



Let's Study

- Definite Integral
- Properties of Definite Integral



Introduction

We know that if $f(x)$ is a continuous function of x , then there exists a function $\phi(x)$ such that $\phi'(x) = f(x)$. In this case, $\phi(x)$ is an integral of $f(x)$ with respect to x and we denote it by $\int f(x) dx = \phi(x) + c$. Now, if we restrict the domain of $f(x)$ to (a, b) , then the difference $\phi(b) - \phi(a)$ is called definite integral of $f(x)$ w.r.t. x on the interval

$[a, b]$ and is denoted by $\int_a^b f(x) dx$.

$$\text{Thus } \int_a^b f(x) dx = \phi(b) - \phi(a)$$

The numbers a and b are called limits of integration, ' a ' is referred to as the lower limit of integral and b is the upper limit of integral.

Note that the domain of the variable x is restricted to the interval (a, b) and a, b are finite numbers.



Let's Learn

6.1 Fundamental theorem of Integral Calculus.

Let f be a continuous function defined on (a, b)

$$\int f(x) dx = \phi(x) + c.$$

$$\begin{aligned} \text{Then } \int_a^b f(x) dx &= [\phi(x) + c]_a^b \\ &= [\phi(b) + c] - [\phi(a) + c] \\ &= \phi(b) - \phi(a) \end{aligned}$$

There is no need of taking the constant of integration c , because it gets eliminated.

SOLVED EXAMPLES

Ex 1 : Evaluate:

$$\text{i) } \int_2^3 x^4 dx$$

$$\text{ii) } \int_0^1 \frac{1}{(2x+5)} dx$$

$$\text{iii) } \int_0^1 \frac{1}{\sqrt{1+x} + \sqrt{x}} dx$$

Solution:

$$\text{i) Here } f(x) = x^4, \phi(x) = \frac{x^5}{5} + c$$

$$\int_2^3 f(x) dx = [\phi(x)]_2^3$$

$$\begin{aligned} \int_2^3 x^4 dx &= \left[\frac{x^5}{5} \right]_2^3 = \frac{3^5}{5} - \frac{2^5}{5} \\ &= \frac{243}{5} - \frac{32}{5} = \frac{211}{5} \end{aligned}$$

$$\begin{aligned} \text{ii) } \int_0^1 \frac{1}{(2x+5)} dx &= \frac{1}{2} [\log|2x+5|]_0^1 \\ &= \frac{1}{2} [\log 7 - \log 5] \\ &= \frac{1}{2} \log \frac{7}{5} \end{aligned}$$

$$\begin{aligned} \text{iii) } \int_0^1 \frac{1}{\sqrt{1+x} + \sqrt{x}} dx &= \int_0^1 \frac{\sqrt{1+x} - \sqrt{x}}{(\sqrt{1+x} + \sqrt{x})(\sqrt{1+x} - \sqrt{x})} dx \\ &= \int_0^1 \frac{\sqrt{1+x} - \sqrt{x}}{1+x-x} dx \end{aligned}$$

$$\begin{aligned}
&= \int_0^1 (\sqrt{1+x} - \sqrt{x}) dx \\
&= \left[\frac{2}{3}(1+x)^{\frac{3}{2}} - \frac{2}{3}x^{\frac{3}{2}} \right]_0^1 \\
&= \left[\frac{2}{3}(1+1)^{\frac{3}{2}} - \frac{2}{3}(1)^{\frac{3}{2}} \right] - \left[\frac{2}{3}(1+0)^{\frac{3}{2}} - \frac{2}{3}(0)^{\frac{3}{2}} \right] \quad \text{ii)} \\
&= \frac{2}{3} \left[2^{\frac{3}{2}} - 1 \right] - \frac{2}{3} [1 - 0] \\
&= \frac{2}{3} \left[2^{\frac{3}{2}} - 2 \right] \\
&= \frac{2}{3} [2\sqrt{2} - 2] \\
&= \frac{4}{3} [\sqrt{2} - 1]
\end{aligned}$$

Ex 2 : Evaluate:

