6 Linear Programming

Let's Study

- Meaning of L.P.P.
- Mathematical formula of L.P.P.

  - Solution of L.P.P. by graphical Method

Let's Recall

- Linear inequations

A linear equation in two variables namely $ax+by+c=0$, where $a,b,c \in R$ and $(a,b) \neq (0,0)$, represents a straight line. A straight line makes three disjoint parts of the plane : the points lying on the straight line and two half planes on either side, which are represented by $ax+by+c<0$ or $ax+by+c>0$

The set of points $\{(x, y)|ax + by + c < 0\}$ and $\{(x,y)\ |ax+by+c>o\}$ are two open half planes The two sets have the common boundary $\{(x,y) \mid ax + by + c = 0\}$.

In the earlier classes, we have studied graphical solution of linear equations and linear inequations in two variables. In this chapter, we shall study these graphical solutions to find the maximum/minimum value of a linear expression.

Let's Learn

6.1 Linear Programming Problem (L.P.P.)

Linear programming is used in industries and government sectors where attempts are made to increase the profitability or efficiency and to reduce wastage. These problems are related to efficient use of limited resources like raw materials, man-power, availability of machine time, cost of material and so on.

Linear Programming is a mathematical technique designed to help for planning and decision making. Linear Programming problems are also known as optimization problems. Mathematical programming involves optimization of a certain function, called objective function, subject to given conditions or restrictions known as constraints.

Meaning of L.P.P.

Linear implies all the mathematical functions containing variables of index one. A L.P.P. may be defined as the problem of maximizing or minimizing a linear function subject to linear constraints.

These constraints may be equations or inequations.

Now, we formally define the terms related to L.P.P. as follows.

1) Decision Variables: The variables involved in L.P.P. are called decision variables

2) Objective function: A linear function of decision variables which is to be optimized, i.e. either maximized or minimized, is called objective function.

3) Constraints: Conditions under which the objective function is to be optimized, are called constraints. These are in the form of equations or inequations.

4) Non-negativity constraints: In some situations, the values of the variables under considerations may be positive or zero due to the imposed conditions. These constraints are referred as non-negativity constraints.

6.2 Mathematical Formulation of L.P.P.

Step 1: Identify the decision variables as (x,y) or (x_1, x_2)

Step 2: Identify the objective function and write it as mathematical expression in terms of decision variables.

Step 3: Identify the different constraints and express them as mathematical equations or inequations.

The general mathematical form of L.P.P.

The L.P.P. can be put in the following form.

Maximize $z = c_1x_1 + c_2x_2$ .... (1).

subject to the constraints.

$$\left.\begin{array}{l} a_{11}x_1 + a_{12}x_2 \le b_1 \\ a_{21}x_1 + a_{22}x_2 \le b_2 \\ \dots\dots\dots\dots\dots\dots \\ \dots\dots\dots\dots\dots\dots \\ a_{m1}x_1 + a_{m2}x_2 \le b_m \end{array}\right\} \quad (2)$$

and each $x_i \ge 0$ for $i = 1, 2$ (3)

1) The linear function in (1) is called the objective function.

2) Conditions in (3) are called non-negativity constraints.

Note:

i) We shall study L.P.P. with only two variables.

ii) We shall restrict ourselves to L.P.P. involving non-negativity constraints.

SOLVED EXAMPLES

Ex. 1:

A manufacturer produces bicycles and tricycles, each of which must be processed through two machines, A and B. Machine A has maximum of 120 hours available and machine B has a maximum of 180 hours available. Manufacturing a bicycle requires 4 hours on machine A and 10 hours on machine B. Manufacturing a tricycle requires 6 hours on machine A and 3 hours on machine B. If profits are Rs. 65 for a bicycle and Rs. 45 for a tricycle, formulate L.P.P. to maximize profit.

Solution:

Let Z be the profit, which can be made by manufacturing and selling x tricycles and y bicycles. $x \ge 0, y \ge 0$.

Total profit = $z = 45x + 65y$

Maximize $Z = 45x + 65y$

Tricycles (x)

6 hours on A — 3 hours on B

Bicycles (y)

4 hours on A — 10 hours on B

Mchine	Tricycles (x)	Bicycles (y)	Availability
A	6	4	120
B	3	10	180

From the above table, remaining conditions are

$$\left.\begin{array}{l} 6x + 4y \le 120, \\ 3x + 10y \le 180, \end{array}\right\}$$

$\therefore$ The required formulated L.P.P. is as follows.

Maximize

$z = 45x + 65y$ (objective function)

Subject to $\left.\begin{array}{l} 6x + 4y \le 120, \\ 3x + 10y \le 180, \end{array}\right\}$ (Constraints)

$x, y \ge 0 \longrightarrow$ (non negativy Constraints)

Ex. 2:

Diet for a sick person must contain at least 4000 units of vitamins, 50 units of minerals and 1500 calories. Two foods F1 and F2 cost Rs. 50 and Rs. 75 per unit respectively. Each unit of food F1 contains 200 units of Vitamins, 1 unit of minerals and 40 calories, whereas each unit of food F2 contain 100 units of vitamins, 2 units of minerals and 30 calories. Formulate the above problem as L.P.P. to satisfy sick person's requirements at minimum cost.

