1 Similarity

Let's study.

- Ratio of areas of two triangles
- Basic proportionality theorem
- Converse of basic proportionality theorem
- Tests of similarity of triangles
- Property of an angle bisector of a triangle
- Property of areas of similar triangles
- The ratio of the intercepts made on the transversals by three parallel lines

Let's recall.

We have studied Ratio and Proportion. The statement, 'the numbers a and b are in the ratio $\frac{m}{n}$' is also written as, 'the numbers a and b are in proportion $m:n$.'
For this concept we consider positive real numbers. We know that the lengths of line segments and area of any figure are positive real numbers.
We know the formula of area of a triangle.

Area of a triangle = $\frac{1}{2}$ Base $\times$ Height

Let's learn.

Ratio of areas of two triangles

Let's find the ratio of areas of any two triangles.

Ex. In Δ ABC, AD is the height and BC is the base.
In Δ PQR, PS is the height and QR is the base

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{\frac{1}{2} \times BC \times AD}{\frac{1}{2} \times QR \times PS}$$

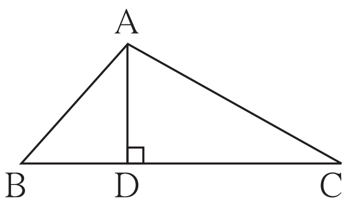

Fig. 1.1

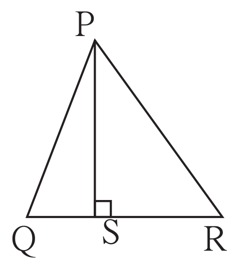

Fig. 1.2

$$\therefore \frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{BC \times AD}{QR \times PS}$$

Hence the ratio of the areas of two triangles is equal to the ratio of the products of their bases and corrosponding heights.

Base of a triangle is b_1 and height is h_1. Base of another triangle is b_2 and height is h_2. Then the ratio of their areas = $\frac{b_1 \times h_1}{b_2 \times h_2}$

Suppose some conditions are imposed on these two triangles,

Condition 1 : If the heights of both triangles are equal then-

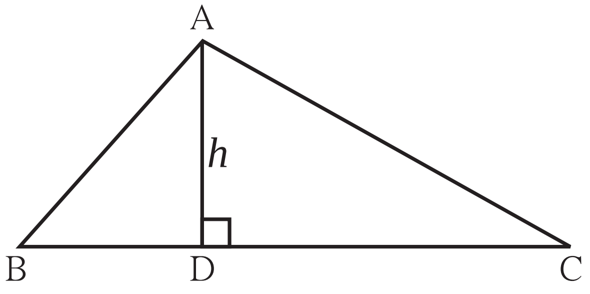

Fig. 1.3

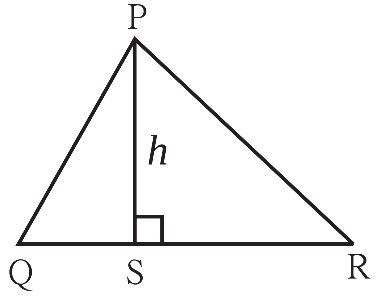

Fig. 1.4

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{BC \times h}{QR \times h} = \frac{BC}{QR}$$

$$\therefore \frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{b_1}{b_2}$$

Property: The ratio of the areas of two triangles with equal heights is equal to the ratio of their corresponding bases.

Condition 2 : If the bases of both triangles are equal then -

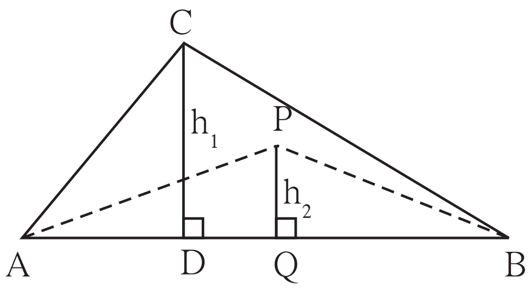

Fig. 1.5

$$\frac{A(\Delta ABC)}{A(\Delta APB)} = \frac{AB \times h_1}{AB \times h_2}$$

$$\frac{A(\Delta ABC)}{A(\Delta APB)} = \frac{h_1}{h_2}$$

Property: The ratio of the areas of two triangles with equal bases is equal to the ratio of their corresponding heights.

Activity :

Fill in the blanks properly.

(i)

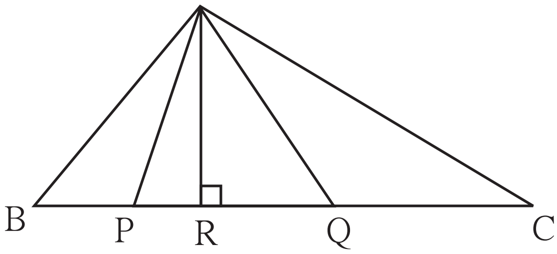

Fig. 1.6

$$\frac{A(\Delta ABC)}{A(\Delta APQ)} = \frac{\boxed{} \times \boxed{}}{\boxed{} \times \boxed{}} = \frac{\boxed{}}{\boxed{}}$$

(ii)

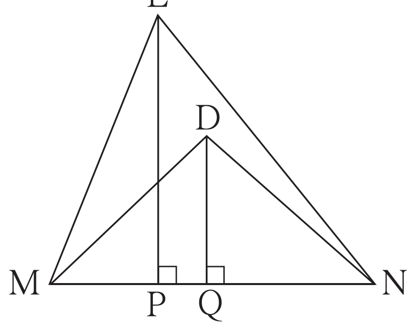

Fig.1.7

$$\frac{A(\Delta LMN)}{A(\Delta DMN)} = \frac{\boxed{} \times \boxed{}}{\boxed{} \times \boxed{}} = \frac{\boxed{}}{\boxed{}}$$

(iii) M is the midpoint of seg AB and seg CM is a median of Δ ABC

$$\therefore \frac{A(\Delta AMC)}{A(\Delta BMC)} = \frac{\boxed{}}{\boxed{}} = \frac{\boxed{}}{\boxed{}} = \boxed{}$$

State the reason.

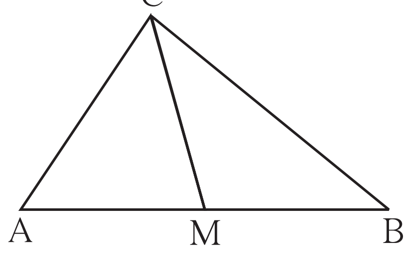

Fig. 1.8

Solved Examples

Ex. (1)

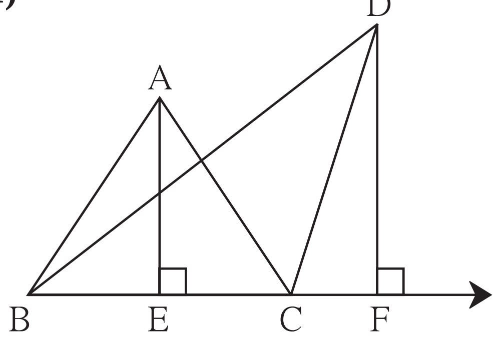

Fig.1.9

In adjoining figure

AE $\perp$ seg BC, seg DF $\perp$ line BC,

AE = 4, DF = 6 , then find $\frac{A(\Delta ABC)}{A(\Delta DBC)}$.

Solution : $\frac{A(\Delta ABC)}{A(\Delta DBC)} = \frac{AE}{DF}$ .... bases are equal, hence areas proportional to heights.

$$= \frac{4}{6} = \frac{2}{3}$$

Ex. (2) In Δ ABC point D on side BC is such that DC = 6, BC = 15. Find A(Δ ABD) : A(Δ ABC) and A(Δ ABD) : A(Δ ADC) .

Solution : Point A is common vertex of Δ ABD, Δ ADC and Δ ABC and their bases are collinear. Hence, heights of these three triangles are equal

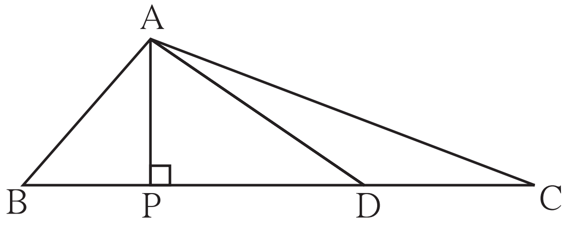

Fig. 1.10

BC = 15, DC = 6 $\therefore$ BD = BC − DC = 15 − 6 = 9

$$\frac{A(\Delta ABD)}{A(\Delta ABC)} = \frac{BD}{BC}$$ .... heights equal, hence areas proportional to bases.

$$= \frac{9}{15} = \frac{3}{5}$$

$$\frac{A(\Delta ABD)}{A(\Delta ADC)} = \frac{BD}{DC}$$ .... heights equal, hence areas proportional to bases.

$$= \frac{9}{6} = \frac{3}{2}$$

Ex. (3)

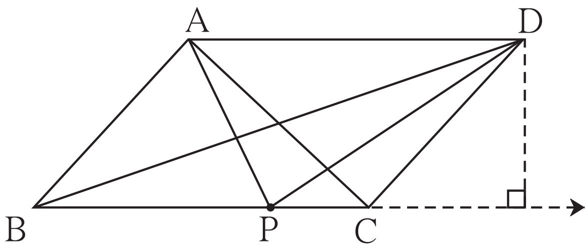

Fig. 1.11

□ ABCD is a parallelogram. P is any point on side BC. Find two pairs of triangles with equal areas.

Solution : □ ABCD is a parallelogram.

$\therefore$ AD || BC and AB || DC

Consider Δ ABC and Δ BDC.

Both the triangles are drawn in two parallel lines. Hence the distance between the two parallel lines is the height of both triangles.

In Δ ABC and Δ BDC, common base is BC and heights are equal.

Hence, A(Δ ABC) = A(Δ BDC)

In Δ ABC and Δ ABD, AB is common base and and heights are equal.