- i) If $\int_0^1 (3x^2 + 2x + a) dx = 0$; find a.
- ii) If $\int_0^a 3x^2 dx = 8$; find the value of a.
- iii) $\int_a^b x^3 dx = 0$ and $\int_a^b x^2 dx = \frac{2}{3}$. Find the values of a and b.
- iv) If $\int_0^a 4x^3 dx = 16$, find α
- v) If $f(x) = a + bx + cx^2$, show that
- $$\int_0^1 f(x) dx = \frac{1}{6} \left[f(0) + 4f\left(\frac{1}{2}\right) + f(1) \right]$$

Solution:

i) $\int_0^1 (3x^2 + 2x + a) dx = 0$

Then $\left[3\frac{x^3}{3} + 2\frac{x^2}{2} + ax \right]_0^1 = 0$

$$\left[x^3 + x^2 + ax \right]_0^1 = 0$$

$$(1+1+a) - 0 = 0$$

$$2 + a - 0 = 0$$

$$a = -2$$

$$\int_0^a 3x^2 dx = 8$$

$$3 \left[\frac{x^3}{3} \right]_0^a = 8$$

$$(a^3 - 0^3) = 8$$

$$a^3 = 8$$

$$a = 2$$

iii) $\int_a^b x^3 dx = 0$ and $\int_a^b x^2 dx = \frac{2}{3}$

$$\therefore \left[\frac{x^4}{4} \right]_a^b = 0 \text{ and } \left[\frac{x^3}{3} \right]_a^b = \frac{2}{3}$$

$$\therefore \frac{1}{4}(b^4 - a^4) = 0 \text{ and } \frac{1}{3}(b^3 - a^3) = \frac{2}{3}$$

$$\therefore b^4 - a^4 = 0 \text{ and } b^3 - a^3 = 2$$

$$\therefore b^4 = a^4 \therefore b = \pm a$$

But b = a does not satisfy $b^3 - a^3 = 2$

$$\therefore b \neq a$$

$$\therefore b = -a$$

Substituting $b = -a$ in $b^3 - a^3 = 2$

$$\text{We get } (-a)^3 - a^3 = 2, -2a^3 = 2$$

We get $a = -1$

$$\therefore b = -a = 1$$

$$\therefore a = -1, b = 1$$

iv) $\int_0^a 4x^3 dx = 16$

$$\therefore 4 \left[\frac{x^4}{4} \right]_0^a = 16$$

$$\frac{4}{4} [a^4 - 0] = 16$$

$$a^4 = 16$$

$$\therefore a = 2$$

$$\begin{aligned} \text{v)} \quad & \int_0^1 f(x) dx \\ &= \int_0^1 (a + bx + cx^2) dx \\ &= a \int_0^1 1 dx + b \int_0^1 x dx + c \int_0^1 x^2 dx \\ &= \left[ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right]_0^1 \\ &= a + \frac{b}{2} + \frac{c}{3} \dots\dots\dots(1) \end{aligned}$$

$$\text{Now } f(0) = a + b(0) + c(0)^2 = a$$

$$f(1/2) = a + b(1/2) + c(1/2)^2 = a + b/2 + c/4$$

$$\text{and } f(1) = a + b + c$$

$$\begin{aligned} \therefore \quad & \frac{1}{6} \left[f(0) + 4f\left(\frac{1}{2}\right) + f(1) \right] \\ &= \frac{1}{6} \left[a + 4\left(a + \frac{b}{2} + \frac{c}{4}\right) + (a + b + c) \right] \\ &= \frac{1}{6} [a + 4a + 2b + c + a + b + c] \\ &= \frac{1}{6} [6a + 3b + 2c] \\ &= a + \frac{b}{2} + \frac{c}{3} \dots\dots\dots(2) \end{aligned}$$

From (1) and (2)

$$= \int_0^1 f(x) dx = \frac{1}{6} \left[f(0) + 4f\left(\frac{1}{2}\right) + f(1) \right]$$

