Chapter 3 Decision Procedure

The concept of decision procedure is predominantly concerned with the concept of decidability.

DO YOU KNOW THAT ....

Logic can help us to determine correctness of certain reasoning by formulating tables.

Some statement forms are always false.

When you ask your friend, would you go to London or Paris, he can choose both.

3.1 CONCEPT OF DECISION PROCEDURE

In the earlier chapter, we have studied about the nature of proposition, its kinds and its basic truth values. In this chapter, we are going to study the procedure for deciding the validity of arguments. In logic, we use the decision procedure (method) to decide whether a truth functional form is Tautologous, Contradictory or Contingent. It also tests whether an argument is valid or invalid. Decision Procedure may be defined as a method of deciding whether an object belongs to a certain class.

There are five types of Decision Procedures:

(1) Truth Table

(2) Shorter Truth Table

(3) Truth Tree

(4) Conjunctive Normal Form

(5) Disjunctive Normal Form

In this text we shall study the method of constructing Truth Table as a decision procedure.

Characteristics of decision procedure :

A decision procedure must be effective, to be an effective decision procedure certain conditions need to be fulfilled. –

(1) Reliable : A decision procedure must be reliable. A reliable procedure is one which always gives a correct answer, provided we use the method and rules correctly.

(2) Mechanical : A decision procedure is mechanical ie just by following certain steps in a certain order one can get an answer. There is no scope for one's imagination and intelligence.

(3) Finite : A decision procedure must be finite ie it should have limited number of steps. There should be a last step for getting the answer.

3.2 NATURE OF TRUTH TABLE

Truth table is one of the decision procedures. **A truth table is defined as a tabular way of expressing the truth value of expressions containing propositional connectives (a truth functionally compound statement).**

Procedure of Construction of a truth table (for truth – functional statement form)

(1) To construct a truth table we shall first make two columns: one on the left hand side for the matrix and the other on the right hand side for the truth – functional form for which the truth table is constructed.

Example : $(q \vee p) \equiv [(p \bullet q) \supset p]$

Matrix	Truth – Functional Form
	$(q \vee p) \equiv [(p \bullet q) \supset p]$

The first step is to write down the Truth functional form in the column for Truth functional form.

(2) The second step is to write down in the matrix column all the distinct variables occurred in the truth functional form.

In above example, there are two, distinct variables 'p' and 'q'. So we write them as follows.

Matrix	Truth – Functional Form
p q	(q ∨ p) ≡ [(p • q) ⊃ p]

(3) The third step is to determine the number of rows the truth table will have. The number of rows depends upon the number of propositional variables, occurred in the truth functional form. The simple formula is,

2^n = Number of rows.

(n = Number of distinct variables occurring in the expression)

No. of distinct variable		No. of rows
2^1	2 x 1	2
2^2	2 x 2	4
2^3	2 x 2 x 2	8
2^4	2 x 2 x 2 x 2	16
2^5	2 x 2 x 2 x 2 x 2	32

Activity : 1

2^6 = ______ = ______

2^7 = ______ = ______

(4) Fourth step is to construct the matrix.

A matrix consists of all the possible combinations of the truth values of the propositional variables in the truth function or argument.

(a) Matrix for one variable

Example : (p • p) ∨ p

Matrix	Truth – Functional Form
p	(p • p) ∨ p
T	
F	

(b) Matrix for two variables:-

Example : (p ∨ q) ⊃ (q ⊃ p)

Matrix	Truth – Functional Form
p q	(p ∨ q) ⊃ (q ⊃ p)
T T	
T F	
F T	
F F	

(c) Matrix for three variables:-

Example : p ≡ (q • r)

Matrix	Truth – Functional Form
p q r	p ≡ (q • r)
T T T	
T T F	
T F T	
T F F	
F T T	
F T F	
F F T	
F F F	

Always Remember

The propositional variables are written in the alphabetical order in the matrix.

E.g. (r ∨ q) • r

Matrix		Truth – Functional Form
q	r	(r ∨ q) • r
T	T	
T	F	
F	T	
F	F	

Activity 2 : 1

Construct matrix for 4 variables i.e. p, q, r, s

Construct matrix for 5 variables i.e, p, q, r, s, t.