$\therefore$ A(Δ ABC) = A(Δ ABD)

Ex.(4)

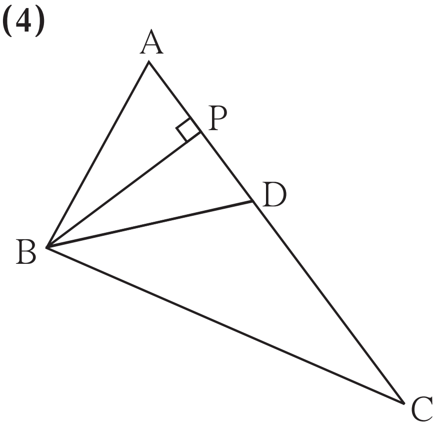

Fig. 1.12

In adjoining figure in Δ ABC, point D is on side AC. If AC = 16, DC = 9 and BP $\perp$ AC, then find the following ratios.

(i) $\dfrac{A(\Delta ABD)}{A(\Delta ABC)}$ (ii) $\dfrac{A(\Delta BDC)}{A(\Delta ABC)}$

(iii) $\dfrac{A(\Delta ABD)}{A(\Delta BDC)}$

Solution : In Δ ABC point P and D are on side AC, hence B is common vertex of Δ ABD, Δ BDC, Δ ABC and Δ APB and their sides AD, DC, AC and AP are collinear. Heights of all the triangles are equal. Hence, areas of these triangles are proportinal to their bases. AC = 16, DC = 9

$\therefore$ AD = 16 − 9 = 7

$\therefore \dfrac{A(\Delta ABD)}{A(\Delta ABC)} = \dfrac{AD}{AC} = \dfrac{7}{16}$ . . . . triangles having equal heights

$\dfrac{A(\Delta BDC)}{A(\Delta ABC)} = \dfrac{DC}{AC} = \dfrac{9}{16}$ . . . . triangles having equal heights

$\dfrac{A(\Delta ABD)}{A(\Delta BDC)} = \dfrac{AD}{DC} = \dfrac{7}{9}$ . . . . triangles having equal heights

Remember this !

- Ratio of areas of two triangles is equal to the ratio of the products of their bases and corresponding heights.
- Areas of triangles with equal heights are proportional to their corresponding bases.
- Areas of triangles with equal bases are proportional to their corresponding heights.

Practice set 1.1

1. Base of a triangle is 9 and height is 5. Base of another triangle is 10 and height is 6. Find the ratio of areas of these triangles.

2. In figure 1.13 $BC \perp AB$, $AD \perp AB$, $BC = 4$, $AD = 8$, then find $\dfrac{A(\Delta ABC)}{A(\Delta ADB)}$.

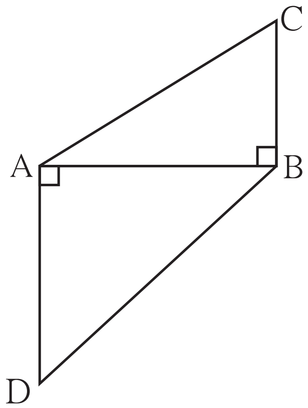

Fig. 1.13

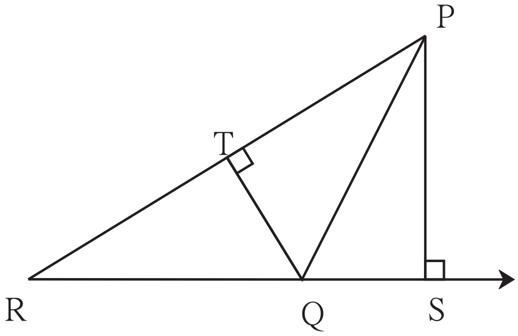

Fig. 1.14

3. In adjoining figure 1.14 seg $PS \perp$ seg RQ seg $QT \perp$ seg PR. If $RQ = 6$, $PS = 6$ and $PR = 12$, then find QT.

4. In adjoining figure, $AP \perp BC$, $AD \parallel BC$, then find $A(\Delta ABC) : A(\Delta BCD)$.

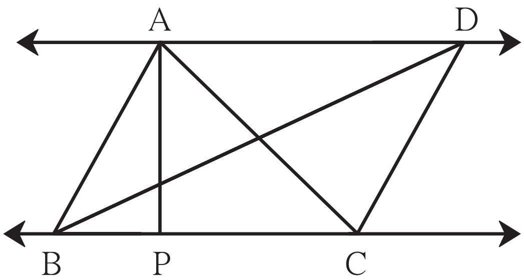

Fig. 1.15

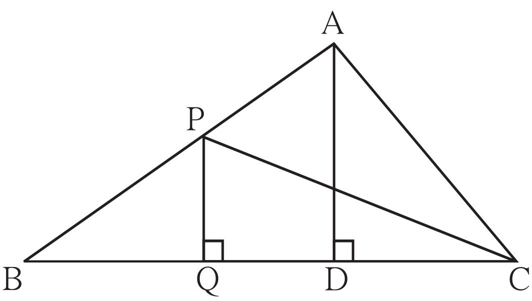

Fig. 1.16

5. In adjoining figure $PQ \perp BC$, $AD \perp BC$ then find following ratios.

(i) $\dfrac{A(\Delta PQB)}{A(\Delta PBC)}$ (ii) $\dfrac{A(\Delta PBC)}{A(\Delta ABC)}$

(iii) $\dfrac{A(\Delta ABC)}{A(\Delta ADC)}$ (iv) $\dfrac{A(\Delta ADC)}{A(\Delta PQC)}$

Let's learn.

Basic proportionality theorem

Theorem : **If a line parallel to a side of a triangle intersects the remaining sides in two distinct points, then the line divides the sides in the same proportion.**

Given : In Δ ABC line l || line BC and line l intersects AB and AC in point P and Q respectively

To prove : $\dfrac{AP}{PB} = \dfrac{AQ}{QC}$

Construction: Draw seg PC and seg BQ

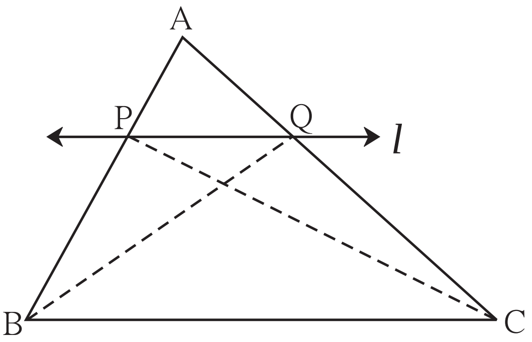

Fig. 1.17

Proof : Δ APQ and Δ PQB have equal heights.

$\therefore \dfrac{A(\Delta APQ)}{A(\Delta PQB)} = \dfrac{AP}{PB}$ .... (I) (areas proportionate to bases)

and $\dfrac{A(\Delta APQ)}{A(\Delta PQC)} = \dfrac{AQ}{QC}$ .... (II) (areas proportionate to bases)

seg PQ is common base of Δ PQB and Δ PQC. seg PQ || seg BC, hence Δ PQB and Δ PQC have equal heights.

$A(\Delta PQB) = A(\Delta PQC)$ .... (III)

$\dfrac{A(\Delta APQ)}{A(\Delta PQB)} = \dfrac{A(\Delta APQ)}{A(\Delta PQC)}$ .... [from (I), (II) and (III)]

$\therefore \dfrac{AP}{PB} = \dfrac{AQ}{QC}$ .... [from (I) and (II)]

Converse of basic proportionality theorem

Theorem : **If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.**

In figure 1.18, line l interesects the side AB and side AC of Δ ABC in the points P and Q respectively and $\dfrac{AP}{PB} = \dfrac{AQ}{QC}$, hence line l || seg BC.

This theorem can be proved by indirect method.

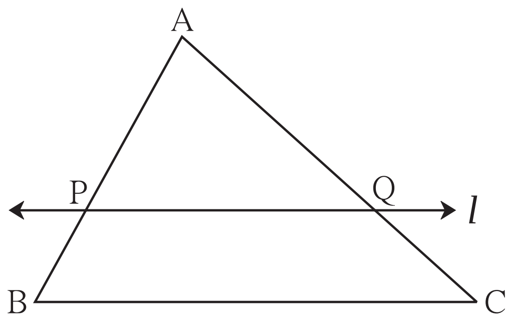

Fig. 1.18

Activity :

- Draw a Δ ABC.
- Bisect $\angle$ B and name the point of intersection of AC and the angle bisector as D.
- Measure the sides.

  AB = ☐ cm BC = ☐ cm

  AD = ☐ cm DC = ☐ cm
- Find ratios $\frac{AB}{BC}$ and $\frac{AD}{DC}$.
- You will find that both the ratios are almost equal.
- Bisect remaining angles of the triangle and find the ratios as above. You can verify that the ratios are equal.

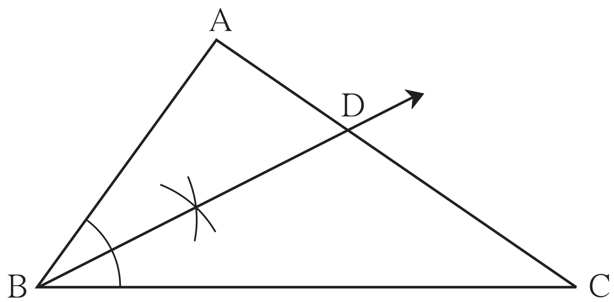

Fig. 1.19

Let's learn.

Property of an angle bisector of a triangle

Theorem : The bisector of an angle of a triangle divides the side opposite to the angle in the ratio of the remaining sides.

Given : In Δ ABC, bisector of $\angle$C interesects seg AB in the point E.

To prove : $\frac{AE}{EB} = \frac{CA}{CB}$

Construction : Draw a line parallel to ray CE, passing through the point B. Extend AC so as to intersect it at point D.

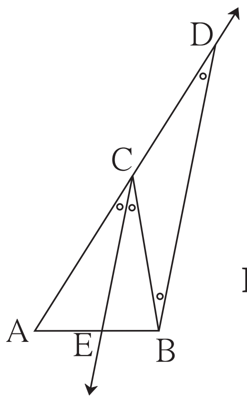

Fig. 1.20

Proof : ray CE || ray BD and AD is transversal,

$\therefore$ $\angle$ ACE = $\angle$ CDB .... (corresponding angles) ...(I)

Now taking BC as transversal

$\angle$ ECB = $\angle$ CBD .... (alternate angle) ...(II)

But $\angle$ ACE $\cong$ $\angle$ ECB .... (given) ...(III)

$\therefore$ $\angle$ CBD $\cong$ $\angle$ CDB .... [from (I), (II) and (III)]

In Δ CBD, side CB $\cong$ side CD ....(sides opposite to congruent angles)

$\therefore$ CB = CD ...(IV)

Now in Δ ABD, seg EC || seg BD .... (construction)

$\therefore$ $\frac{AE}{EB} = \frac{AC}{CD}$ ....(Basic proportionality theorem)..(V)

$\therefore$ $\frac{AE}{EB} = \frac{AC}{CB}$ .... [from (IV) and (V)]

For more information :

Write another proof of the theorem yourself.