Ex 3 : Evaluate:

$$\text{i)} \quad \int_0^2 \frac{1}{4+x-x^2} dx$$

$$\text{ii)} \quad \int_0^4 \frac{dx}{\sqrt{x^2+2x+3}}$$

Solution:

$$\begin{aligned} \text{i)} \quad & \int_0^2 \frac{1}{4+x-x^2} dx \\ &= \int_0^2 \frac{1}{-x^2+x+4} dx \\ &= \int_0^2 \frac{-1}{x^2-x+\frac{1}{4}-\frac{1}{4}-4} dx \\ &= \int_0^2 \frac{-1}{\left(x-\frac{1}{2}\right)^2 - \left(\frac{\sqrt{17}}{2}\right)^2} dx \\ &= \int_0^2 \frac{1}{\left(\frac{\sqrt{17}}{2}\right)^2 - \left(x-\frac{1}{2}\right)^2} dx \\ &= \frac{1}{\sqrt{17}} \left[\log \left| \frac{\frac{\sqrt{17}}{2} + \left(x-\frac{1}{2}\right)}{\frac{\sqrt{17}}{2} - \left(x-\frac{1}{2}\right)} \right| \right]_0^2 \\ &= \frac{1}{\sqrt{17}} \left[\log \left| \frac{\sqrt{17} + 2x - 1}{\sqrt{17} - 2x + 1} \right| \right]_0^2 \\ &= \frac{1}{\sqrt{17}} \left\{ \log \left(\frac{\sqrt{17} + 4 - 1}{\sqrt{17} - 4 + 1} \right) - \log \left(\frac{\sqrt{17} - 1}{\sqrt{17} + 1} \right) \right\} \\ &= \frac{1}{\sqrt{17}} \left\{ \log \left(\frac{\sqrt{17} + 3}{\sqrt{17} - 3} \right) - \log \left(\frac{\sqrt{17} - 1}{\sqrt{17} + 1} \right) \right\} \\ &= \frac{1}{\sqrt{17}} \log \left(\frac{\sqrt{17} + 3}{\sqrt{17} - 3} \times \frac{\sqrt{17} + 1}{\sqrt{17} - 1} \right) \\ &= \frac{1}{\sqrt{17}} \log \left(\frac{20 + 4\sqrt{17}}{20 - 4\sqrt{17}} \right) \\ &= \frac{1}{\sqrt{17}} \log \left(\frac{5 + \sqrt{17}}{5 - \sqrt{17}} \right) \end{aligned}$$

$$\begin{aligned}
\text{ii)} \quad & \int_0^4 \frac{dx}{\sqrt{x^2 + 2x + 3}} \\
&= \int_0^4 \frac{1}{\sqrt{x^2 + 2x + 1 - 1 + 3}} dx \\
&= \int_0^4 \frac{1}{\sqrt{(x+1)^2 + (\sqrt{2})^2}} dx \\
&= \left[\log \left| (x+1) + \sqrt{(x+1)^2 + (\sqrt{2})^2} \right| \right]_0^4 \\
&= \left[\log \left| (x+1) + \sqrt{x^2 + 2x + 3} \right| \right]_0^4 \\
&= \log(5 + \sqrt{16 + 8 + 3}) - \log(1 + \sqrt{3}) \\
&= \log(5 + 3\sqrt{3}) - \log(1 + \sqrt{3}) \\
&= \log \left[\frac{5 + 3\sqrt{3}}{(1 + \sqrt{3})} \right]
\end{aligned}$$

Ex. 4: Evaluate:

$$\text{i)} \quad \int_1^2 \log x \, dx$$

$$\text{ii)} \quad \int_1^2 \frac{\log x}{x^2} dx$$

Solution:

$$\begin{aligned}
\text{i)} \quad I &= \int_1^2 \log x \, dx \\
I &= \int_1^2 \log x \cdot 1 \cdot dx \\
I &= \left[\log x \cdot x \right]_1^2 - \int_1^2 \frac{1}{x} x \, dx \\
&= \left[x \log x - x \right]_1^2 \\
&= \left[(2 \log 2 - 1 \log 1) \right] - [2 - 1] \\
&= (\log 4 - 0) - 1 \\
&= \log 4 - 1
\end{aligned}$$

