. It is written as $A \cup B$ and read as 'A union B'.

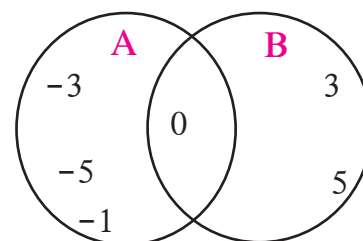
$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

Ex (1) $A = \{-1, -3, -5, 0\}$

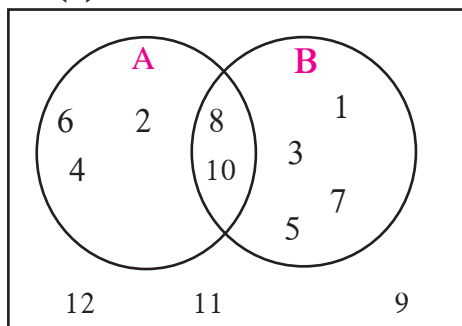
$$B = \{0, 3, 5\}$$

$$A \cup B = \{-3, -5, 0, -1, 3, 5\}$$

Note that, $A \cup B = B \cup A$



Ex (2)



Observe the Venn diagram and write the following sets using listing method.

(i) U (ii) A (iii) B (iv) $A \cup B$ (v) $A \cap B$

(vi) A' (vii) B' (viii) $(A \cup B)'$ (ix) $(A \cap B)'$

Solution :

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

$$A = \{2, 4, 6, 8, 10\},$$

$$B = \{1, 3, 5, 7, 8, 10\}$$

$$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 10\}$$

$$A \cap B = \{8, 10\}$$

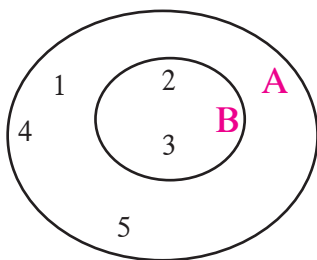
$$A' = \{1, 3, 5, 7, 9, 11, 12\}$$

$$B' = \{2, 4, 6, 9, 11, 12\}$$

$$(A \cup B)' = \{9, 11, 12\}$$

$$(A \cap B)' = \{1, 2, 3, 4, 5, 6, 7, 9, 11, 12\}$$

Ex (3)



$$A = \{1, 2, 3, 4, 5\} \quad B = \{2, 3\}$$

Let us draw its Venn diagram.

$$A \cup B = \{1, 2, 3, 4, 5\}$$

Observe that set A and $A \cup B$ have the same elements.

Hence, if $B \subseteq A$ then $A \cup B = A$



Remember this !

Properties of Union of sets

$$(1) A \cup B = B \cup A$$

$$(2) \text{ If } A \subseteq B \text{ then } A \cup B = B$$

$$(3) A \subseteq A \cup B, B \subseteq A \cup B$$

$$(4) A \cup A' = U$$

$$(5) A \cup A = A$$

$$(6) A \cup \phi = A$$



Let's learn.

Number of elements in a set

Let $A = \{3, 6, 9, 12, 15\}$ is a given set with 5 elements.

Number of elements in set A is denoted as $n(A)$. $\therefore n(A) = 5$

Let $B = \{6, 12, 18, 24, 30, 36\}$ $\therefore n(B) = 6$

Number of elements in Union and Intersection of sets.

Let us consider the set A and set B as given above,

$$n(A) + n(B) = 5 + 6 = 11 \text{ -----(I)}$$

$$A \cup B = \{3, 6, 9, 12, 15, 18, 24, 30, 36\} \therefore n(A \cup B) = 9 \text{ -----(II)}$$

To find $A \cap B$ means to find common elements of set A and set B.

$$A \cap B = \{6, 12\} \therefore n(A \cap B) = 2 \text{ -----(III)}$$

In $n(A)$ and $n(B)$ elements in $A \cap B$ are counted twice.

$$\therefore n(A) + n(B) - n(A \cap B) = 5 + 6 - 2 = 9 \text{ and } n(A \cup B) = 9$$

From equations (I), (II) and (III), we can write it as follows.

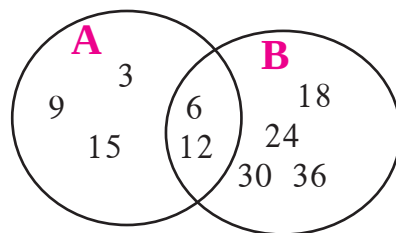
$$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Verify the above rule for the given Venn diagram.

$$n(A) = \boxed{}, n(B) = \boxed{}$$

$$n(A \cup B) = \boxed{}, n(A \cap B) = \boxed{}$$

$$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B)$$



Remember this !

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$\text{means } n(A) + n(B) = n(A \cup B) + n(A \cap B)$$

Ex. Let $A = \{1, 2, 3, 5, 7, 9, 11, 13\}$

$B = \{1, 2, 4, 6, 8, 12, 13\}$

Verify the above rule for the given set A and set B.