Activity 2:2 Complete the matrix column

Activity : 1

r	(r ⊃ r) ∨ (r • r)

Activity : 2

q ☐	(t • q) ≡ (q ∨ t)

Activity : 3

☐ ☐	(p ∨ s) ≡ (p ⊃ s)

Activity : 4

☐ ☐ ☐ ☐	(r ⊃ s) • (p ≡ t)

Let us continue with same example

Matrix		Truth – Functional Form
p	q	(q ∨ p) ≡ [(p • q) ⊃ p]
T	T	
T	F	
F	T	
F	F	

(5) Let us construct the truth table. In the above truth – functional form, there are two distinct variables i. e,. 'p' and 'q', wherever 'p' occurs in the truth functional form we shall write down the truth values written under 'p' in the matrix. Then wherever 'q' occurs in the truth function we shall write down the truth – values written under 'q' in the matrix. After assigning the values to 'p' and 'q' variables, the truth table will be as follows.

Matrix		Truth – Functional Form				
p	q	(q ∨ p) ≡ [(p • q) ⊃ p]				
T	T	T	T	T	T	T
T	F	F	T	T	F	T
F	T	T	F	F	T	F
F	F	F	F	F	F	F

(6) In the previous chapter, we have learnt the basic truth values of compound propositions. Accordingly we shall now determine the truth value of the truth functional form.

Example : (q ∨ p) ≡ [(p • q) ⊃ p]

❖ In our expression, '≡' is the main connective.

(q ∨ p) ≡ [(p • q) ⊃ p]

❖ First we shall find out the truth value of the expression on the left – hand side of the truth functional form, i.e. disjunction between 'q' and 'p' as follows.

Matrix	Truth – Functional Form
P q	(q ∨ p) ≡ [(p • q) ⊃ p]
T T	T **T** T T T T
T F	F **T** T T F T
F T	T **T** F F T F
F F	F **F** F F F F

❖ Then, we shall determine the truth value of the expression on the right – hand side of the truth functional form i.e the value of conjunction between 'p' and 'q' as follows.

Matrix	Truth – Functional Form
P q	(q ∨ p) ≡ [(p • q) ⊃ p]
T T	T **T** T T T T T
T F	F **T** T T F F T
F T	T **T** F F F T F
F F	F **F** F F F F F

❖ Let us determine the truth value of the conditional statement between conjunction i.e., p • q and variable 'p' to the right side.

Matrix	Truth – Functional Form
P q	(q ∨ p) ≡ [(p • q) ⊃ p]
T T	T **T** T T T T **T** T
T F	F **T** T T F F **T** T
F T	T **T** F F F T **T** F
F F	F **F** F F F F **T** F

❖ Finally, let us determine the truth value of material equivalent statement which is the main connective i.e. between (q ∨ p) and [(p • q) ⊃ p] which will give us the truth value of truth – functional form under all possibilities. We need to consider disjunction in the left bracket and implication in the right bracket. Taking these two values, we will determine the value of equivalence which is the main connective.

Thus, the final truth table will be as follows:

Matrix	Truth – Functional Form
p q	(q ∨ p) ≡ [(p • q) ⊃ p]
T T	T **T** T **T** T T T **T** T
T F	F **T** T **T** T F F **T** T
F T	T **T** F **T** F F T **T** F
F F	F **F** F **F** F F F **T** F

This truth table shows that under the main connective, only in one possibility i.e, in the fourth row, the truth functional form is false. In remaining possibilities it is true.

❖ Let us understand truth table with more examples.

Example 2 : (~ r • ~ p) ⊃ (r ∨ ~ p)

Matrix	Truth – Functional Form
p r	(~ r • ~ p) ⊃ (r ∨ ~ p)
T T	F T **F** F T **T** T **T** F T
T F	T F **F** F T **T** F **F** F T
F T	F T **F** T F **T** T **T** T F
F F	T F **T** T F **T** F **T** T F

Example 3 : ~ (t ∨ q) • ~ (~ t • ~ q)

Matrix	Truth – Functional Form
q t	~ (t ∨ q) • ~ (~ t • ~ q)
T T	**F** T T T **F** **T** F T F F T
T F	**F** F T T **F** **T** T F F F T
F T	**F** T T F **F** **T** F T F T F
F F	**T** F F F **F** **F** T F T T F