Draw DM $\perp$ AB and DN $\perp$ AC. Use the following properties and write the proof.

(1) The areas of two triangles of equal heights are proportional to their bases.

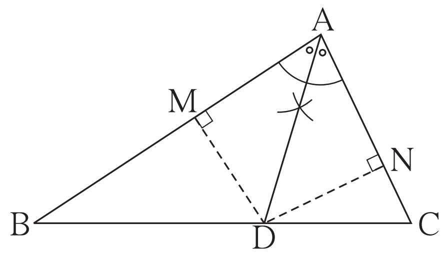

Fig. 1.21

(2) Every point on the bisector of an angle is equidistant from the sides of the angle.

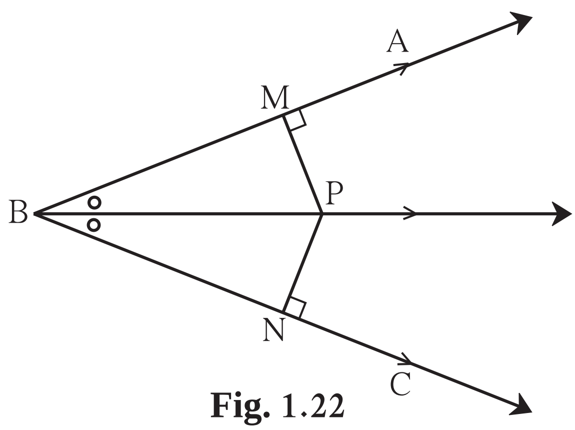

Fig. 1.22

Converse of angle bisector theorem

If in Δ ABC, point D on side BC such that $\frac{AB}{AC} = \frac{BD}{DC}$, then ray AD bisects $\angle$ BAC.

Property of three parallel lines and their transversals

Activity:

- Draw three parallel lines.
- Label them as *l*, *m*, *n*.
- Draw transversals t_1 and t_2.
- AB and BC are intercepts on transversal t_1.
- PQ and QR are intercepts on transversal t_2.

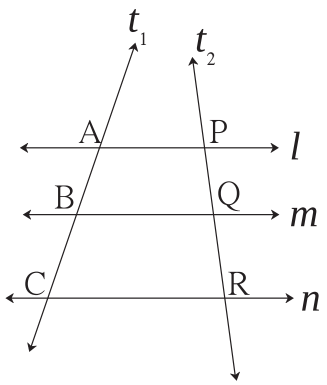

Fig. 1.23

- Find ratios $\frac{AB}{BC}$ and $\frac{PQ}{QR}$. You will find that they are almost equal.

Theorem : The ratio of the intercepts made on a transversal by three parallel lines is equal to the ratio of the corrosponding intercepts made on any other transversal by the same parallel lines.

Given : line *l* || line *m* || line *n*

t_1 and t_2 are transversals. Transversal t_1 intersects the lines in points A, B, C and t_2 intersects the lines in points P, Q, R.

To prove : $\frac{AB}{BC} = \frac{PQ}{QR}$

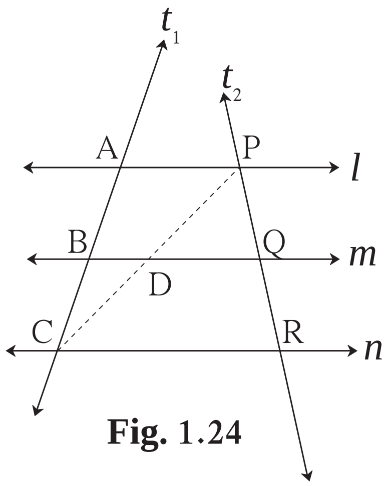

Fig. 1.24

Proof : Draw seg PC , which intersects line *m* at point D.

In Δ ACP, BD || AP

$\therefore \frac{AB}{BC} = \frac{PD}{DC}$. . . . . (I) (Basic proportionality theorem)

In Δ CPR, DQ || CR

$\therefore \frac{PD}{DC} = \frac{PQ}{QR}$. . . . . (II) (Basic proportionality theorem)

$\therefore \frac{AB}{BC} = \frac{PD}{DC} = \frac{PQ}{QR}$. . . . . . from (I) and (II). $\qquad \therefore \frac{AB}{BC} = \frac{PQ}{QR}$

Remember this!

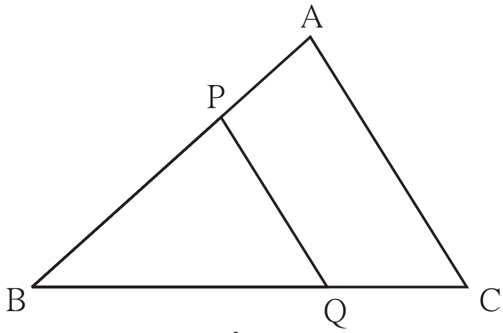

Fig. 1.25

(1) Basic proportionality theorem.

In Δ ABC, if seg PQ || seg AC

then $\dfrac{AP}{BP} = \dfrac{QC}{BQ}$

(2) Converse of basic proportionality theorem.

In ΔPQR, if $\dfrac{PS}{SQ} = \dfrac{PT}{TR}$

then seg ST || seg QR.

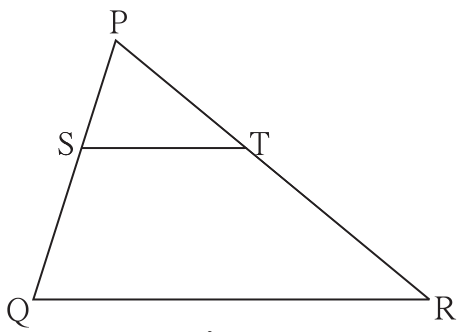

Fig. 1.26

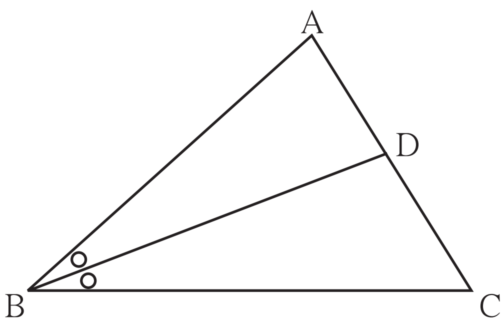

Fig. 1.27

(3) Theorem of bisector of an angle of a triangle.

If in Δ ABC, BD is bisector of $\angle$ ABC,

then $\dfrac{AB}{BC} = \dfrac{AD}{DC}$

(4) Property of three parallel lines and their transversals.

If line AX || line BY || line CZ

and line l and line m are their

transversals then $\dfrac{AB}{BC} = \dfrac{XY}{YZ}$

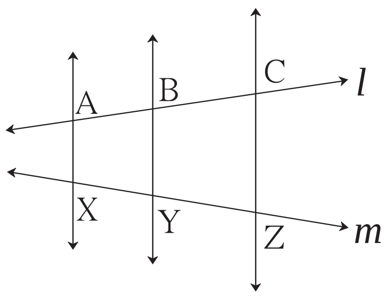

Fig. 1.28

Solved Examples

Ex. (1) In Δ ABC, DE || BC

If DB = 5.4 cm, AD = 1.8 cm

EC = 7.2 cm then find AE.

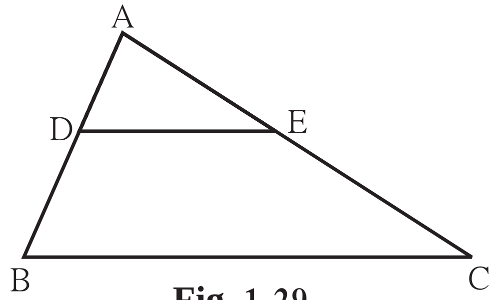

Fig. 1.29

Solution : In Δ ABC, DE || BC

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ ..... Basic proportionality theorem

$\therefore \frac{1.8}{5.4} = \frac{AE}{7.2}$

$\therefore$ AE $\times$ 5.4 = 1.8 $\times$ 7.2

$\therefore$ AE = $\frac{1.8 \times 7.2}{5.4}$ = 2.4

AE = 2.4 cm

Ex. (2) In Δ PQR, seg RS bisects $\angle$ R.

If PR = 15, RQ = 20 PS = 12

then find SQ.

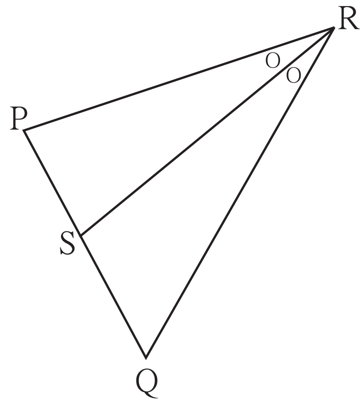

Fig. 1.30

Solution : In Δ PRQ, seg RS bisects $\angle$ R.