$$\begin{aligned}
\text{ii)} \quad & \int_1^2 \frac{\log x}{x^2} dx \\
&= \int_1^2 \log x \cdot \frac{1}{x^2} \cdot dx \\
&= \int \log x \cdot \frac{1}{x^2} \cdot dx = \left[\left(\log x \left(\frac{-1}{x} \right) \right) \right]_1^2 - \int_1^2 \frac{1}{x} \left(\frac{x^{-1}}{-1} \right) dx \\
&= \left[\log x \left(\frac{-1}{x} \right) - \frac{1}{x} \right]_1^2 \\
&= \left(\frac{-1}{2} \log 2 + \log 1 \right) - \left(\frac{1}{2} - \frac{1}{1} \right) \\
&= \frac{-1}{2} \log 2 + \frac{1}{2} \\
&= \frac{1}{2} (-\log 2 + 1) \\
&= \frac{1}{2} (-\log 2 + \log e) \\
&= \frac{1}{2} \log \frac{e}{2}
\end{aligned}$$

Ex. 5: Evaluate:

$$\text{i)} \quad \int_1^2 \frac{1}{(x+1)(x+3)} dx$$

$$\text{ii)} \quad \int_1^3 \frac{1}{x(1+x^2)} dx$$

Solution:

$$\begin{aligned}
\text{i)} \quad & \int_1^2 \frac{1}{(x+1)(x+3)} dx \\
\text{Let } \frac{1}{(x+1)(x+3)} &= \frac{A}{(x+1)} + \frac{B}{(x+3)} \\
1 &= A(x+3) + B(x+1) \dots\dots\dots (1) \\
\text{Putting } x+1 &= 0 \\
\text{i.e. } x &= -1 \text{ in equation (i) we get } A = \frac{1}{2} \\
\text{Putting } x+3 &= 0 \\
\text{i.e. } x &= -3 \text{ in equation (i) we get } B = \frac{-1}{2}
\end{aligned}$$

$$\begin{aligned}
\frac{1}{(x+1)(x+3)} &= \frac{\frac{1}{2}}{(x+1)} + \frac{-\frac{1}{2}}{(x+3)} \\
\int_1^2 \frac{1}{(x+1)(x+3)} dx &= \frac{1}{2} \int_1^2 \frac{dx}{x+1} - \frac{1}{2} \int_1^2 \frac{dx}{x+3} \\
&= \frac{1}{2} [\log|x+1| - \log|x+3|]_1^2 \\
&= \frac{1}{2} (\log 3 - \log 2) - \frac{1}{2} (\log 5 - \log 4) \\
&= \frac{1}{2} \left[\log \frac{3}{2} - \log \frac{5}{4} \right] \\
&= \frac{1}{2} \log \left[\frac{6}{5} \right]
\end{aligned}$$

ii) $\int_1^3 \frac{1}{x(1+x^2)} dx$

Let $\frac{1}{x(1+x^2)} = \frac{A}{x} + \frac{Bx+c}{1+x^2}$

$1 = A(1+x^2) + (Bx+c)x \dots\dots\dots(1)$

Putting $x = 0$ in equation (i) we get $A = 1$
Comparing the coefficient of x^2 and x , we get $A + B = 0$, $B = -1$ & $C = 0$

$$\begin{aligned}
\frac{1}{x(1+x^2)} &= \frac{1}{x} - \frac{x}{1+x^2} \\
\int_1^3 \frac{dx}{x(1+x^2)} &= \int_1^3 \frac{1}{x} dx - \int_1^3 \frac{x}{1+x^2} dx \\
&= \left[\log|x| \right]_1^3 - \frac{1}{2} \left[\log|1+x^2| \right]_1^3 \\
&= (\log 3 - \log 1) - \frac{1}{2} (\log 10 - \log 2) \\
&= \log \left(\frac{3}{1} \right) - \frac{1}{2} \log \left(\frac{10}{2} \right)
\end{aligned}$$

$$\begin{aligned}
&= (\log 3 - \frac{1}{2} \log 5) \\
&= \log 3 - \log 5^{1/2} = \log 3 - \log \sqrt{5} \\
&= \log \left(\frac{3}{\sqrt{5}} \right)
\end{aligned}$$