Activity : 3

Complete the following tables

Matrix	Truth – Functional Form
q	(q ⊃ ~ q) • ~ q
T	☐ **F** ☐ ☐ F ☐ T
F	F ☐ ☐ ☐ ☐ ☐ ☐

Matrix	Truth – Functional Form
p s t	t ≡ (p ∨ s)
T T T	☐ ☐ T **T** ☐
T T F	☐ **F** ☐ ☐ ☐
T F T	**T** ☐ ☐ ☐ ☐
T F F	☐ ☐ T ☐ ☐
F T T	☐ ☐ F ☐ ☐
F T F	**F** ☐ ☐ **T** T
F F T	☐ ☐ ☐ ☐ F
F F F	☐ **T** ☐ ☐ ☐

Concept of Tautology, Contradiction and Contingency

The truth functional statement forms are broadly classified into three kinds. They are Tautology, Contradiction and Contingency.

(1) Tautology :

A tautology is a truth functional statement form which is "True" under all truth possibilities of its components. It means that in the truth table for a tautology truth value "True" appears under the main connective in all the rows. Thus, tautology is a statement form which has all true substitution instances.

Example : (p • ~ p) ⊃ ~ p

Matrix	Truth - Functional Form
p	(p • ~ p) ⊃ ~ p
T	T **F** F T **T** **F** T
F	F **F** T F **T** **T** F

In the above statement form, truth value T's appear under the main connective, so the given truth functional form is tautology.

(2) Contradiction :

A contradiction is defined as a truth functional statement form which is 'False' under all truth possibilities of its components. It means that in the truth table for a contradiction truth value 'False' appears under the main connective in all the rows. A contradiction is a statement form which has only 'False' substitution instances.

Example : (p ≡ ~ p) • (~ p ⊃ p)

Matrix	Truth - Functional Form
P	(p ≡ ~ p) • (~ p ⊃ p)
T	T **F** F T **F** F T **T** T
F	F **F** T F **F** T F **F** F

In the above statement form, truth value F' s appears under the main connective, so the given truth functional form is contradiction.

(3) Contingency :

A contingency is defined as a truth functional statement form which is 'True' as well as 'False' under some truth possibilities of its components. It means that in the truth table for contingency truth value 'True' as well as 'False' appears under the main connectives in truth possibilities. Thus contingency is a statement form which has some true as well as false substitution instances.

Example : (p • ~ p) ≡ (p ⊃ ~ p)

Matrix	Truth - Functional Form
P	(p • ~ p) ≡ (p ⊃ ~ p)
T	T **F** F T **T** T **F** F T
F	F **F** T F **F** F **T** T F

In the above truth functional statement form T's and F' s appears under the main connective. So, the given propositional form is a contingency.

Relation between Tautology, Contradiction and Contingency:

(1) Denial of tautology leads to contradiction.

For Example : the truth – functional form '(p • p) ⊃ p ' is a tautology. Its denial i.e. ~ [(p • p) ⊃ p]is a contradiction.

(2) Denial of contradiction leads to tautology.

For Example : the truth – functional form ' (p • ~ p)' is a contradiction. Its denial i.e. ~ (p • ~ p) is a tautology.

(3) Denial of contingency leads to contingency.

For Example : the truth -functional form (~p •q) is a contingency. Its denial i.e. ~ (~p •q) is a contigency.

Let us now construct truth table for some truth – functional statement forms and determine whether they are tautology, contradiction or contingency.

Example 1 : : ~ [p • (p ∨ ~ p)] ⊃ (p ⊃ p)

Matrix	Truth Functional Form
p	~ [p • (p ∨ ~ p)] ⊃ (p ⊃ p)
T	**F** T T T T F T **T** T **T** T
F	**T** F F F T T F **T** F **T** F

Example 2 :

Matrix	Truth - Functional Form
p q r	(p ⊃ q) ∨ r
T T T	T **T** T **T** **T**
T T F	T **T** T **T** **F**
T F T	T **F** F **T** **T**
T F F	T **F** F **F** **F**
F T T	F **T** T **T** **T**
F T F	F **T** T **T** **F**
F F T	F **T** F **T** **T**
F F F	F **T** F **T** **F**

Example 3 :

~ (q ∨ p) • ~ (~ q • ~ p)

Matrix	Truth 0 Functional Form
p q	~ (q ∨ p) • ~ (~ q • ~ p)
T T	**F** T T T **F** **T** F T F F T
T F	**F** F T T **F** **T** T F F F T
F T	**F** T T F **F** **T** F T F T F
F F	**T** F F F **F** **F** T F T T F

Activity : 4

State whether the above statement forms are tautology, contradiction or contingency with reason.