Solution: Let x units of food F1 and y units of food F2 be fed to sick persons to meet his requirements at minimum cost

$\therefore\ x \geq 0\ ,\ y \geq 0$

F_1

200 units Vitamins | 1 unit minerals | 40 Calories

F_2

100 units Vitamins | 2 unit minerals | 30 Calories

Food/ Product	F_1 (x) Per Unit	F_2 (y) Per Unit	Minimum requirement
Vitamins	200	100	4000
Minerals	1	2	50
Calories	40	30	1500
Cost/Unit Rs.	50	75	

sick person's problem is to determine x and y so as to minimize the total cost.

Total cost = $z = 50x + 75y$

Minimize $z = 50x + 75y$

The remaining conditions are

$200x + 100y \geq 4000$

$x + 2y \geq 50$

$40x + 30y \geq 1500$

where x, y denote units of food F_1 and F_2 respectively.

$\therefore$ $x, y \geq 0$

$\therefore$ The L.P.P. is as follows.

Minimize $z = 50x + 75y$ subject to the constraints

$200x + 100y \geq 4000$,

$x + 2y \geq 50$,

$40x + 30y \geq 1500$,

$x \geq 0, y \geq 0$.

Ex. 3:

Rakesh wants to invest at most Rs. 45000/- in savings certificates and fixed deposits. He wants to invest at least Rs. 5000/- in savings certificates and at least Rs. 15000/- in fixed deposits. The rate of interest on savings certificates is 4% p. a. and that on fixed deposits is 7% p.a. Formulate the above problem as L.P.P. to determine maximum yearly income.

Solution:

Let Rakesh invest Rs. x in savings certificate and Rs. y in fixed deposits

$\therefore\ x \geq 0,\ y \geq 0$

Since he has at most Rs. 45,000/- to invest, from the given conditions, $x + y \leq 45{,}000$.

$x \geq 5000$ and $y \geq 15000$

The rate of interest on savings certificate is 4% p.a. and that on fixed deposits is 7% p.a.

$\therefore$ Total annual income = $z = 0.04x + 0.07y$

$\therefore$ The L.P.P. is

Maximize $z = 0.04x + 0.07y$

subject to

$x \geq 5000,\ y \geq 15000,$

$x + y \leq 45000,$

$x \geq 0,\ y \geq 0.$

EXERCISE 6.1

1) A manufacturing firm produces two types of gadgets A and B, which are first processed in the foundry and then sent to machine shop for finishing. The number of man hours of labour required in each shop for production of A and B and the number of man hours available for the firm are as follows.

Gadgets	Foundry	Machine Shops
A	10	5
B	6	4
Time available (hours)	60	35

Profit on the sale of A is Rs. 30 and B is Rs. 20 per unit Formulate the LPP to have maximum profit.

2) In a cattle breeding firm, it is prescribed that the food ration for one animal must contain 14, 22 and 1 unit of nutrients A, B and C respectively Two different kinds of fodder are available. Each unit weight of these two contains the following amounts of these three nutrients.

Fodder / Nutrients	Fodder 1	Fodder 2
Nutrients A	2	1
Nutrients B	2	3
Nutrients C	1	1

The cost of fodder 1 is Rs. 3 per unit and that of fodder 2 is Rs. 2 per unit. Formulate the LPP to minimize the cost.

3) A Company manufactures two types of chemicals A and B. Each chemical requires two types of raw material P and Q. The table below shows number of units of P and Q required to manufacture one unit of A and one unit of B.

Chemical / Raw Material	A	B	Availability
P	3	2	120
Q	2	5	160

The company gets profits of Rs. 350/- and Rs. 400/- by selling one unit of A and one unit of B respectively. Formulate the problem as LPP to maximize the profit.

4) A printing company prints two types of magazines A and B. The company earns Rs. 10 and Rs. 15 on magazine A and B per copy. These are processed on three machine I, II, III. Magazine A requires 2 hours on machine I, 5 hours on machine II, and 2 hours on machine III. Magazine B requires 3 hours on machine I, 2 hours on machine II and 6 hours on machine III, Machines I, II, III are available for 36, 50, 60 hours per week respective.

Formulate the Linear programming problem to maximize the profit.

5) A manufacturer produces bulbs and tubes. Each of these must be processed through two machines. M_1 and M_2. A package of bulbs require 1 hour of work on machine M_1 and 3 hours of work on M_2. A package of tubes require 2 hours on machine M_1 and 4 hours machine M_2. He earns a profit of Rs. 13.5 per package of bulbs and Rs. 55 per package of tubes. If he operates machine M_1 for atmost 10 hours a day and machine M_2 for atmost 12 hours a day then formulate the LPP to maximize the profit.