EXERCISE 6.1

Evaluate the following definite integrals:

- $\int_4^9 \frac{1}{\sqrt{x}} dx$
- $\int_{-2}^3 \frac{1}{x+5} dx$
- $\int_2^3 \frac{x}{x^2-1} dx$
- $\int_0^1 \frac{x^2+3x+2}{\sqrt{x}} dx$
- $\int_2^3 \frac{x}{(x+2)(x+3)} dx$
- $\int_1^2 \frac{dx}{x^2+6x+5}$
- If $\int_0^a (2x+1) dx = 2$, find the real value of a .
- If $\int_1^a (3x^2+2x+1) dx = 11$, find a .
- $\int_0^1 \frac{1}{\sqrt{1+x}+\sqrt{x}} dx$
- $\int_1^2 \frac{3x}{(9x^2-1)} dx$
- $\int_1^3 \log x dx$

6.2 Properties of definite integrals

In this section we will study some properties of definite integrals which are very useful in evaluating integrals.

Property 1 : $\int_a^a f(x) dx = 0$

Property 2 : $\int_a^b f(x) dx = -\int_b^a f(x) dx$

Property 3 : $\int_a^b f(x) dx = \int_a^b f(t) dt$

Property 4 : $\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$, where $a < c < b$

Property 5 : $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$

Property 6 : $\int_0^a f(x) dx = \int_0^a f(a-x) dx$

Property 7 : $\int_0^{2a} f(x) dx = \int_0^a f(x) dx + \int_0^a f(2a-x) dx$

[If $f(-x) = f(x)$, $f(x)$ is an even function.
If $f(-x) = -f(x)$, $f(x)$ is an odd function.]

Property 8 : $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$ if f is an even function
 $= 0$ if f is an odd function

SOLVED EXAMPLES

Ex. Evaluate the following integrals:

1. $\int_{-1}^1 f(x) dx$ where $f(x) = \begin{cases} 1-2x; x \leq 0 \\ 1+2x; x \geq 0 \end{cases}$

2. $\int_0^1 x(1-x)^n dx$

3. $\int_0^3 \frac{\sqrt[3]{x+4}}{\sqrt[3]{x+4} + \sqrt[3]{7-x}} dx$

4. $\int_a^b \frac{f(x)}{f(x) + f(a+b-x)} dx$

5. $\int_4^7 \frac{(11-x^2)}{x^2 + (11-x^2)} dx$

Solution:

1.
$$\begin{aligned} \int_{-1}^1 f(x) dx &= \int_{-1}^0 f(x) dx + \int_0^1 f(x) dx \\ &= \int_{-1}^0 (1-2x) dx + \int_0^1 (1+2x) dx \\ &= \left[x - x^2 \right]_{-1}^0 + \left[x + x^2 \right]_0^1 \\ &= [0 - (-1 - 1)] + [(1 + 1) - 0] \\ &= 2 + 2 = 4 \end{aligned}$$

2. $\int_0^1 x(1-x)^n dx$
By property $\int_0^a f(x) dx = \int_0^a f(a-x) dx$

$$\begin{aligned} I &= \int_0^1 (1-x)[1-(1-x)]^n dx \\ &= \int_0^1 (1-x)x^n dx \\ &= \int_0^1 (x^n - x^{n+1}) dx \\ &= \left[\frac{x^{n+1}}{n+1} \right]_0^1 - \left[\frac{x^{n+2}}{n+2} \right]_0^1 \\ &= \frac{1}{n+1} - \frac{1}{n+2} = \frac{(n+2) - (n+1)}{(n+1)(n+2)} \\ &= \frac{1}{(n+1)(n+2)} \end{aligned}$$

3. Let $I = \int_0^3 \frac{\sqrt[3]{x+4}}{\sqrt[3]{x+4} + \sqrt[3]{7-x}} dx$ (1)

By property $\int_0^a f(x) dx = \int_0^a f(a-x) dx$

$$\begin{aligned}
 I &= \int_0^3 \frac{\sqrt[3]{(3-x)+4}}{\sqrt[3]{(3-x)+4} + \sqrt[3]{7-(3-x)}} dx \\
 &= \int_0^3 \frac{\sqrt[3]{7-x}}{\sqrt[3]{7-x} + \sqrt[3]{x+4}} dx \quad \dots\dots\dots (2)
 \end{aligned}$$