Activity : 5

Construct the truth table for the following truth – functional forms and determine whether they are tautologies, contradictions or contingencies.

1. (~ q ⊃ ~ p) ≡ (p ⊃ q)
2. p ∨ (q • r)
3. (~p • p) ∨ p

3.3 Truth table as a decision procedure for arguments

An a argument is a group of statements. An argument consists of simple and truth – functionally compound propositions.

Let's now examine a truth table method to determine an argument form to be valid or invalid.

Example :

Amita is intelligent and courageous.

Amita is intelligent.

Amita is courageous.

Therefore either Amita is intelligent or courageous.

(I, C)

Let's symbolize the given argument.

❖ Symbolization of an argument

(1) I • C

(2) I

(3) C

∴ I V C

❖ Now we will convert the given symbolized argument in argument form.

(1) p • q

(2) p

(3) q

∴ p ∨ q

❖ Now let us construct a truth table for a given argument form. Write down the matrix and then column for premises and conclusion in a single row in order.

Matrix	Premise 1	Premise 2	Premise 3	Conclusion
p q	p • q	p	q	p ∨q

❖ Construct a matrix for a given argument form, then assign the truth - values under each variables.

Matrix p q	Premise 1 p • q	Premise 2 P	Premise 3 q	Conclusion p ∨ q
T T	T T	T	T	T T
T F	T F	T	F	T F
F T	F T	F	T	F T
F F	F F	F	F	F F

• By using the truth values of propositions, assign the truth value to the premises and conclusion separately.

Matrix p q	Premise 1 p • q	Premise 2 P	Premise 3 q	Conclusion p ∨ q
T T	T **T** T	**T**	**T**	T **T** T
T F	T **F** F	**T**	**F**	T **T** F
F T	F **F** T	**F**	**T**	F **T** T
F F	F **F** F	**F**	**F**	F **F** F

Also, highlight the column under the main connective of each premise and conclusion.

❖ Next step is the criteria of deciding the validity of an argument. In the first chapter, we have learnt that in case of a valid deductive argument, if all the premises are true, its conclusion is also true. It cannot be false.

Accordingly, to determine whether the given argument form is valid, one should see all the rows in which all the premises are true. If in these rows, the conclusion is also true, then the argument is valid. **Even if in one such row where all the premises are true, and the conclusion is false. Then the argument is invalid.**

❖ We need to select those rows where premises are true. In our example, only in the first row, all the three premises are true and conclusion is also true. Therefore the given argument form is valid. The argument being substitution instance of this form is also valid.

Let's Determine the validity of some more arguments:

(1) Macro Economics and Micro Economics are sub branches of Economics.

Macro Economics is a sub branch of Economics.

Therefore, the Micro Economics is not a sub branch of Economics

(M, I)

❖ Symbolization of an argument

(1) M • I

(2) M

(3) ~ I

❖ Argument form :

(1) p • q

(2) p

∴ ~ q

Matrix p q	Premise 1 p • q	Premise 2 P	Conclusion ~ q
T T	T **T** T	**T**	**F** T
T F	T **F** F	**T**	**T** F
F T	F **F** T	**F**	**F** T
F F	F **F** F	**F**	**T** F

All the premises are true only in row no. 1 wherein the conclusion in false. Therefore the given argument is substitution instance of this argument form and therefore the above argument is invalid.

(2) Either Nainital is a city or it is a beautiful hill station.

Nainital is not a city.

Therefore it is a beautiful hill station

(C, H)

❖ Symbolization of an argument

(1) C ∨ H

(2) ~ C

∴ H

❖ Argument form :

(1) p ∨ q

(2) ~ p

∴ q

Matrix	Premise 1	Premise 2	Conclusion
p q	p ∨ q	~ P	q
T T	T **T** T	**F** T	**T**
T F	T **T** F	**F** T	**F**
F T	F **T** T	**T** F	**T**
F F	F **F** F	**T** F	**F**

All the premises are true only in row no. 3 wherein even the conclusion is 'true'. Therefore the argument form is valid. The given argument is substitution instance of this argument form and therefore the above argument is valid.