$\frac{PR}{RQ} = \frac{PS}{SQ}$ . . . . . . property of angle bisector

$\frac{15}{20} = \frac{12}{SQ}$

SQ = $\frac{12 \times 20}{15}$

$\therefore$ SQ = 16

Activity :

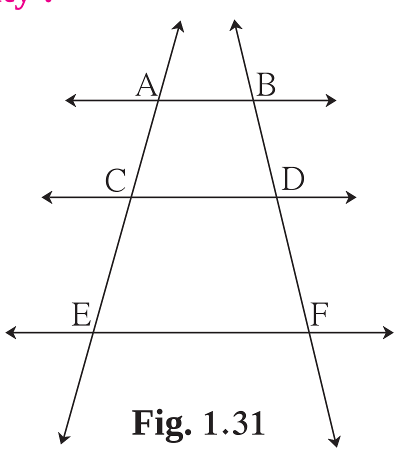

Fig. 1.31

In the figure 1.31, AB || CD || EF

If AC = 5.4, CE = 9, BD = 7.5

then find DF

Solution : AB || CD || EF

$\frac{AC}{\square} = \frac{\square}{DF}$ . . . . . ( )

$\frac{5.4}{9} = \frac{\square}{DF}$ $\therefore$ DF = $\square$

Activity :

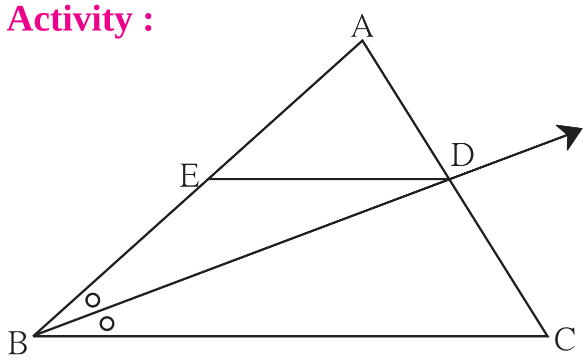

Fig. 1.32

In Δ ABC, ray BD bisects $\angle$ABC. A-D-C, side DE || side BC, A-E-B then prove that, $\frac{AB}{BC} = \frac{AE}{EB}$

Proof : In Δ ABC, ray BD bisects $\angle$ B.

$\therefore \frac{AB}{BC} = \frac{AD}{DC}$ . . (I)(Angle bisector theorem)

In Δ ABC, DE || BC

$\frac{AE}{EB} = \frac{AD}{DC}$ . . . . . . (II) (. . . . .)

$\frac{AB}{\square} = \frac{\square}{EB}$ . . . . . . from (I) and (II)

Practice set 1.2

1. Given below are some triangles and lengths of line segments. Identify in which figures, ray PM is the bisector of $\angle$ QPR.

(1)

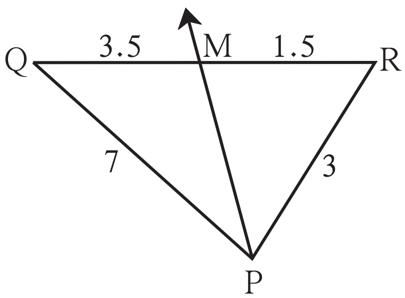

Fig. 1.33

(2)

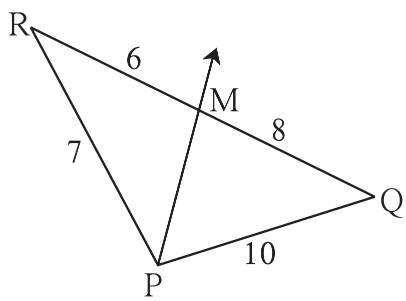

Fig. 1.34

(3)

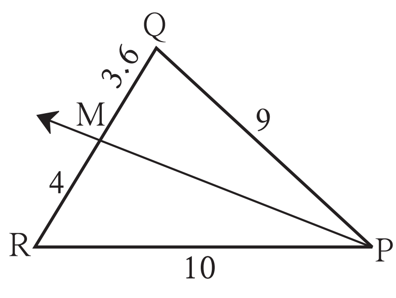

Fig. 1.35

2. In Δ PQR, PM = 15, PQ = 25 PR = 20, NR = 8. State whether line NM is parallel to side RQ. Give reason.

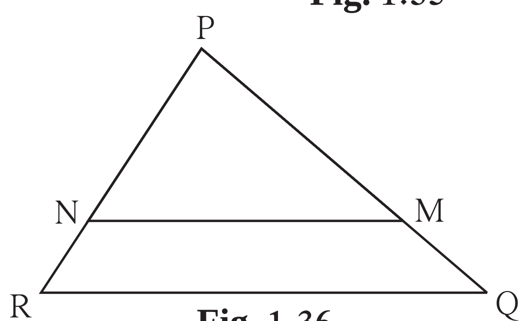

Fig. 1.36

3. In Δ MNP, NQ is a bisector of $\angle$ N. If MN = 5, PN = 7 MQ = 2.5 then find QP .

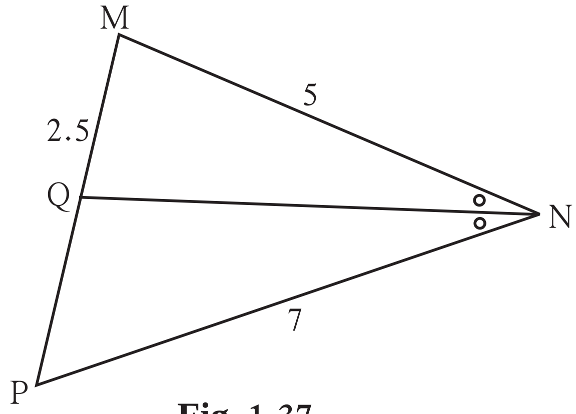

Fig. 1.37

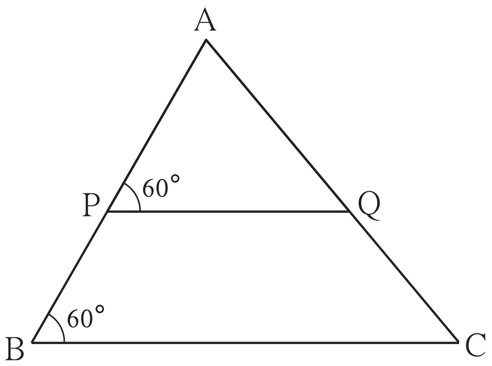

Fig. 1.38

4. Measures of some angles in the figure are given. Prove that

$$\frac{AP}{PB} = \frac{AQ}{QC}$$

5. In trapezium ABCD, side AB || side PQ || side DC, AP = 15, PD = 12, QC = 14, find BQ.

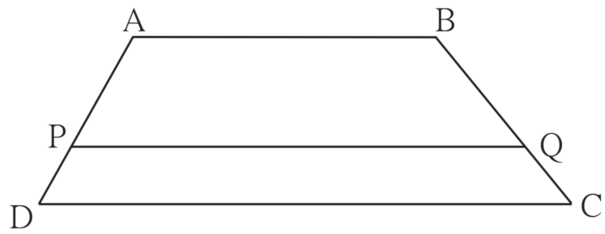

Fig. 1.39

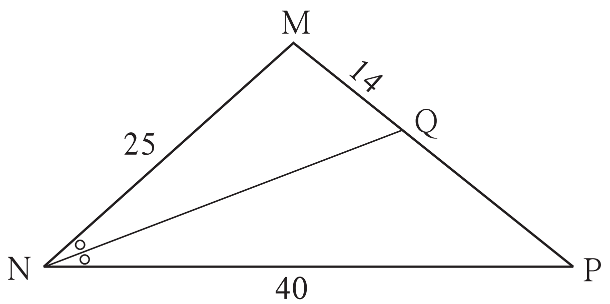

Fig. 1.40

6. Find QP using given information in the figure.

7. In figure 1.41, if AB || CD || FE then find x and AE.

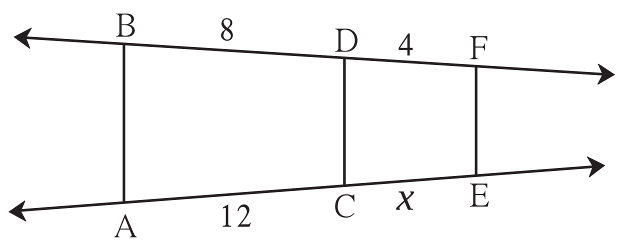

Fig. 1.41

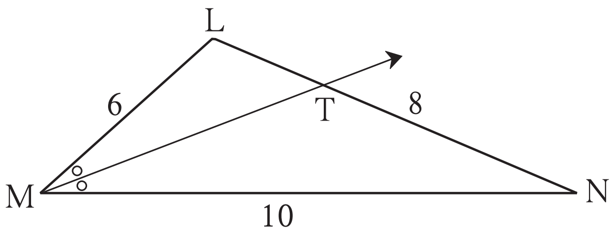

Fig. 1.42

8. In Δ LMN, ray MT bisects $\angle$ LMN
If LM = 6, MN = 10, TN = 8,
then find LT.

9. In Δ ABC, seg BD bisects $\angle$ ABC.
If AB = x, BC = $x + 5$,
AD = $x - 2$, DC = $x + 2$, then find
the value of x.

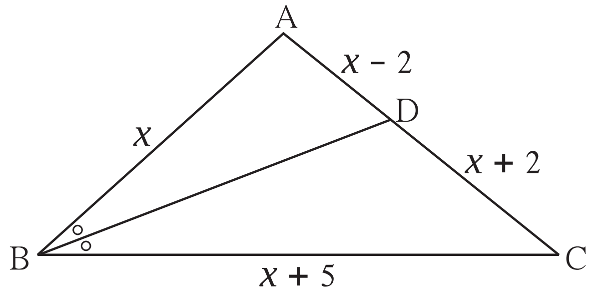

Fig. 1.43

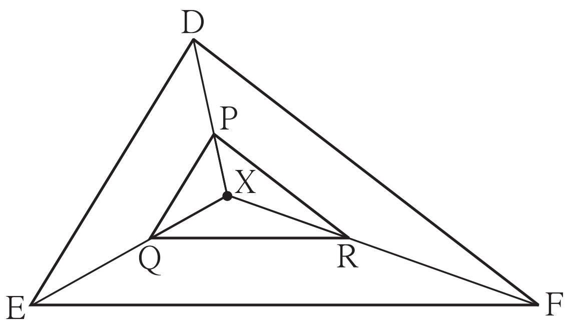

Fig. 1.44

10. In the figure 1.44, X is any point in the interior of triangle. Point X is joined to vertices of triangle. Seg PQ || seg DE, seg QR || seg EF. Fill in the blanks to prove that,

seg PR || seg DF.