Let's learn.

Word problems based on sets

Ex. In a class of 70 students, 45 students like to play Cricket. 52 students like to play Kho-kho. All the students like to play atleast one of the two games. How many students like to play Cricket or Kho-kho? How many students like to play only Cricket?

Solution : We will solve this example in two ways.

Method I : Total number of students = 70

Let A be the set of students who likes to play Cricket.

Let B be the set of students who likes to play Kho-kho.

Hence the number of students who likes to play Cricket or Kho-kho is $n(A \cup B)$

$$\therefore n(A \cup B) = 70$$

Number of students who likes to play both Cricket and Kho-kho = $n(A \cap B)$

$$n(A) = 45, \quad n(B) = 52$$

We know, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

$$\begin{aligned} \therefore n(A \cap B) &= n(A) + n(B) - n(A \cup B) \\ &= 45 + 52 - 70 = 27 \end{aligned}$$

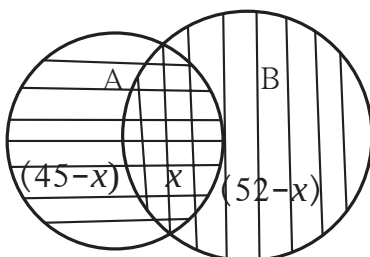
$\therefore$ Number of students who likes to play both the games are 27,

Number of students who likes to play Kho-kho are 45.

$\therefore$ Number of students who likes to play only Cricket = $45 - 27 = 18$

$\therefore A \cap B$ is the set of students who play both the games. $\therefore n(A \cap B) = 27$

Method II : Given information can be shown by Venn diagrams as follows.



Let $n(A \cap B) = x$, $n(A) = 45$, $n(B) = 52$,

We know that, $n(A \cup B) = 70$

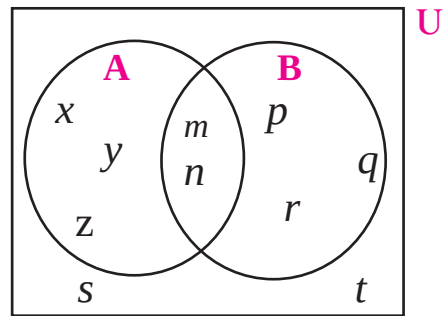
$$\begin{aligned} \therefore n(A \cap B) = x &= n(A) + n(B) - n(A \cup B) \\ &= 45 + 52 - 70 = 27 \end{aligned}$$

$$\begin{aligned} \text{Students who like to play only cricket} &= 45 - 27 \\ &= 18 \end{aligned}$$

Practice set 1.4

- (1) If $n(A) = 15$, $n(A \cup B) = 29$, $n(A \cap B) = 7$ then $n(B) = ?$
- (2) In a hostel there are 125 students, out of which 80 drink tea, 60 drink coffee and 20 drink tea and coffee both. Find the number of students who do not drink tea or coffee.
- (3) In a competitive exam 50 students passed in English. 60 students passed in Mathematics. 40 students passed in both the subjects. None of them fail in both the subjects. Find the number of students who passed at least in one of the subjects ?
- (4*) A survey was conducted to know the hobby of 220 students of class IX. Out of which 130 students informed about their hobby as rock climbing and 180 students informed about their hobby as sky watching. There are 110 students who follow both the hobbies. Then how many students do not have any of the two hobbies ? How many of them follow the hobby of rock climbing only ? How many students follow the hobby of sky watching only ?
- (5) Observe the given Venn diagram and write the following sets.

- (i) A (ii) B (iii) $A \cup B$ (iv) U
 (v) A' (vi) B' (vii) $(A \cup B)'$



Problem set 1

- (1) Choose the correct alternative answer for each of the following questions.
 - (i) If $M = \{1, 3, 5\}$, $N = \{2, 4, 6\}$, then $M \cap N = ?$
 (A) $\{1, 2, 3, 4, 5, 6\}$ (B) $\{1, 3, 5\}$ (C) ϕ (D) $\{2, 4, 6\}$
 - (ii) $P = \{x \mid x \text{ is an odd natural number, } 1 < x \leq 5\}$
 How to write this set in roster form?
 (A) $\{1, 3, 5\}$ (B) $\{1, 2, 3, 4, 5\}$ (C) $\{1, 3\}$ (D) $\{3, 5\}$
 - (iii) $P = \{1, 2, \dots, 10\}$, What type of set P is ?
 (A) Null set (B) Infinite set (C) Finite set (D) None of these
 - (iv) $M \cup N = \{1, 2, 3, 4, 5, 6\}$ and $M = \{1, 2, 4\}$ then which of the following represent set N ?
 (A) $\{1, 2, 3\}$ (B) $\{3, 4, 5, 6\}$ (C) $\{2, 5, 6\}$ (D) $\{4, 5, 6\}$