6) A Company manufactures two types of fertilizers F1 and F_2. Each type of fertilizer requires two raw materials A and B. The number of units of A and B required to manufacture one unit of fertilizer F_1 and F_2 and availability of the raw materials A and B per day are given in the table below

Fertilizer / Raw Materials	F_1	F_2	Availability
A	2	3	40
B	1	4	70

By selling one unit of F_1 and one unit of F_2, company get a profit of Rs. 500 and Rs. 750 respectively. Formulate the problem as LPP to maximize the profit.

7) A doctor has prescribed two different kinds of foods A and B to form a weekly diet for sick person. The minimum requirement of fats, carbohydrates and proteins are 18, 28, 14 units respectively. One unit of food A has 4 units of fat, 14 units of carbohydrates and 8 units of protein. One unit of food B has 6 units of fat, 12 units of carbohydrates and 8 units of protein. The price of food A is Rs. 4.5 per unit and that of food B is Rs. 3.5 per unit. Form the LPP so that the sick person's diet meets the requirements at a minimum cost.

8) If John drives a car at a speed of 60 kms/hour he has to spend Rs. 5 per km on petrol. If he drives at a faster speed of 90 km/hour, the cost of petrol increases to Rs. 8 per km. He has Rs. 600 to spend on petrol and wishes to travel the maximum distance within an hour. Formulate the above problem as LPP.

9) The company makes concrete bricks made up of cement and sand. The weight of a concrete brick has to be at least 5 kg. cement costs Rs. 20 per kg. and sand cost Rs. 6 per kg. strength consideration dictate that a concrete brick should contain minimum 4kg of cement and not more that 2 kg of sand. Formulate the LPP for the cost to be minimum.

6.2.1 Convex set and feasible region.

Definition: A set of points in a plane is said to be a convex set if the line segment joining any two points of the set entirely lies within the same set.

The following sets are convex sets

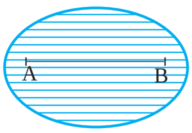

Fig. 6.1

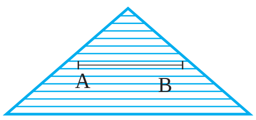

Fig. 6.2

The following sets are not convex sets

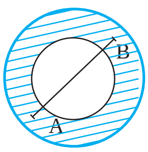

Fig. 6.3

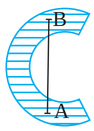

Fig. 6.4

Note :

i) The convex sets may be bounded. Following are bounded convex sets.

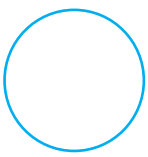
Fig. 6.5

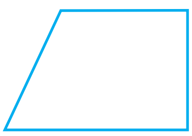
Fig. 6.6

ii) Convex sets may be unbounded Following are unbounded conves sets

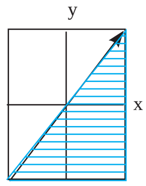

Fig. 6.7

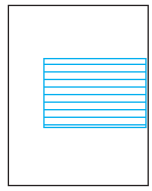
Fig. 6.8

Solution of LLP:

There are two methods to find the solution of L.P.P. 1) Graphical method 2) Simplex method.

Note : We shall restrict ourselves to graphical method.

Some definitions:

1) **Solution :** A set of values of variable which satisfies all the constraints of the LPP, is called the solultion of the LPP.

2) **Feasible Solution:** Solution which satisfy all constraints is called feasible solution.

3) **Optimum feasible solution:** A feasible solution which optimizes i.e. either maximizes or minimizes the objective function of LPP is called optimum feasible solution.

4) **Feasible Region:** The common region determined by all the constraints and non-negativity restrictions of the linear programming problem is called the feasible region.

Note: The boundaries of the region may or may not be included in the feasible region.

Theorems (without proof)

Theorem 1 : The set of all feasible solutions of LPP is a convex set.

Convex polygon theorem:

Theorem 2 : The objective function of LPP attains its optimum value (either maximum or minimum) at, at least one of the vertices of convex polygon.

Note: If a LPP has optimum solutions at more than one point then the entire line joining those two points will give optimum solutions. Hence the problem will have infinite solutions.

Solution of LPP by Corner point method (convex polygon theorem) Algorithm:

Step:

i) Convert all inequation of the constraints into equations.

ii) Draw the lines in xy plane, by using x intercept and y intercept of the line from its equation.

iii) Locate common region indicated by the constraints. This common region is called feasible region.

iv) Find the vertices of the feasible region.

v) Find the value of the objective function z at all vertices of the feasible region.

vi) If the objective function is of maximization (or minimization) type, then the co-ordinates, of the vertex (Vertices) for which z is maximum (or minimum) gives (give) the optimum solution/solutions.

SOLVED EXAMPLES

Ex.1. Maximize $z = 9x + 13y$ Subject to

$$2x + 3y \le 18,\ 2x + y \le 10,\ x \ge 0,\ y \ge 0$$

Solution: To draw $2x + 3y \le 18$, and $2x + y \le 10$

Draw lines $2x + 3y = 18$, and $2x + y = 10$.