Proof : In Δ XDE, PQ || DE .... $\square$

$\therefore \frac{XP}{\square} = \frac{\square}{QE}$ .... (I) (Basic proportionality theorem)

In Δ XEF, QR || EF .... $\square$

$\therefore \frac{\square}{\square} = \frac{\square}{\square}$ ....(II) $\square$

$\therefore \frac{\square}{\square} = \frac{\square}{\square}$ .... from (I) and (II)

$\therefore$ seg PR || seg DE .... (converse of basic proportionality theorem)

11★. In Δ ABC, ray BD bisects $\angle$ ABC and ray CE bisects $\angle$ ACB. If seg AB $\cong$ seg AC then prove that ED || BC.

Let's recall.

Similar triangles

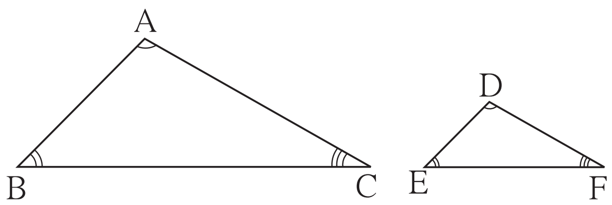

Fig. 1.45

In Δ ABC and Δ DEF, if $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$

and $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

then ΔABC and ΔDEF are similar triangles.

'Δ ABC and Δ DEF are similar' is expressed as '$\Delta ABC \sim \Delta DEF$'

Let's learn.

Tests of similarity of triangles

For similarity of two triangles, the necessary conditions are that their corresponding sides are in same proportion and their corresponding angles are congruent. Out of these conditions; when three specifc conditions are fulfilled, the remaining conditions are automatically fulfilled. This means for similarity of two triangles, only three specific conditions are sufficient. Similarity of two triangles can be confirmed by testing these three conditions. The groups of such sufficient conditions are called tests of similarity, which we shall use.

AAA test for similarity of triangles

For a given correspondence of vertices, when corresponding angles of two triangles are congruent, then the two triangles are similar.

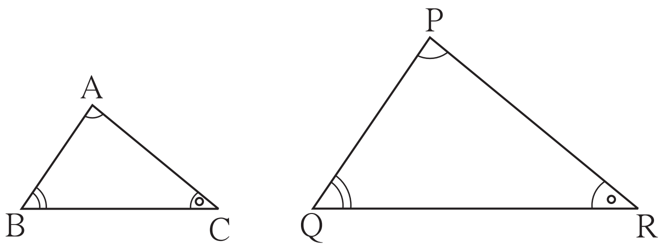

Fig. 1.46

In Δ ABC and Δ PQR, in the correspondence $ABC \leftrightarrow PQR$ if

$\angle A \cong \angle P$, $\angle B \cong \angle Q$ and $\angle C \cong \angle R$

then $\Delta ABC \sim \Delta PQR$.

For more information :

Proof of AAA test

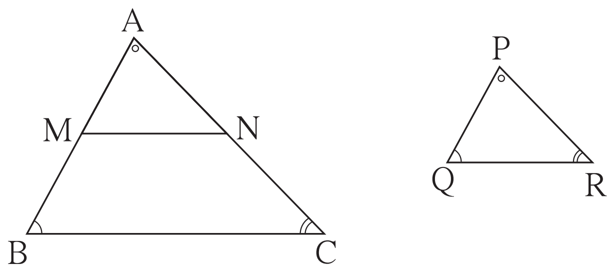

Fig. 1.47

Given : In Δ ABC and Δ PQR, $\angle A \cong \angle P$, $\angle B \cong \angle Q$, $\angle C \cong \angle R$.

To prove : Δ ABC $\sim$ Δ PQR

Let us assume that Δ ABC is bigger than Δ PQR. Mark point M on AB, and point N on AC such that AM = PQ and AN = PR.

Show that Δ AMN $\cong$ Δ PQR. Hence show that MN || BC.

Now using basic proportionality theorem, $\frac{AM}{MB} = \frac{AN}{NC}$

That is $\frac{MB}{AM} = \frac{NC}{AN}$ .... (by invertendo)

$\frac{MB+AM}{AM} = \frac{NC+AN}{AN}$ .... (by componendo)

$\therefore \frac{AB}{AM} = \frac{AC}{AN}$

$\therefore \frac{AB}{PQ} = \frac{AC}{PR}$

Similarly it can be shown that $\frac{AB}{PQ} = \frac{BC}{QR}$

$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ $\quad\quad$ $\therefore$ Δ ABC $\sim$ Δ PQR

A A test for similarity of triangles:

We know that for a given correspondence of vertices, when two angles of a triangle are congruent to two corresponding angles of another triangle, then remaining angle of first triangle is congruent to the remaining angle of the second triangle.

This means, when two angles of one triangle are congruent to two corresponding angles of another triangle then this condition is sufficient for similarity of two triangles. This condition is called AA test of similarity.

SAS test of similarity of triangles

For a given correspondence of vertices of two triangles, if two pairs of corresponding sides are in the same proportion and the angles between them are congruent, then the two triangles are similar.

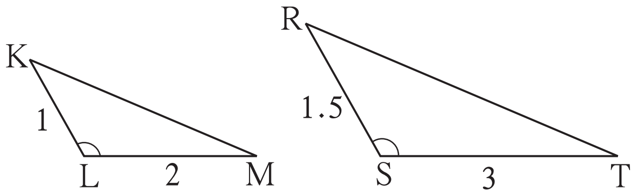

Fig. 1.48

For example, if in Δ KLM and Δ RST,

$\angle KLM \cong \angle RST$

$$\frac{KL}{RS} = \frac{LM}{ST} = \frac{2}{3}$$

Therefore, $\Delta KLM \sim \Delta RST$

SSS test for similarity of triangles

For a given correspondence of vertices of two triangles, when three sides of a triangle are in proportion to corresponding three sides of another triangle, then the two triangles are similar.

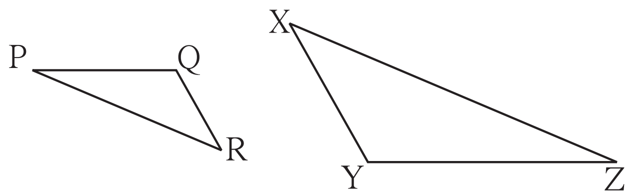

Fig. 1.49

For example, if in Δ PQR and Δ XYZ,

If $\frac{PQ}{YZ} = \frac{QR}{XY} = \frac{PR}{XZ}$

then $\Delta PQR \sim \Delta ZYX$

Properties of similar triangles :

(1) $\Delta ABC \sim \Delta ABC$ - Reflexivity

(2) If $\Delta ABC \sim \Delta DEF$ then $\Delta DEF \sim \Delta ABC$ - Symmetry

(3) If $\Delta ABC \sim \Delta DEF$ and $\Delta DEF \sim \Delta GHI$, then $\Delta ABC \sim \Delta GHI$ - Transitivity

Solved Examples

Ex. (1) In Δ XYZ,

$\angle Y = 100°$, $\angle Z = 30°$,

In Δ LMN,

$\angle M = 100°$, $\angle N = 30°$,

Are Δ XYZ and Δ LMN similar? If yes, by which test?

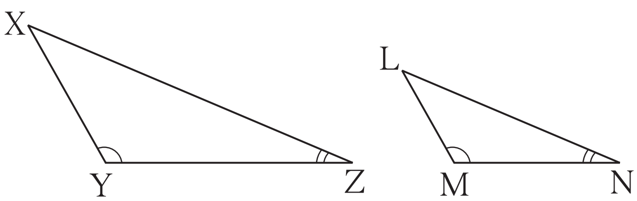

Fig. 1.50

Solution : In Δ XYZ and Δ LMN,

$\angle Y = 100°$, $\angle M = 100°$, $\therefore \angle Y \cong \angle M$

$\angle Z = 30°$, $\angle N = 30°$, $\therefore \angle Z \cong \angle N$

$\therefore \Delta XYZ \sim \Delta LMN$ ..... by AA test.

Ex. (2) Are two triangles in figure 1.51 similar, according to the information given? If yes, by which test?

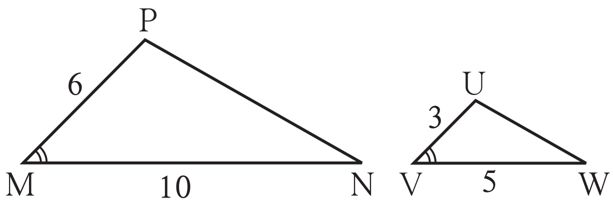

Fig. 1.51

Solution : In Δ PMN and Δ UVW

$\frac{PM}{UV} = \frac{6}{3} = \frac{2}{1}$, $\frac{MN}{VW} = \frac{10}{5} = \frac{2}{1}$

$\therefore \frac{PM}{UV} = \frac{MN}{VW}$

and $\angle M \cong \angle V$ .... Given

$\Delta PMN \sim \Delta UVW$ .... SAS test of similarity

Ex. (3) Can we say that the two triangles in figure 1.52 similar, according to information given? If yes, by which test ?

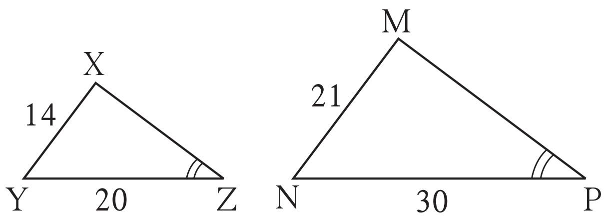

Fig. 1.52

Solution : Δ XYZ and Δ MNP,

$\frac{XY}{MN} = \frac{14}{21} = \frac{2}{3}$,

$\frac{YZ}{NP} = \frac{20}{30} = \frac{2}{3}$

It is given that $\angle Z \cong \angle P$.

But $\angle Z$ and $\angle P$ are not included angles by sides which are in proportion.

$\therefore \Delta$ XYZ and Δ MNP can not be said to be similar.

Ex. (4) In the adjoining figure BP ⊥ AC, CQ ⊥ AB, A – P – C, A – Q – B , then prove that Δ APB and Δ AQC are similar.