(3) If either mobile games are helpful in development of personality or in achieving knowledge then it is useful in securing jobs.

Mobile games are **neither** helpful in development of personality **nor** in achieving knowledge.

Therefore mobile games are **not** useful in securing job.

(P, K, J)

❖ Symbolization of an argument

(1) (P ∨ K) ⊃ J

(2) ~ P • ~ K

∴ ~ J

❖ Argument from :

(1) (p ∨ q) ⊃ r

(2) p • ~ q

∴ ~ r

Matrix	Premise 1	Premise 2	Conclusion
p q r	(p ∨ q) ⊃ r	~ p • ~ q	~ r
T T T	T T T **T** T	F T **F** F T	**F** T
T T F	T T T **F** F	F T **F** F T	**T** F
T F T	T T F **T** T	F T **F** T F	**F** T
T F F	T T F **T** F	F T **F** T F	**T** F
F T T	F T T **T** T	T F **F** F T	**F** T
F T F	F T T **F** F	T F **F** F T	**T** F
F F T	F F F **T** T	T F **T** T F	**F** T
F F F	F F F **T** F	T F **T** T F	**T** F

All the premises are true in 7th and 8th row wherein the 8th row conclusion is true. But in 7th row where the premises are true but the conclusion is false. Therefore the given argument form is invalid. The given argument is substitution instance of this argument form. Therefore the above argument is invalid.

(4) Dr. Krishnan was a teacher and a philosopher.

If Dr. Krishnan was not a politician then he was not a philosopher.

Therefore, Dr. Krishanan was not a politician. (T, P, O)

❖ Symbolization of an argument

(1) T • P

(2) ~ O ⊃ ~ P

∴ ~ O

❖ Argument form :

(1) p • q

(2) ~ r ⊃ ~ q

∴ ~ r

Matrix p q r	Premise 1 p • q	Premise 2 ~ r ⊃ ~ q	Conclusion ~ r
T T T	T **T** T	F T **T** F T	**F** T
T T F	T **T** T	T F **F** F T	**T** F
T F T	T **F** F	F T **T** T F	**F** T
T F F	T **F** F	T F **T** T F	**T** F
F T T	F **F** T	F T **T** F T	**F** T
F T F	F **F** T	T F **F** FT	**T** F
F F T	F **F** F	F T **T** T F	**F** T
F F F	F **F** F	T F **T** T F	**T** F

All the premises are true in the 1st row and the conclusion is false. Therefore the given argument form is invalid. The given argument is substitution instance of this argument form. Therefore the above argument is invalid.

Activity : 6

With the help of truth table method determine whether the following arguments are valid or invalid.

(1) If examinations are held on time then the results will not be delayed.

It is not true that examinations are not held on time.

Therefore the results will not be delayed. (E, R)

(2) If workers join the strike then the production will suffer.

Either workers do not join the strike or production will not suffer.

Production does not suffer.

Therefore workers do not join the strike. (W, P)

(3) If Hiteksha studies hard then her mother will be happy and if she joins games then her friends will be happy.

Either she studies hard or she joins games.

Therefore either her mother will be happy or her friends will not be happy. (S, M, G, F)

3.4 Truth table as decision procedure :

Truth table method is one of the effective decision procedures by which we can solve the problem of deciding whether a propositional form is tautology, contradiction or contingency. And also decides whether an argument form is valid or invalid.

It satisfies all the conditions of effective decision procedure. i.e. reliable, mechanical and finite.

Truth table method is **reliable**. It always gives correct answer. The method never fails if one follows the basic truth values of propositions, the instructions for construction of matrix and the order of constructing the rows of truth values then the method will always be correct.

Truth table method is also **mechanical**. It goes step by step in a systematic manner. It does not require any imagination or intelligence or abstract principles to solve the problem.

Truth table method is **finite**. It has a limited number of steps. There is a last step in truth table for getting the answer.

Summary

- A decision Procedure is a method which decides whether a proposition belongs to certain class.
- Truth table is a tabular way of expressing the truth values of the truth functional statements.
- Truth table method is a decision procedure which helps us to decide whether propositional form is tautology, contradictory, contingent.
- Truth table tests the validity and invalidity of arguments.
- Truth table method is an effective decision procedure as it is reliable, mechanical and finite.