Equation of line	Intercept	Constraint type	Feasible Region
$2x + 3y = 18$	$x : 9$ $y : 6$	$\le$	Originside
$2x + y = 10$	$x : 5$ $y : 10$	$\le$	Originside

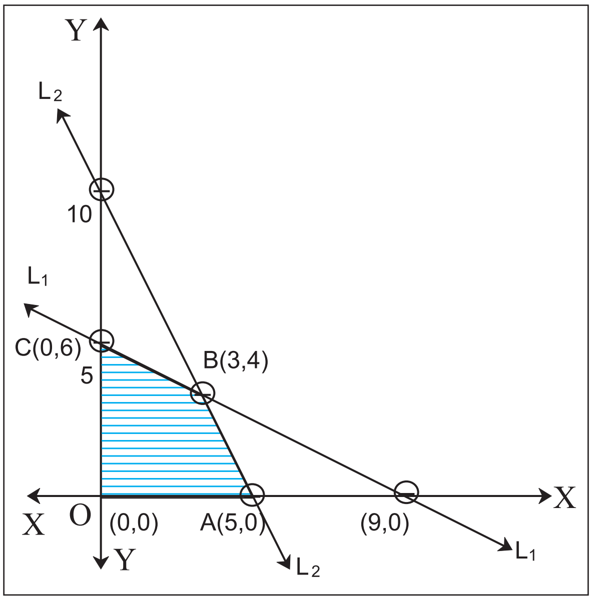

Fig. 6.9

The common shaded region OABCO is the feasible region with vertices 0(0,0), A(5,0), B(3,4), C (0,6).

Vertex	Lines through vertex	Value of objective
A (5, 0)	$2x + y = 10$ $y = 0$	45
B (3, 4)	$2x + 3y = 18$ $2x + y = 10$	79 Maximum
C (0, 6)	$2x + 3y = 18$ $x = 0$	78
O (0, 0)	$x = 0$ $y = 0$	0

From the table, maximum value of $z = 79$ occurs at B (3, 4) i.e. when $x = 3$, $y = 4$.

Ex. 2. Solve graphically the following LPP Minimize $z = 5x + 2y$ subject to

$$5x + y \ge 10,\ x + y \ge 6,\ x \ge 0,\ y \ge 0.$$

Solution: To draw $5x + y \ge 10$, and $x + y \ge 6$

Draw lines $5x + y = 10$, $x + y = 6$.

Equation of line	Intercept	Constraint type	Feasible Region
$5x + y = 10$	$x : 2$ $y : 10$	$\ge$	Non-originside
$x + y = 6$	$x : 6$ $y : 6$	$\ge$	Non-originside

The common shaded region is feasible region with vertices A(6,0), B (1, 5), C (0,10)

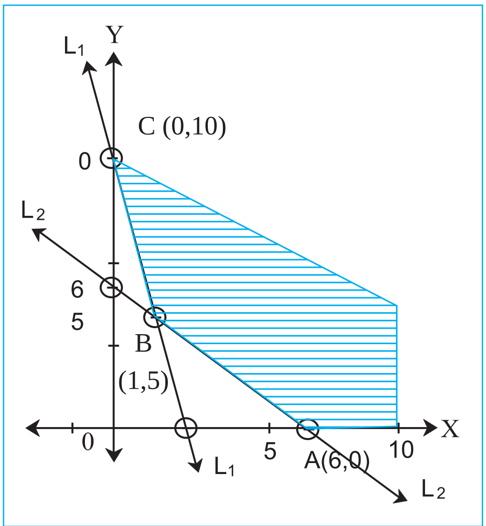

Fig. 6.10

(x, y)	Value of $z = 5x + 2y$ at (x, y)
A (6, 0)	30
B (1, 5)	15
C (0, 10)	20

From the table, we observe the following.

The minimum value of $z = 15$ occurs at B (1,5) ie. when $x = 1, y = 5$.

Ex. 3. Maximize $z = 3x + 4y$ Subject to

$x - y \geq -1,\ 2x - y \leq 2,\ x \geq 0,\ y \geq 0.$

Solution: To draw $x - y \geq -1$, $2x - y \leq 2$,

Draw lines $x - y = -1$, $2x - y = 2$.

Vertex	Lines through vertex	Value of objective
(3, 4)	$x - y = -1$ $2x - y = 2$	25
(1, 0)	$2x - y = 2$ $y = 0$	3

From graph, we can see that the common shaded area is feasible region.

which is unbounded (not a closed polygon)

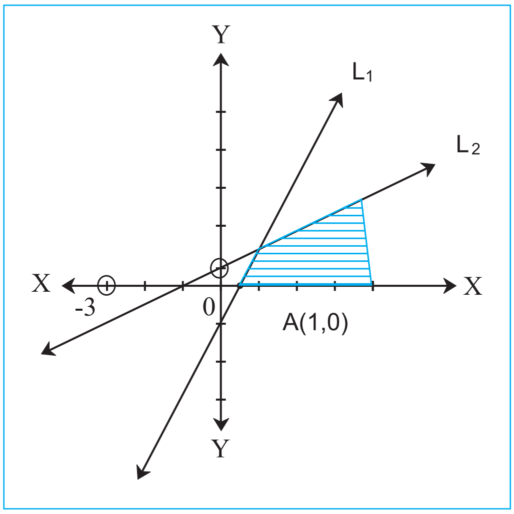

Fig. 6.11

∴ There is no finite maximum value of z since the feasible region is unbounded.