On adding equations (1) and (2)

$$\begin{aligned}
 2I &= \int_0^3 \left[\frac{\sqrt[3]{x+4}}{\sqrt[3]{7-x} + \sqrt[3]{x+4}} + \frac{\sqrt[3]{7-x}}{\sqrt[3]{7-x} + \sqrt[3]{x+4}} \right] dx \\
 &= \int_0^3 \frac{\sqrt[3]{x+4} + \sqrt[3]{(7-x)}}{\sqrt[3]{x+4} + \sqrt[3]{(7-x)}} dx \\
 &= \int_0^3 1 dx \\
 &= [x]_0^3
 \end{aligned}$$

$$2I = 3$$

$$I = \frac{3}{2}$$

$$\therefore \int_0^3 \frac{\sqrt[3]{(x+4)}}{\sqrt[3]{(x+4)} + \sqrt[3]{(7-x)}} dx = \frac{3}{2}$$

$$4. \int_a^b \frac{f(x)}{f(x) + f(a+b-x)} dx \quad \dots\dots\dots (1)$$

$$\text{By property } \int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$I = \int_a^b \frac{f(a+b-x)}{f(a+b-x) + f[(a+b-(a+b-x))]} dx$$

$$I = \int_a^b \frac{f(a+b-x)}{f(a+b-x) + f(x)} dx \quad \dots\dots\dots (2)$$

Adding equations (1) and (2) we get,

$$2I = \int_a^b \frac{f(x)}{f(x) + f(a+b-x)} dx +$$

$$\int_a^b \frac{f(a+b-x)}{f(a+b-x) + f(x)} dx$$

$$= \int_a^b \frac{f(x) + f(a+b-x)}{f(x) + f(a+b-x)} dx$$

$$= \int_a^b 1 dx$$

$$= [x]_a^b$$

$$2I = b - a$$

$$I = \frac{b-a}{2}$$

$$\therefore \int_a^b \frac{f(x)}{f(x) + f(a+b-x)} dx = \frac{b-a}{2}$$

$$5. \int_4^7 \frac{(11-x^2)}{x^2 + (11-x^2)} dx \quad \dots\dots\dots (1)$$

$$= \int_4^7 \frac{x^2}{(11-x^2) + x^2} dx \quad \dots\dots\dots (2)$$

By Property

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

Adding equations (1) and (2)

$$2I = \int_4^7 \frac{(11-x^2)}{x^2 + (11-x^2)} dx + \int_4^7 \frac{x^2}{(11-x^2) + x^2} dx$$

$$= \int_4^7 \frac{(x^2) + (11-x^2)}{(x^2) + (11-x^2)} dx$$

$$= \int_4^7 1 dx$$

$$= [x]_4^7$$

$$2I = 3$$

$$I = \frac{3}{2}$$

$$\therefore \int_4^7 \frac{(11-x^2)}{x^2 + (11-x^2)} dx = \frac{3}{2}$$

EXERCISE 6.2

Evaluate the following integrals:

1) $\int_{-9}^9 \frac{x^3}{4-x^2} dx$

2) $\int_0^a x^2 (a-x)^{3/2} dx$

3) $\int_1^3 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5} + \sqrt[3]{9-x}} dx$

4) $\int_2^5 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{7-x}} dx$

5) $\int_1^2 \frac{\sqrt{x}}{\sqrt{3-x} + \sqrt{x}} dx$

6) $\int_2^7 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{9-x}} dx$

7) $\int_0^1 \log\left(\frac{1}{x} - 1\right) dx$

8) $\int_0^1 x(1-x)^5 dx$



Let's Remember

● **Rules for evaluating definite integrals.**

1) $\int_a^b [f(x) \pm g(x)] dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx$

2) $\int_a^b k f(x) dx = k \int_a^b f(x) dx$

● **Properties of definite integrals**

1) $\int_a^a f(x) dx = 0$

2) $\int_a^b f(x) dx = -\int_b^a f(x) dx$

3) $\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$

4) $\int_a^b f(x) dx = \int_a^b f(t) dt$

5) $\int_0^a f(x) dx = \int_0^a f(a-x) dx$

6) $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$

7) $\int_0^{2a} f(x) dx = \int_0^a f(x) dx + \int_0^a f(2a-x) dx$

8) $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$, if f is even function,
 $= 0$, if f is odd function

MISCELLANEOUS EXERCISE - 6

I) Choose the correct alternative.