Exercises

Q. 1. Fill in the blanks with suitable words given in the brackets.

(1) ………...… is a tabular way of expressing the truth value of any truth functional compound proposition. *(Truth table, Truth tree)*

(2) A tautology is a truth – functional propositional form which is ………...… under all truth possibilities of its components. *(True, False)*

(3) A contradiction is a truth – functional propositional form which is ………...… under all truth possibilities of its components. *(True, False)*

(4) A …….. is a truth – functional proposition which is true under some and false under some truth possibilities of its components. *(Contradiction, Contingency)*

(5) By denying a tautology, we get a ………...…

(Contingency, Contradiction)

(6) By denying a contradiction, we can get a ………...… *(Tautology, Contingency)*

(7) By denying a contingency, we can get a ………...… *(Tautology, Contingency)*

(8) p ∨ ~ p is a ………...… *(Tautology, Contradiction)*

(9) ~ (p • ~ p) is a ………...… *(Tautology, Contingency)*

(10) p • ~ p is a ………...…

(Contingency, Contradiction)

(11) The truth table method can also be used for testing the ………...… of arguments.

(Validity, Reliability)

(12) ~ (p ∨ ~ p) is a ………...…

(Tautology, Contradiction)

(13) p ∨ q is a ………...…

(Contingency, Contradiction)

Q. 2. State whether the following statement are true or false.

(1) There are many decision procedures.

(2) The truth table method is an effective decision procedure.

(3) The truth table method is mechanical.

(4) A contradiction is a truth functional propositional form which is true under all truth possibilities of its components.

(5) A contingency is a truth – functional propositional form which is true under some and false under some truth possibilities of its components.

(6) A tautology is a truth – functional propositional form which is true under all the truth – possibilities of its components.

(7) The method of truth table requires use of intelligence.

(8) In the truth – table method, the matrix is written on the left hand side.

(9) Propositional form contains propositional variables.

(10) The method of truth table can be used to test the validity of all types of arguments.

Q. 3. Match the column.

	(A)		(B)
1.	Tautology	a.	Always false
2.	Decision procedure	b.	Sometimes true and sometimes false
3.	Contradiction	c.	Truth table
4.	Contingency	d.	Always true

Q. 4. Give logical terms for the following.

(1) A method of deciding whether an object belongs to a certain class.

(2) A tabular way of expressing the truth value of expressions containing propositional connectives.

(3) A column consists of all possible combinations of the truth values of the propositional variables in the truth function or argument.

(4) A truth functional statement form which is 'True' under all truth possibilities of its components.

(5) A truth functional statement form which is 'False' under all truth-possibilities of its components.

(6) A truth-functional statement form which is 'True' as well as 'False' under some truth possibilities of its components.

Q. 5. Give reasons for the following :

(1) Truth table is an effective decision procedure.

(2) By denying tautology, we get contradiction.

(3) By denying contradiction, we get tautology.

(4) By denying contingency, we get contingency.

Q. 6. Explain the following :

(1) Decision procedure

(2) Tautology

(3) Contradiction

(4) Contingency

(5) Truth table method as an effective decision procedure

Q. 7. Answer the following questions :

(1) What is decision procedure? What are the conditions of an effective decision procedure?

(2) Differentiate between statement form and argument form.

(3) What is truth table? How to construct a truth table?

(4) Differentiate between tautology and contradiction.

(5) How do we determine the number of rows in the truth table?

(6) Differentiate between contradiction and continguency.

(7) Why is truth table method called mechanical?

(8) Differentiate between tautology and contingency.

Q. 8. Construct truth – table to determine whether the following statement forms are tautologous, contradictory or contingent