EXERCISE 6.2

Solve the following LPP by graphical method

1. Maximize $z = 11x + 8y$ Subject to
   $x \leq 4, y \leq 6\ x + y\ \leq 6, x \geq 0, y \geq 0.$
2. Maximize $z = 4x + 6y$ Subject to
   $3x + 2y \leq 12\ x + y \geq 4, x, y \geq 0.$
3. Maximize $z = 7x + 11y$ Subject to
   $3x + 5y \leq 26,\ \ 5x + 3y \leq 30, x \geq 0, y \geq 0.$
4. Maximize $z = 10x + 25y$
   Subject to $0 \leq x \leq 30 \leq y \leq 3, x + y \leq 5.$
5. Maximize $z = 3x + 5y$
   Subject to $x + 4y \leq 24, 3x + y \leq 21$
   $x + y \leq 9, x \geq 0, y \geq 0.$
6. Minimize $z = 7x + y$ Subject to
   $5x + y \geq 5, x + y \geq 3, x \geq 0, y \geq 0$
7. Minimize $z = 8x + 10y$ Subject to
   $2x + y \geq 7, 2x + 3y \geq 15, y \geq 2, x \geq 0, y \geq 0$
8. minimize $z = 6x + 2y$ Subject to $x + 2y \geq 3$,
   $x + 4y \geq 4\ 3x + y \geq 3\ x \geq 0, y \geq 0,$

Working Rule to formulate the LPP.

Step1: Identify the decision variables and assign the symbols x, y or x_1, x_2 to them. Introduce non-negativity constraints.

Step2: Identify the set of constraints and express them as linear inequations in terms of the decision variables.

Step3: Identify the objective function to be optimized (ie. maximized on minimized) and express it as a linear function of decisions variables.

* Let R be the feasible region (convex polygon) for a LPP and let $z = ax + by$ be the objective function then the optimum value (maximum or minimum) of z occurs at, at least one of the corner points (vertex) of the feasible region.

Corner point method for solving the LPP graphically.

Step1: Find the feasible region of the LPP.

Step2: Determine the vertices of the feasible region either by inspection or by solving the two equations of the lines intersecting at that points.

Step 3: Find the value of the objective function z, at all vertices of feasible region.

Step 4: Determine the feasible solution which optimizes the value of the objective function.

Working rule to formulate and solve the LPP Graphically.

Identify the decision variables and assign the symbols x, y or x_1, x_2 to them.

Identify the objective function (maximized or minimized) and express it as a linear function of decision variables.

Convert inequations (constraints) into equations, find out intercept points on them.

Draw the graph.

Identify the feasible region (convex polygon) of the L.P.P. and shade it.

Find all corner points of the feasible region.

Find the value of z at all the corner points.

State the optimum value of z (maximum or minimum).

MISCELLANEOUS EXERCISE - 6

I Choose the correct alternative.

1. The value of objective function is maximize under linear constraints.
   a) at the centre of feasible region
   b) at (0, 0)
   c) at any vertex of feasible region.
   d) The vertex which is at maximum distance from (0, 0).

2. Which of the following is correct?
   a) Every LPP has an optimal solution
   b) Every LPP has unique optimal solution.
   c) If LPP has two optimal solutions then it has infinitely many optimal solutions.
   d) The set of all feasible solutions of LPP may not be a convex set.

3. Objective function of LPP is
   a) a constraint
   b) a function to be maximized or minimized
   c) a relation between the decision variables
   d) a feasible region.

4. The maximum value of $z = 5x + 3y$. subject to the constraints $3x + 5y = 15$; $5x + 2y \leq 10$, $x, y \geq 0$ is.
   a) 235 b) 235/9
   c) 235/19 d) 235/3

5. The maximum value of $z = 10x + 6y$, subjected to the constraints $3x + y \leq 12$, $2x + 5y \leq 34$, $x \geq 0, y \geq 0$ is.
   a) 56 b) 65
   c) 55 d) 66

6. The point at which the maximum value of $z = x + y$ subject to the constraints

$x + 2y \leq 70, \; 2x + y \leq 95, \; x \geq 0, \; y \geq 0$ is

a) (36, 25) b) (20, 35)
c) (35, 20) d) (40, 15)

7. Of all the points of the feasible region the optimal value of z is obtained at a point

a) inside the feasible region.
b) at the boundary of the feasible region.
c) at vertex of feasible region.
d) on x - axis.

8. Feasible region; the set of points which satisfy.

a) The objective function.
b) All of the given function.
c) Some of the given constraints
d) Only non-negative constrains

9. Solution of LPP to minimize $z = 2x + 3y$ st. $x \geq 0, y \geq 0, 1 \leq x + 2y \leq 10$ is

a) $x = 0, y = 1/2$ b) $x = 1/2, y = 0$
c) $x = 1, y = -2$ d) $x = y = 1/2$.