1) $\int_{-9}^9 \frac{x^3}{4-x^2} dx =$

- a) 0 b) 3 c) 9 d) -9

2) $\int_{-2}^3 \frac{dx}{x+5} =$

- a) $-\log\left(\frac{8}{3}\right)$ b) $\log\left(\frac{8}{3}\right)$
 c) $\log\left(\frac{3}{8}\right)$ d) $-\log\left(\frac{3}{8}\right)$

3) $\int_2^3 \frac{x}{x^2-1} dx =$

- a) $\log\left(\frac{8}{3}\right)$ b) $-\log\left(\frac{8}{3}\right)$
 c) $\frac{1}{2} \log\left(\frac{8}{3}\right)$ d) $-\frac{1}{2} \log \frac{8}{3}$

4) $\int_4^9 \frac{dx}{\sqrt{x}} =$
 a) 9 b) 4 c) 2 d) 0

5) If $\int_0^a 3x^2 dx = 8$ then $a = ?$
 a) 2 b) 0 c) $\frac{8}{3}$ d) a

6) $\int_2^3 x^4 dx =$
 a) $\frac{1}{2}$ b) $\frac{5}{2}$ c) $\frac{5}{211}$ d) $\frac{211}{5}$

7) $\int_0^2 e^x dx =$
 a) $e - 1$ b) $1 - e$
 c) $1 - e^2$ d) $e^2 - 1$

8) $\int_a^b f(x) dx =$
 a) $\int_b^a f(x) dx$ b) $-\int_a^b f(x) dx$
 c) $-\int_b^a f(x) dx$ d) $\int_0^a f(x) dx$

9) $\int_{-7}^7 \frac{x^3}{x^2 + 7} dx =$
 a) 7 b) 49 c) 0 d) $\frac{7}{2}$

10) $\int_2^7 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{9-x}} dx =$
 a) $\frac{7}{2}$ b) $\frac{5}{2}$ c) 7 d) 2

II) Fill in the blanks.

1) $\int_0^2 e^x dx = \dots\dots\dots$

2) $\int_2^3 x^4 dx = \dots\dots\dots$

3) $\int_0^1 \frac{dx}{2x+5} = \dots\dots\dots$

4) If $\int_0^a 3x^2 dx = 8$ then $a = \dots\dots\dots$

5) $\int_4^9 \frac{1}{\sqrt{x}} dx = \dots\dots\dots$

6) $\int_2^3 \frac{x}{x^2-1} dx = \dots\dots\dots$

7) $\int_{-2}^3 \frac{dx}{x+5} = \dots\dots\dots$

8) $\int_{-9}^9 \frac{x^3}{4-x^2} dx = \dots\dots\dots$

III) State whether each of the following is True or False

1) $\int_a^b f(x) dx = \int_{-b}^{-a} f(x) dx$

2) $\int_a^b f(x) dx = \int_a^b f(t) dt$

3) $\int_0^a f(x) dx = \int_a^0 f(a-x) dx$

4) $\int_a^b f(x) dx = \int_a^b f(x-a-b) dx$

5) $\int_{-5}^5 \frac{x^3}{x^2+7} dx = 0$

$$6) \int_1^2 \frac{\sqrt{x}}{\sqrt{3-x} + \sqrt{x}} dx = \frac{1}{2}$$

$$7) \int_2^7 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{9-x}} dx = \frac{9}{2}$$

$$8) \int_4^7 \frac{(11-x)^2}{(11-x)^2 + x^2} dx = \frac{3}{2}$$

IV Solve the following.