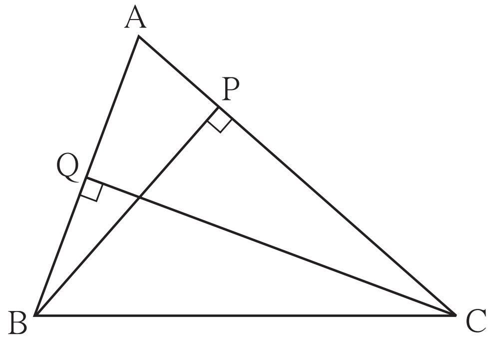

Fig. 1.53

Solution : In Δ APB and Δ AQC

∠ APB = ☐° (I)

∠ AQC = ☐° (II)

∴ ∠ APB ≅ ∠ AQC ....from(I) and (II)

∠ PAB ≅ ∠ QAC .... (☐)

∴ Δ APB ~ Δ AQC ..... AA test

Ex. (5) Diagonals of a quadrilateral ABCD intersect in point Q. If 2QA = QC , 2QB = QD, then prove that DC = 2AB.

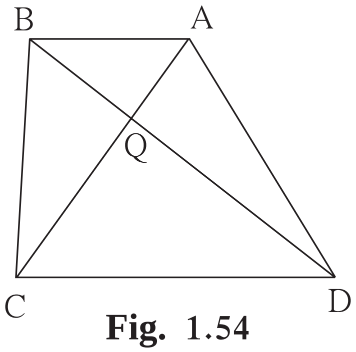

Fig. 1.54

Given : 2QA = QC

2QB = QD

To prove : CD = 2AB

Proof : 2QA = QC $\therefore \frac{QA}{QC} = \frac{1}{2}$ .... (I)

2QB = QD $\therefore \frac{QB}{QD} = \frac{1}{2}$ .... (II)

$\therefore \frac{QA}{QC} = \frac{QB}{QD}$ ....from (I) and (II)

In Δ AQB and Δ CQD,

$\frac{QA}{QC} = \frac{QB}{QD}$ .... proved

∠ AQB ≅ ∠ DQC .... opposite angles

∴ Δ AQB ~Δ CQD .... (SAS test of similarity)

$\therefore \frac{AQ}{CQ} = \frac{QB}{QD} = \frac{AB}{CD}$ .... correponding sides are proportional

But $\frac{AQ}{CQ} = \frac{1}{2}$ $\therefore \frac{AB}{CD} = \frac{1}{2}$

∴ 2AB = CD

Practice set 1.3

1. In figure 1.55, $\angle ABC = 75°$, $\angle EDC = 75°$ state which two triangles are similar and by which test? Also write the similarity of these two triangles by a proper one to one correspondence.

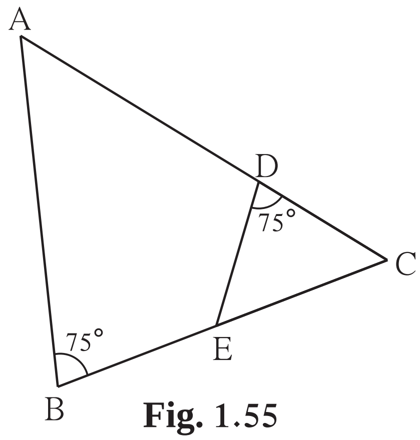

Fig. 1.55

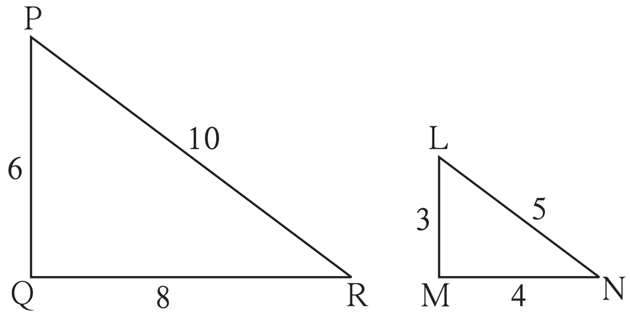

Fig. 1.56

2. Are the triangles in figure 1.56 similar? If yes, by which test ?

3. As shown in figure 1.57, two poles of height 8 m and 4 m are perpendicular to the ground. If the length of shadow of smaller pole due to sunlight is 6 m then how long will be the shadow of the bigger pole at the same time ?

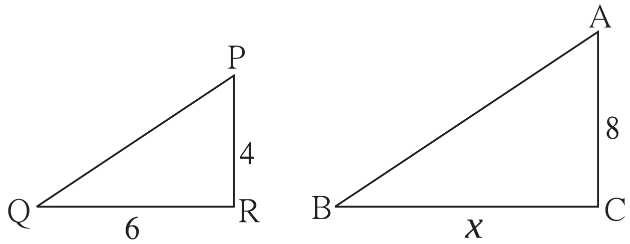

Fig. 1.57

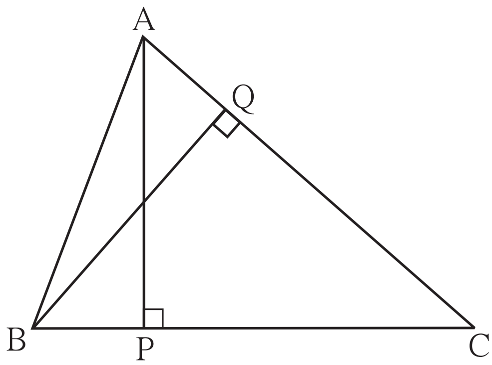

Fig. 1.58

4. In Δ ABC, AP $\perp$ BC, BQ $\perp$ AC B–P–C, A–Q–C then prove that, $\Delta CPA \sim \Delta CQB$.

If AP = 7, BQ=8, BC=12 then find AC.

5. **Given :** In trapezium PQRS, side PQ || side SR, AR = 5AP, AS = 5AQ then prove that, SR = 5PQ

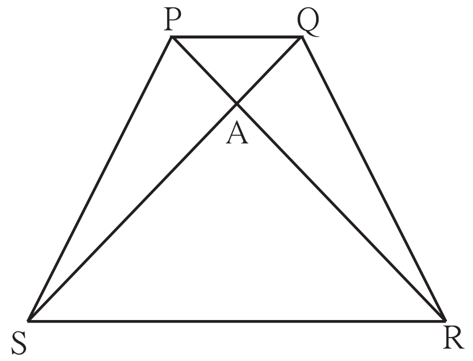

Fig. 1.59

6. In trapezium ABCD, (Figure 1.60) side AB || side DC, diagonals AC and BD intersect in point O. If AB = 20, DC = 6, OB = 15 then find OD.

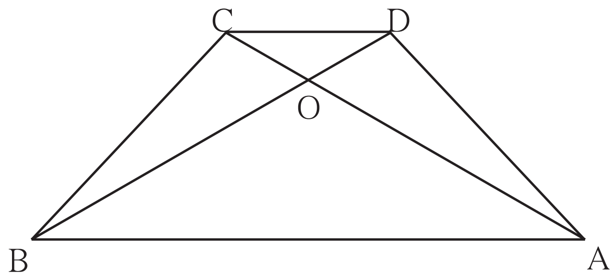

Fig. 1.60

7. □ABCD is a parallelogram point E is on side BC. Line DE intersects ray AB in point T. Prove that $DE \times BE = CE \times TE$.

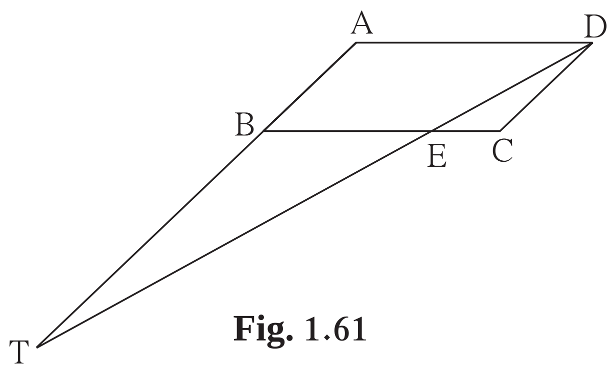

Fig. 1.61

8. In the figure, seg AC and seg BD intersect each other in point P and $\frac{AP}{CP} = \frac{BP}{DP}$. Prove that, Δ ABP ~ Δ CDP

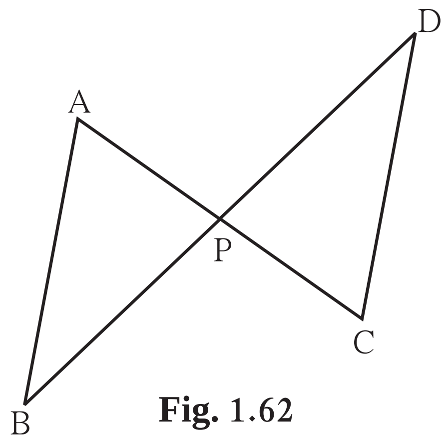

Fig. 1.62

9. In the figure, in Δ ABC, point D on side BC is such that, $\angle BAC = \angle ADC$.

Prove that, $CA^2 = CB \times CD$

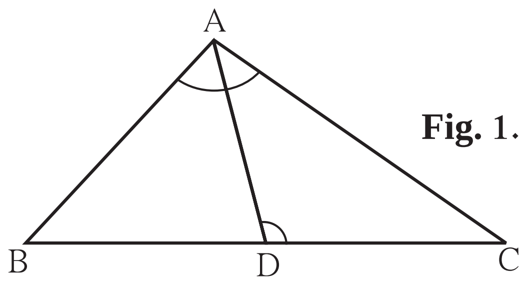

Fig. 1.63

Let's learn.