10. The corner points of the feasible region given by the inequations $x + y \leq 4$,

$2x + y \leq 7, x \geq 0, y \geq 0$, are

a) (0, 0), (4, 0), (3, 1), (0, 4).
b) (0, 0), (7/2, 0), (3, 1), (0, 4).
c) (0, 0), (7/2, 0), (3, 1), (5, 7).
d) (6, 0), (4, 0), (3, 1), (0, 7).

11. The corner points of the feasible region are (0, 0), (2, 0), (12/7, 3/7) and (0,1) then the point of maximum $z = 6.5x + y$

a) (0, 0) b) (2, 0)
c) (11/7, 3/7) d) (0, 1)

12. If the corner points of the feasible region are (0, 0), (3, 0), (2, 1) and (0, 7/3) the maximum value of $z = 4x + 5y$ is .

a) 12 b) 13
c) 35/2 d) 0

13. If the corner points of the feasible region are (0, 10), (2, 2), and (4, 0) then the point of minimum $z = 3x + 2y$ is.

a) (2, 2) b) (0, 10)
c) (4, 0) d) (2, 4)

14. The half plane represented by $3x + 2y \leq 0$ contains the point.

a) (1, 5/2) b) (2,1)
c) (0, 0) d) (5, 1)

15. The half plane represented by $4x + 3y \geq 24$ contains the point

a) (0, 0) b) (2, 2)
c) (3, 4) d) (1, 1)

II Fill in the blanks.

1) Graphical solution set of the in equations

$x \geq 0, y \geq 0$ is in ....quadrant

2) The region represented by the in equations $x \geq 0, \; y \geq 0$ lies in .... quadrants

3) The optimal value of the objective function is attained at the .....points of feasible region.

4) The region represented by the inequality

$y \leq 0$ lies in ....quadrants

5) The constraint that a factory has to employ more women (y) than men (x) is given by........

6) A garage employs eight men to work in its showroom and repair shop. The constants that there must be not least 3 men in showroom and repair shop. The constrains that there must be at least 3 men in showroom and at least 2 men in repair shop are.........and ....respectively

7) A train carries at least twice as many first class passengers (y) as second class passengers (x) The constraint is given by......

8) A dish washing machine holds up to 40 pieces of large crockery (x) This constraint is given by...........

III State whether each of the following is True or False.

1) The region represented by the inequalities $x \geq 0$, $y \geq 0$ lies in first quadrant.

2) The region represented by the in qualities $x \leq 0$, $y \leq 0$ lies in first quadrant.

3) The optimum value of the objective function of LPP occurs at the center of the feasible region.

4) Graphical solution set of $x \leq 0, y \geq 0$ in xy system lies in second quadrant.

5) Saina wants to invest at most Rs. 24000 in bonds and fixed deposits. Mathematically this constraints is written as $x + y \leq 24000$ where x is investment in bond and y is in fixed deposits.

6) The point (1, 2) is not a vertex of the feasible region bounded by $2x + 3y \leq 6$, $5x + 3y \leq 15$, $x \geq 0$, $y \geq 0$.

7) The feasible solution of LPP belongs to only quadrant I The Feasible region of graph $x + y \leq 1$ and $2x + 2y \geq 6$ exists.

IV) Solve the following problems.

1) Maximize $z = 5x_1 + 6x_2$ Subject to
$2x_1 + 3x_2 \leq 18$, $2x_1 + x_2 \leq 12$, $x \geq 0$, $y \geq 0$

2) Minimize $z = 4x + 2y$ Subject to
$3x + y \geq 27$, $x + y \geq 21$, $x \geq 0$, $y \geq 0$

3) Maximize $z = 6x + 10y$ Subject to
$3x + 5y \leq 10$, $5x + 3y \leq 15$, $x \geq 0$, $y \geq 0$

4) Minimize $z = 2x + 3y$ Subject to
$x - y \leq 1$, $x + y \geq 3$, $x \geq 0$, $y \geq 0$

5) Maximize $z = 4x_1 + 3x_2$ Subject to
$3x_1 + x_2 \leq 15$, $3x_1 + 4x_2 \leq 24$, $x \geq 0$, $y \geq 0$

6) Maximize $z = 60x + 50y$ Subject to
$x + 2y \leq 40$, $3x + 2y \leq 60$, $x \geq 0$, $y \geq 0$

7) Minimize $z = 4x + 2y$ Subject to
$3x + y \geq 27$, $x + y \geq 21$, $x + 2y \geq 30$
$x \geq 0$, $y \geq 0$

8) A carpenter makes chairs and tables profits are Rs. 140 per chair and Rs. 210 per table Both products are processed on three machine, Assembling, Finishing and Polishing the time required for each product in hours and availability of each machine is given by following table.

Product / Machines	Chair (x)	Table (y)	Available time (hours)
Assembling	3	3	36
Finishing	5	2	50
Polishing	2	6	60

Formulate and solve the following Linear programming problems using graphical method.