- (v) If $P \subseteq M$, then Which of the following set represent $P \cap (P \cup M)$?
 (A) P (B) M (C) $P \cup M$ (D) $P' \cap M$
- (vi) Which of the following sets are empty sets ?
 (A) set of intersecting points of parallel lines (B) set of even prime numbers.
 (C) Month of an english calendar having less than 30 days.
 (D) $P = \{x \mid x \in I, -1 < x < 1\}$
- (2) Find the correct option for the given question.
- (i) Which of the following collections is a set ?
 (A) Colours of the rainbow (B) Tall trees in the school campus
 (C) Rich people in the village (D) Easy examples in the book
- (ii) Which of the following set represent $N \cap W$?
 (A) $\{1, 2, 3, \dots\}$ (B) $\{0, 1, 2, 3, \dots\}$ (C) $\{0\}$ (D) $\{ \}$
- (iii) $P = \{x \mid x \text{ is a letter of the word 'indian'}\}$ then which one of the following is set P in listing form ?
 (A) $\{i, n, d\}$ (B) $\{i, n, d, a\}$ (C) $\{i, n, d, i, a\}$ (D) $\{n, d, a\}$
- (iv) If $T = \{1, 2, 3, 4, 5\}$ and $M = \{3, 4, 7, 8\}$ then $T \cup M = ?$
 (A) $\{1, 2, 3, 4, 5, 7\}$ (B) $\{1, 2, 3, 7, 8\}$
 (C) $\{1, 2, 3, 4, 5, 7, 8\}$ (D) $\{3, 4\}$
- (3) Out of 100 persons in a group, 72 persons speak English and 43 persons speak French. Each one out of 100 persons speak at least one language. Then how many speak only English ? How many speak only French ? How many of them speak English and French both ?
- (4) 70 trees were planted by Parth and 90 trees were planted by Pradnya on the occasion of Tree Plantation Week. Out of these; 25 trees were planted by both of them together. How many trees were planted by Parth or Pradnya ?
- (5) If $n(A) = 20$, $n(B) = 28$ and $n(A \cup B) = 36$ then $n(A \cap B) = ?$
- (6) In a class, 8 students out of 28 have only dog as their pet animal at home, 6 students have only cat as their pet animal. 10 students have dog and cat both, then how many students do not have a dog or cat as their pet animal at home ?
- (7) Represent the union of two sets by Venn diagram for each of the following.
- (i) $A = \{3, 4, 5, 7\}$ $B = \{1, 4, 8\}$
- (ii) $P = \{a, b, c, e, f\}$ $Q = \{l, m, n, e, b\}$

(iii) $X = \{x \mid x \text{ is a prime number between } 80 \text{ and } 100\}$

$Y = \{y \mid y \text{ is an odd number between } 90 \text{ and } 100\}$

(8) Write the subset relations between the following sets

$X =$ set of all quadrilaterals

$Y =$ set of all rhombuses

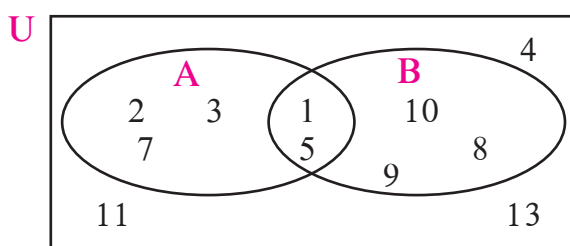
$S =$ set of all squares

$T =$ set of all parallelograms

$V =$ set of all rectangles

(9) If M is any set, then write $M \cup \phi$ and $M \cap \phi$.

(10*)



Observe the Venn diagram and write the given sets $U, A, B, A \cup B$ and $A \cap B$.

(11) If $n(A) = 7, n(B) = 13, n(A \cap B) = 4$, then $n(A \cup B) = ?$

Activity I : Fill in the blanks with elements of that set.

$U = \{1, 3, 5, 8, 9, 10, 11, 12, 13, 15\}$

$A = \{1, 11, 13\}$ $B = \{8, 5, 10, 11, 15\}$

$A' = \{\dots\dots\dots\}$ $B' = \{\dots\dots\dots\}$

$A \cap B = \{\dots\dots\dots\}$

$A' \cap B' = \{\dots\dots\dots\}$

$A \cup B = \{\dots\dots\dots\}$

$A' \cup B' = \{\dots\dots\dots\}$

$(A \cap B)' = \{\dots\dots\dots\}$

$(A \cup B)' = \{\dots\dots\dots\}$

Verify : $(A \cap B)' = A' \cup B'$, $(A \cup B)' = A' \cap B'$

Activity II : Collect the following information from 20 families nearby your house.

(i) Number of families subscribing for Marathi Newspaper.

(ii) Number of families subscribing for English Newspaper.

(iii) Number of families subscribing for both English as well as Marathi Newspaper.

Show the collected information using Venn diagram.