Theorem of areas of similar triangles

Theorem : **When two triangles are similar, the ratio of areas of those triangles is equal to the ratio of the squares of their corresponding sides.**

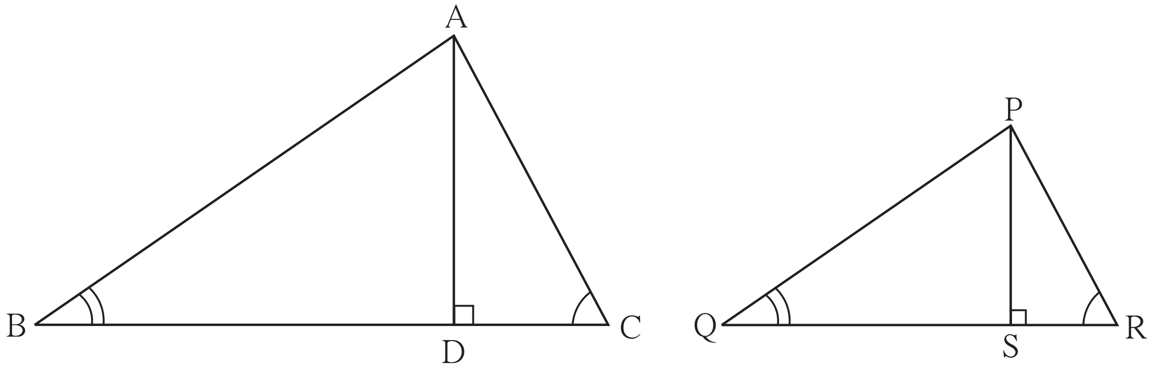

Fig. 1.64

Given : $\Delta ABC \sim \Delta PQR$, $AD \perp BC$, $PS \perp QR$

To prove: $\dfrac{A(\Delta ABC)}{A(\Delta PQR)} = \dfrac{AB^2}{PQ^2} = \dfrac{BC^2}{QR^2} = \dfrac{AC^2}{PR^2}$

Proof : $\dfrac{A(\Delta ABC)}{A(\Delta PQR)} = \dfrac{BC \times AD}{QR \times PS} = \dfrac{BC}{QR} \times \dfrac{AD}{PS}$ .... (I)

In Δ ABD and Δ PQS,

$\angle B = \angle Q$ .... given

$\angle ADB = \angle PSQ = 90^\circ$

$\therefore$ According to AA test $\Delta ABD \sim \Delta PQS$

$\therefore \dfrac{AD}{PS} = \dfrac{AB}{PQ}$ .... (II)

But $\Delta ABC \sim \Delta PQR$

$\therefore \dfrac{AB}{PQ} = \dfrac{BC}{QR}$ .... (III)

From (I), (II) and (III)

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{BC}{QR} \times \frac{AD}{PS} = \frac{BC}{QR} \times \frac{BC}{QR} = \frac{BC^2}{QR^2} = \frac{AB^2}{PQ^2} = \frac{AC^2}{PR^2}$$

Solved Examples

Ex. (1) : $\Delta ABC \sim \Delta PQR$, $A(\Delta ABC) = 16$, $A(\Delta PQP) = 25$, then find the value of ratio $\frac{AB}{PQ}$.

Solution : $\Delta ABC \sim \Delta PQR$

$$\therefore \frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB^2}{PQ^2}$$ .... theorem of areas of similar triangles

$$\therefore \frac{16}{25} = \frac{AB^2}{PQ^2} \quad \therefore \frac{AB}{PQ} = \frac{4}{5}$$ .... taking square roots

Ex. (2) Ratio of corresponging sides of two similar triangles is 2:5, If the area of the small triangle is 64 sq.cm. then what is the area of the bigger triangle ?

Solution : Assume that $\Delta ABC \sim \Delta PQR$.

ΔABC is smaller and ΔPQR is bigger triangle.

$$\therefore \frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{(2)^2}{(5)^2} = \frac{4}{25}$$ .... ratio of areas of similar triangles

$$\therefore \frac{64}{A(\Delta PQR)} = \frac{4}{25}$$

$4 \times A(\Delta PQR) = 64 \times 25$

$$A(\Delta PQR) = \frac{64 \times 25}{4} = 400$$

$\therefore$ area of bigger triangle = 400 sq.cm.

Ex. (3) In trapezium ABCD, side AB || side CD, diagonal AC and BD intersect each other at point P. Then prove that $\frac{A(\Delta ABP)}{A(\Delta CPD)} = \frac{AB^2}{CD^2}$.

Solution : In trapezium ABCD side AB || side CD

In ΔAPB and ΔCPD

$\angle PAB \cong \angle PCD$ ...... alternate angles

$\angle APB \cong \angle CPD$ .... opposite angles

$\therefore \Delta APB \sim \Delta CPD$ .... AA test of similarity

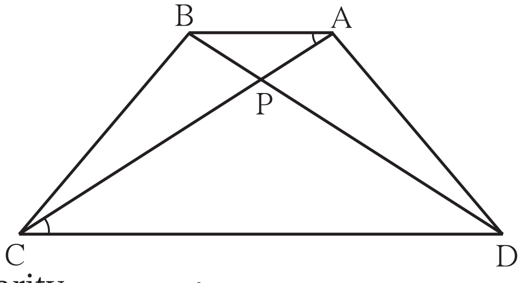

Fig. 1.65

$$\frac{A(\Delta APB)}{A(\Delta CPD)} = \frac{AB^2}{CD^2}$$ . . . . . . theorem of areas of similar triangles

Practice set 1.4

1. The ratio of corresponding sides of similar triangles is 3 : 5; then find the ratio of their areas .

2. If $\Delta ABC \sim \Delta PQR$ and AB: PQ = 2:3, then fill in the blanks.

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB^2}{\square} = \frac{2^2}{3^2} = \frac{\square}{\square}$$

3. If Δ ABC $\sim$ Δ PQR, A (Δ ABC) = 80, A (Δ PQR) = 125, then fill in the blanks.

$$\frac{A(\Delta ABC)}{A(\Delta \ldots\ldots)} = \frac{80}{125} \qquad \therefore \frac{AB}{PQ} = \frac{\square}{\square}$$

4. Δ LMN $\sim$ Δ PQR, $9 \times A(\Delta PQR) = 16 \times A(\Delta LMN)$. If QR = 20 then find MN.

5. Areas of two similar triangles are 225 sq.cm. 81 sq.cm. If a side of the smaller triangle is 12 cm, then find corresponding side of the bigger triangle .

6. ΔABC and Δ DEF are equilateral triangles. If A(ΔABC) : A (Δ DEF) = 1 : 2 and AB = 4, find DE.

7. In figure 1.66, seg PQ || seg DE, A(Δ PQF) = 20 units, PF = 2 DP, then find A($\square$ DPQE) by completing the following activity.

A(Δ PQF) = 20 units, PF = 2 DP, Let us assume DP = x. $\therefore$ PF = $2x$

DF = DP + $\square$ = $\square$ + $\square$ = $3x$

In Δ FDE and Δ FPQ,

$\angle$ FDE $\cong$ $\angle$ .... corresponding angles

$\angle$ FED $\cong$ $\angle$ .... corresponding angles

$\therefore$ Δ FDE $\sim$ Δ FPQ .... AA test

$$\therefore \frac{A(\Delta FDE)}{A(\Delta FPQ)} = \frac{\square}{\square} = \frac{(3x)^2}{(2x)^2} = \frac{9}{4}$$

$$A(\Delta FDE) = \frac{9}{4} A(\Delta FPQ) = \frac{9}{4} \times \square = \square$$

A($\square$ DPQE) = A(Δ FDE) – A(Δ FPQ)

= $\square$ – $\square$

= $\square$

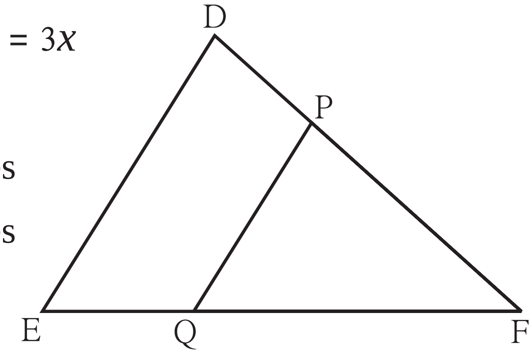

Fig. 1.66

Problem set 1

1. Select the appropriate alternative.

(1) In Δ ABC and Δ PQR, in a one to one correspondence $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$ then

(A) $\Delta PQR \sim \Delta ABC$
(B) $\Delta PQR \sim \Delta CAB$
(C) $\Delta CBA \sim \Delta PQR$
(D) $\Delta BCA \sim \Delta PQR$

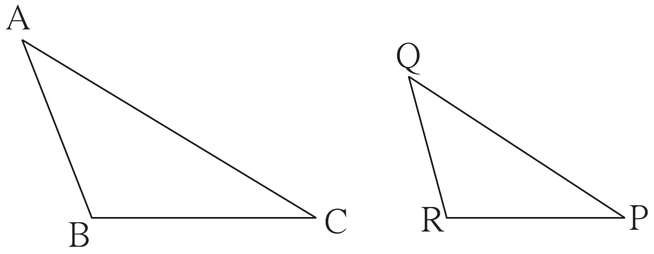

Fig. 1.67

(2) If in Δ DEF and Δ PQR, $\angle D \cong \angle Q$, $\angle R \cong \angle E$ then which of the following statements is false ?

(A) $\frac{EF}{PR} = \frac{DF}{PQ}$ (B) $\frac{DE}{PQ} = \frac{EF}{RP}$
(C) $\frac{DE}{QR} = \frac{DF}{PQ}$ (D) $\frac{EF}{RP} = \frac{DE}{QR}$

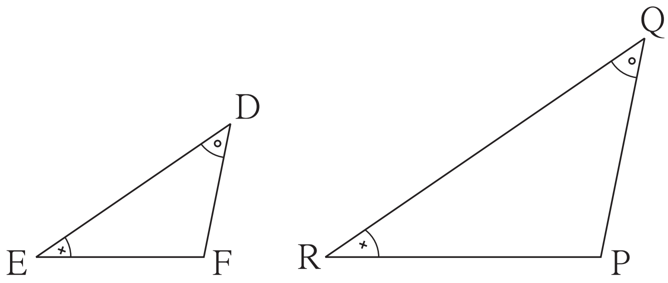

Fig. 1.68

(3) In Δ ABC and Δ DEF $\angle B = \angle E$, $\angle F = \angle C$ and AB = 3DE then which of the statements regarding the two triangles is true ?

(A) The triangles are not congruent and not similar
(B) The triangles are similar but not congruent.
(C) The triangles are congruent and similar.
(D) None of the statements above is true.

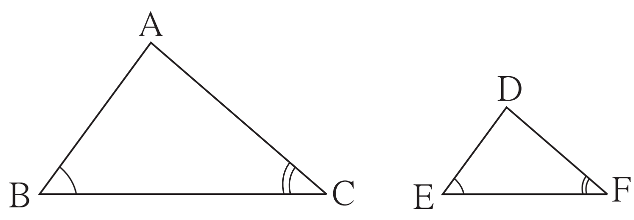

Fig. 1.69

(4) Δ ABC and Δ DEF are equilateral triangles, $A(\Delta ABC):A(\Delta DEF) = 1:2$
If AB = 4 then what is length of DE ?