9) A company manufactures bicyles and tricycles, each of which must be processed through two machines A and B Maximum availability of machine A and B is respectively 120 and 180 hours. Manufacturing a bicycle requires 6 hours on machine A and 3 hours on machine B Manufacturing a tricycle requires 4 hours on machine A and 10 hours on machine B. If profits are Rs. 180 for a bicycle and Rs. 220 on a tricycle, determine the number of bicycles and tricycles that should be manufacturing in order to maximize the profit.

10) A factory produced two types of chemicals A and B The following table gives the units of ingredients P & Q (per kg) of Chemicals A and B as well as minimum requirements of P and Q and also cost per kg. of chemicals A and B.

Chemicals Units / Ingredients per kg.	A (x)	B (y)	Minimum requirements in
P	1	2	80
Q	3	1	75
Cost (in Rs.)	4	6	

Find the number of units of chemicals A and B should be produced so as to minimize cost.

11) A Company produces mixers and processors Profit on selling one mixer and one food processor is Rs. 2000 and Rs. 3000 respectively. Both the products are processed through three machines A, B, C The time required in hours by each product and total time available in hours per week on each machine are as follows:

Product / Machine	Mixer per unit	Food processor per unit	Available time
A	3	3	36
B	5	2	50
C	2	6	60

How many mixers and food processors should be produced to maximize the profit?

12) A Chemical company produces a chemical containing three basic elements A, B, C so that it has at least 16 liters of A, 24 liters of B and 18 liters of C. This chemical is made by mixing two compounds I and II. Each unit of compound I has 4 liters of A, 12 liters of B, 2 liters of C Each unit of compound II has 2 liters of A, 2 liters of B and 6 liters of C. The cost per unit of compound I is Rs. 800 and that of compound II is Rs. 640 Formulate the problem as LPP. and solve it to minimize the cost.

13) A person makes two types of gift items A and B requiring the services of a cutter and a finisher. Gift item A requires 4 hours of cutter's time and 2 hours of finisher's time. B requires 2 hours of cutters time, 4 hours of finishers time. The cutter and finisher have 208 hours and 152 hours available times respectively every month. The profit of one gift item of type A is Rs. 75 and on gift item B is Rs. 125. Assuming that the person can sell all the items produced, determine how many gift items of each type should be make every month to obtain the best returns?

14) A firm manufactures two products A and B on which profit earned per unit is Rs. 3 and Rs. 4 respectively. The product A requirs one minute of processing time on M_1 and 2 minutes on M_2. B requires one minutes on M_1 and one minute on M_2. Machine M_1 is available for use for 450 minutes while M_2 is available for 600 minutes during any working day. Find the number of units of product A and B to be manufactured to get the maximum profit.

15) A firm manufacturing two types of electrical items A and B, can make a profit of Rs. 20 per unit of A and Rs. 30 per unit of B. Both A and B make use of two essential components, a motor and a transformer. Each unit of A requires 3 motors and 2 transformers and each unit of B requires 2 motors and 4 transformers. The total supply of components per month is restricted to 210 motors and 300 transformers. How many units of A and B should be manufacture per month to maximize profit? How much is the maximum profit?

Activities

1) Find the graphical solution for the following system of linear inequations.

$8x + 5y \leq 40,\ \ 4x + 5y \leq 40,\ \ x \geq 0, y \geq 0$

Solution to draw $8x + 5y \leq 40$

Draw line L_1 $8x + 5y = 40$

x	y	(x, y)	Sign	Region
	0	(, 0)	$\leq$	On origin side of line
0		(0,)		

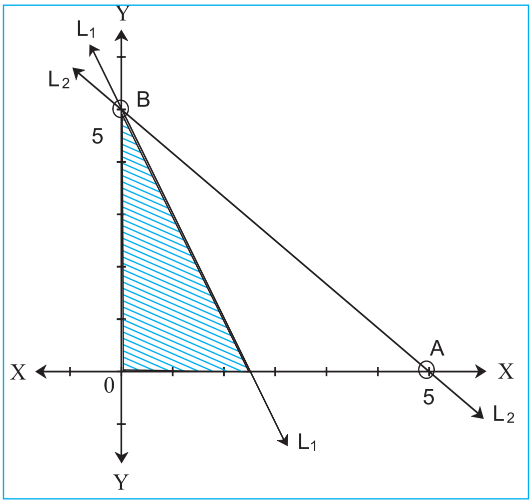

Fig. 6.12

To draw $4x + 5y \le 40$

Draw line $L_2 : 4x + 5y = 40$

x	y	(x, y)	Sign	Region
	0	(, 0)	$\le$	On origin side of line
0		(0,)		

The common shaded region OABO is graphical solution, with vertices 0(,), A (,) , B (,)

2) Find the graphical solution for the following system of linear inequations $3x + 5y \ge 15$, $2x + 5y \le 15$, $2x + 3y \le 18$, $x \ge 0$, $y \ge 0$