$$1) \int_2^3 \frac{x}{(x+2)(x+3)} dx$$

$$2) \int_1^2 \frac{x+3}{x(x+2)} dx$$

$$3) \int_1^3 x^2 \log x dx$$

$$4) \int_0^1 e^{x^2} x^3 dx$$

$$5) \int_1^2 e^{2x} \left(\frac{1}{x} - \frac{1}{2x^2} \right) dx$$

$$6) \int_4^9 \frac{1}{\sqrt{x}} dx$$

$$7) \int_{-2}^3 \frac{1}{x+5} dx$$

$$8) \int_2^3 \frac{x}{x^2-1} dx$$

$$9) \int_0^1 \frac{x^2 + 3x + 2}{\sqrt{x}} dx$$

$$10) \int_3^5 \frac{dx}{\sqrt{x+4} + \sqrt{x-2}}$$

$$11) \int_2^3 \frac{x}{x^2+1} dx$$

$$12) \int_1^2 x^2 dx$$

$$13) \int_{-4}^{-1} \frac{1}{x} dx$$

$$14) \int_0^1 \frac{1}{\sqrt{1+x} + \sqrt{x}} dx$$

$$15) \int_0^4 \frac{1}{\sqrt{x^2 + 2x + 3}} dx$$

$$16) \int_2^4 \frac{x}{x^2+1} dx$$

$$17) \int_0^1 \frac{1}{2x-3} dx$$

$$18) \int_1^2 \frac{5x^2}{x^2+4x+3} dx$$

$$19) \int_1^2 \frac{dx}{x(1+\log x)^2}$$

$$20) \int_0^9 \frac{1}{1+\sqrt{x}} dx$$

Activities

1) Complete the following activity.

$$\text{If } \int_a^b x^3 dx = 0 \text{ then}$$

$$\left(\frac{x^4}{\square} \right)_a^b = 0$$

$$\therefore \frac{1}{4}(\square - \square) = 0$$

$$\therefore b^4 - \square = 0$$

$$\therefore (b^2 - a^2)(\square + \square) = 0$$

$$\therefore b^2 - \square = 0 \quad \text{as } a^2 + b^2 \neq 0$$

$$\therefore b = \pm \square$$

$$\begin{aligned}
2) \quad & \int_0^2 \frac{dx}{4+x-x^2} \\
&= \int_0^2 \frac{dx}{-x^2 + \boxed{} + \boxed{}} \\
&= \int_0^2 \frac{dx}{-x^2 + x + \frac{1}{4} - \boxed{} - 4} \\
&= -\int_0^2 \frac{dx}{\left(x - \frac{1}{2}\right)^2 - (\boxed{})^2} \\
&= \frac{1}{\sqrt{17}} \log \left(\frac{20+4\sqrt{17}}{20-4\sqrt{17}} \right)
\end{aligned}$$

$$\begin{aligned}
3) \quad & \int_0^1 \log \left(\frac{1}{x} - 1 \right) dx \\
&= \int_0^1 \log \left(\frac{1-x}{\boxed{}} \right) dx \quad \dots\dots\dots(1) \\
&= \int_0^1 \log \left(\frac{1-(1-x)}{\boxed{}} \right) dx
\end{aligned}$$

$$= \int_0^1 \log \left(\frac{\boxed{}}{1-x} \right) dx \quad \dots\dots\dots(2)$$

Adding (i) and (ii), we get

$$\begin{aligned}
2I &= \int_0^1 \log \left[\frac{1-x}{x} \times \frac{x}{\boxed{}} \right] dx \\
&= \int_0^1 \log \boxed{} dx = \int_0^1 0 dx =
\end{aligned}$$

$$\begin{aligned}
4) \quad & \int_{-8}^8 \frac{x^5}{1-x^2} dx \\
&f(x) = \frac{x^5}{1-x^2} \\
&f(-x) = \frac{(-x)^5}{1-x^2} = \frac{\boxed{}}{1-x^2}
\end{aligned}$$

Hence f is $\boxed{}$ function

$$\therefore \int_{-8}^8 \frac{x^5}{1-x^2} dx = \boxed{}$$