(A) $2\sqrt{2}$ (B) 4 (C) 8 (D) $4\sqrt{2}$

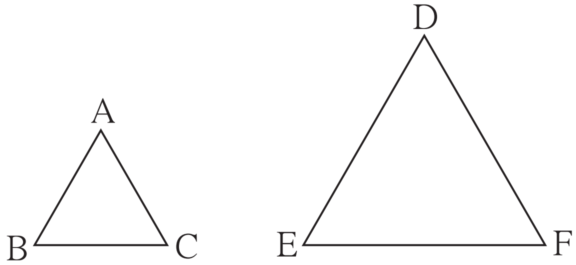

Fig. 1.70

(5) In figure 1.71, seg XY || seg BC, then which of the following statements is true?

(A) $\frac{AB}{AC} = \frac{AX}{AY}$ (B) $\frac{AX}{XB} = \frac{AY}{AC}$

(C) $\frac{AX}{YC} = \frac{AY}{XB}$ (D) $\frac{AB}{YC} = \frac{AC}{XB}$

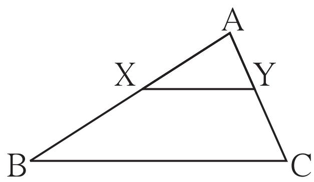

Fig. 1.71

2. In Δ ABC, B – D – C and BD = 7, BC = 20 then find following ratios.

(1) $\frac{A(\Delta ABD)}{A(\Delta ADC)}$

(2) $\frac{A(\Delta ABD)}{A(\Delta ABC)}$

(3) $\frac{A(\Delta ADC)}{A(\Delta ABC)}$

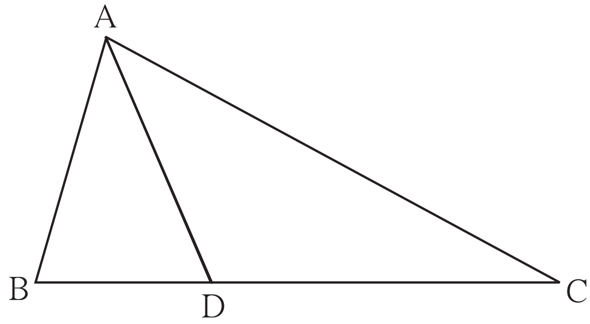

Fig. 1.72

3. Ratio of areas of two triangles with equal heights is 2 : 3. If base of the smaller triangle is 6 cm then what is the corresponding base of the bigger triangle ?

4.

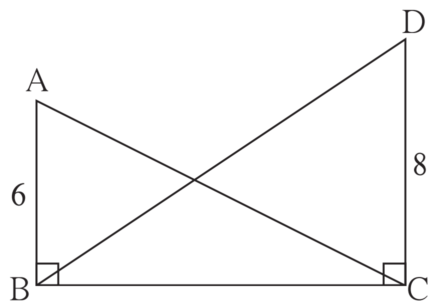

Fig. 1.73

In figure 1.73, $\angle ABC = \angle DCB = 90°$

AB = 6, DC = 8

then $\frac{A(\Delta ABC)}{A(\Delta DCB)} = ?$

5. In figure 1.74, PM = 10 cm

$A(\Delta PQS)$ = 100 sq.cm

$A(\Delta QRS)$ = 110 sq.cm

then find NR.

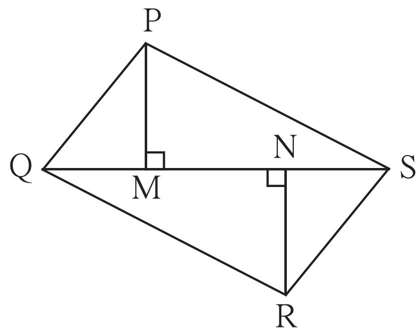

Fig. 1.74

6. Δ MNT ~ Δ QRS. Length of altitude drawn from point T is 5 and length of altitude drawn from point S is 9. Find the ratio $\frac{A(\Delta MNT)}{A(\Delta QRS)}$.

7. In figure 1.75, A–D–C and B–E–C seg DE || side AB If AD = 5, DC = 3, BC = 6.4 then find BE.

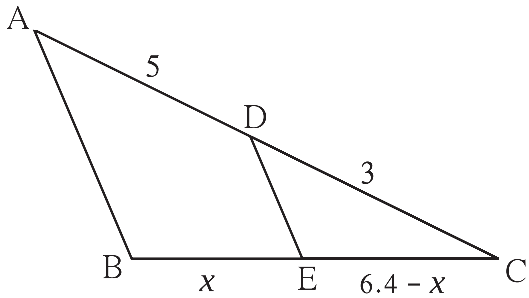

Fig. 1.75

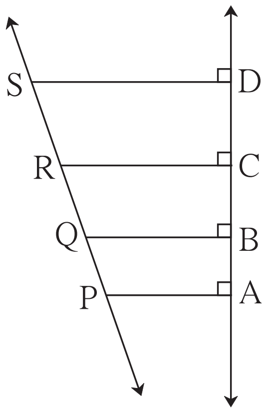

Fig. 1.76

8. In the figure 1.76, seg PA, seg QB, seg RC and seg SD are perpendicular to line AD.

AB = 60, BC = 70, CD = 80, PS = 280 then find PQ, QR and RS.

9. In Δ PQR seg PM is a median. Angle bisectors of ∠PMQ and ∠PMR intersect side PQ and side PR in points X and Y respectively. Prove that XY || QR.

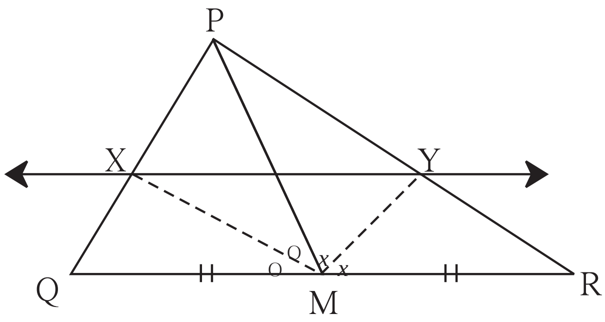

Fig. 1.77

Complete the proof by filling in the boxes.

In Δ PMQ, ray MX is bisector of ∠PMQ.

$\therefore \quad \frac{\square}{\square} = \frac{\square}{\square}$ .... (I) theorem of angle bisector.

In Δ PMR, ray MY is bisector of ∠PMR.

$\therefore \quad \frac{\square}{\square} = \frac{\square}{\square}$ .... (II) theorem of angle bisector.

But $\frac{MP}{MQ} = \frac{MP}{MR}$ .... M is the midpoint QR, hence MQ = MR.

$\therefore \quad \frac{PX}{XQ} = \frac{PY}{YR}$

∴ XY || QR .... converse of basic proportionality theorem.

10. In fig 1.78, bisectors of $\angle B$ and $\angle C$ of Δ ABC intersect each other in point X. Line AX intersects side BC in point Y. AB = 5, AC = 4, BC = 6 then find $\frac{AX}{XY}$.

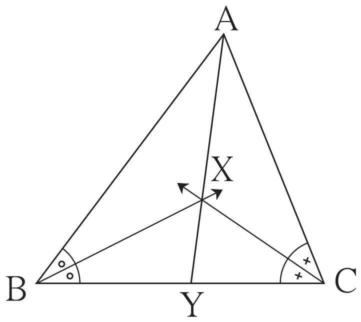

Fig. 1.78

11. In $\square$ ABCD, seg AD || seg BC. Diagonal AC and diagonal BD intersect each other in point P. Then show that $\frac{AP}{PD} = \frac{PC}{BP}$

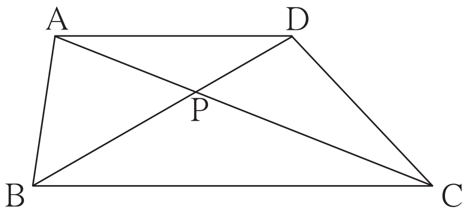

Fig. 1.79

12. In fig 1.80, XY || seg AC.

If 2AX = 3BX and XY = 9. Complete the activity to find the value of AC.

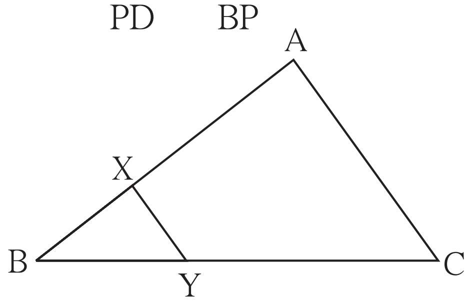

Fig. 1.80

Activity : 2AX = 3BX $\therefore \frac{AX}{BX} = \frac{\square}{\square}$

$\frac{AX + BX}{BX} = \frac{\square + \square}{\square}$ .... by componendo.

$\frac{AB}{BX} = \frac{\square}{\square}$ .... (I)

Δ BCA ~ Δ BYX .... $\square$ test of similarity.

$\therefore \frac{BA}{BX} = \frac{AC}{XY}$ .... corresponding sides of similar triangles.

$\therefore \frac{\square}{\square} = \frac{AC}{9}$ $\therefore$ AC = $\square$ ...from (I)

13*. In figure1.81, the vertices of square DEFG are on the sides of Δ ABC. $\angle A = 90°$. Then prove that $DE^2 = BD \times EC$

(Hint : Show that Δ GBD is similar to Δ CFE. Use GD = FE = DE.)

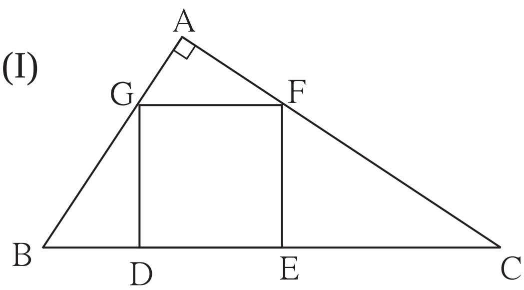

Fig. 1.81