Solution : To Draw $3x + 5y \ge 15$

Draw line $3x + 5y = 15$.

x	y	(x, y)	Sign	Region
	0	(, 0)	$\le$	On origin side of line
0		(0,)		

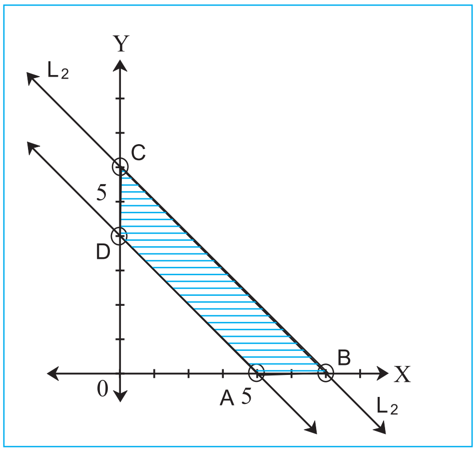

Fig. 6.13

The common shaded region OABO is graphical solution, with vertices 0 (,), A (,), B (,)

3) Shraddha wants to invest at most 25,000/- in savings certificates and fixed deposits. She wants to invest at least Rs. 10,000/- in savings certificate and at least Rs. 15,000/- in fixed deposits. The rate of interest on saving certificate is 5% per annum and that on fixed deposits is 7% per annum. Formulate the above problem as LPP to determine maximum yearly income.

Solution: Let x_1 amount (in Rs.) invest in saving cerficate

x_2: amount (in Rs.) invest in fixed deposits.

$x_1 \ge 0$, $x_2 \ge 0$

From given conditions $x_1 + x_2$ ☐ 25,000

She wants to invest at least Rs. 10000/- in saving certificate

$\therefore x_1$ ☐ 10,000

Shradha want to invest at least Rs. 15,000/- in fixed deposits.

x_2 ☐ 15,000

Total interest = z = ...

Maximize z = ... Subject to.

....

....

4) The graphical solution of LPP is shown by following figure. Find the maximum value of $z = 3x + 2y$ subject to the conditions given in graphical solution.

Solution: From Fig. 6.14. The common Shaded region OABCO is feasible region with vertices 0(,), A(,), B(4, 3) C(,)

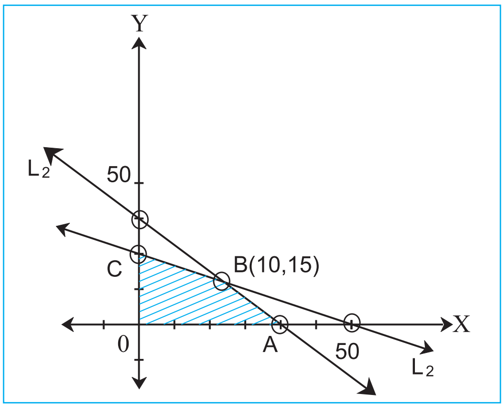

Fig. 6.14

Sr. No	(x, y)	Value of $z = 3x + 2y$ at (x, y)
1)	0(0, 0)	$z = 6$
2)	A(5, 0)	$z =$
3)	B(,)	$z =$
4)	C(0, 3)	$z = 10$

From above table, maximum value of

z = ...occurs at point .... that is when x = ..., y = ...

5) Formulate and solve the following LPP. A company manufactures bicycles and tricycles, each of which must be processed through two machines A and B. Machine A has maximum of 120 hours available and machine B has a maximum of 180 hours available. Manufacturing a Bicycle require 6 hours on machine A and 3 hours on machine B. Manufacturing a tricycle requires 4 hours on machine A and 10 hours on machine B. If profits are Rs. 180/- for a bicycle and Rs. 220/- for a tricycle, determine the number of bicycles and tricycles that should be manufactured in order to maximize the profit.

Sol. Let x no of bicycles and y no. of tricycles be manufactured $x \geq 0$, $y \geq 0$ 1

Total profit = z =

Maximize z =

The remaining conditions are............

$\therefore$ LPP is maximize z = ...

subject to $x \geq 0$, $y \geq 0$, ...

To draw $6x + 4y \leq 120$

Draw line L_1 : $6x + 4y = 120$

x	y	(x, y)	Sign	Region
	0	(, 0)	$\leq$	origin side of line L_1
0		(0,)		

To draw $3x + 10y \leq 180$

Draw line L_2 : $3x + 10y = 180$

x	y	(x, y)	Sign	Region
	0	(, 0)	$\leq$	--
0		(0,)		

The common shaded region ☐ is feasible region with vertices 0(0, 0), A(,), B(10, 15), C(0, 18).

Sr. No	(x, y)	Value of $z = 3x + 2y$ at (x, y)
1)	0(0, 0)	$z = 0$
2)	A(, 0)	$z =$
3)	B(,)	$z =$
4)	C(0, 18)	$z =$

Maximum value of z = ☐ occurs at point .... that is when x = , y =

Thus company gets maximum profit z = Rs. ☐ when x = ☐ no of bicycles and y = ☐ no of tricycles are manufactured.

✦✦✦