(1) $p \bullet \sim p$

(2) $p \supset (q \supset p)$

(3) $P \vee (r \bullet p)$

(4) $(r \vee q) \equiv r$

(5) $(\sim t \bullet q) \supset (q \supset t)$

(6) $(p \supset \sim p) \bullet (\sim p \supset q)$

(7) $p \supset (p \vee r)$

(8) $\sim q \supset (q \bullet q)$

(9) $(t \supset t) \bullet (t \supset \sim t)$

(10) $[(p \supset s) \bullet p] \supset s$

(11) $[q \vee (p \bullet \sim q)] \equiv [\sim p \bullet (q \vee p)]$

(12) $(p \supset t) \bullet \sim (\sim p \vee p)$

(13) $(\sim p \bullet p) \supset [(s \vee \sim p) \bullet (\sim s \vee \sim p)]$

(14) $(p \bullet p) \vee \sim p$

(15) $\sim \{ \sim p \supset [(p \bullet q) \vee p] \}$

(16) $\sim (p \vee q) \bullet \sim (\sim p \bullet \sim q)$

(17) $[(p \bullet (q \bullet r)] \equiv [(p \bullet q) \bullet r]$

(18) $[(p \vee q) \bullet \sim p] \supset q$

(19) $(t \equiv \sim q) \supset (\sim q \supset t)$

(20) $[p \supset (r \bullet q)] \equiv [(p \supset q) \bullet (p \supset r)]$

Q. 9. With the help of truth table method, test the validity of the following arguments:

(1) $\sim M \supset N$

$\sim N$

$\therefore M \bullet N$

(2) (P ∨ Q) • p

∴ P

(3) P ⊃ (Q • R)

∼ Q ∨ ∼ R

∴ ∼ P

(4) Q ⊃ p

∼ P

∴ Q

(5) (P • Q) ⊃ R

∼ R

∴ Q

(6) (∼ P V Q) ⊃ P

P ⊃ R

∴ (P ⊃ Q) ⊃ R

(7) ∼ Q V P

∴ P ⊃ Q

(8) (P ≡ Q) ⊃ R

R

∴ ∼ P V Q

(9) ∼ Q ≡ S

P ≡ Q

∴ Q V ∼ P

(10) ∼ (A • B)

∼ B

∴ A

(11) J V K

∼ J

∴ ∼ K

(12) M ⊃ ∼ B

∼ B V M

∴ B • M

(13) ∼ E • M

∼ (M ≡ E)

∴ ∼ M

(14) C ⊃ F

∼ F • C

∴ ∼ C

(15) G ≡ W

∼ W

∼ G

∴ W ⊃ G

Q. 10. Symbolize the following arguments using propositional constants given in the bracket and state whether they are valid or invalid by using the method of truth table.

(1) Either Germans are disciplined or they are not progressive. Germans are disciplined. Therefore they are not progressive.

(D, P)

(2) Nitin Shankar is a rhythm aranger. Therefore it is false that Nitin Shankar is both a rhythm arranger and a singer.

(R, S)

(3) If Picasso was not an Italian artist, then he was an explorer. Picasso was not an explorer. Therefore, Picasso was either an Italian artist or a dancer.

(A, E, D)

(4) It is not the case that Kalansh is serious and humorous. Kalansh is humorous. Therefore he is not serious.

(S, H)

(5) It is false that if Sparsh opts for mathematics, then he cannot offer history. Sparsh does not opt history. Therefore he opts for mathematics, but not history.

(M, H)

(6) Durvansh plays vollyball. Hence Durvansh plays vollyball but not football.

(V, F)

(7) If a man overeats, then he either invites diabetes or develops heart problems.

Some men have both diabetes and heart problems. Therefore, some men overeat.

(O, D, H)

(8) If Zoey has a strong will- power, then she can achieve many things. Zoey has a strong will power. Therefore she can achieve many things.

(W, A)

(9) Riddhi took either taxi or a bus.

If she takes a taxi, then she would be on time.

She was not on time. Therefore Riddhi took a bus.

(T, B, M)

(10) If the family planning program is effective then population can be cotrolled. The family planning program is not effective. Therefore, population cannot be controlled.

(F, P)

(11) If Het is a batsman, then Smit is a bowler. Smit is not a bowler. Hence Het is a batsman.

(B, O)

(12) Either the books are interesting or informative. If the books are informative, then they improve one's knowledge. Therefore, if the books are interesting then it improves one's knowledge.

(I, F, K)

(13) Either luck or courage is needed for success. He does not have courage. Therefore he has luck.

(L, C)

(14) If it rains, then the crops will be good. The crops are good. Therefore, it rains.

(R, C)

(15) If and only if Mann is a government servant, then he is called a public servant. Mann is not a government servant. Therefore he is not a public servant.

(G, P)

(16) Shruti loves her brothers, if and only if they work for her.

If Vinayak and Vaibhav are Shruti's brothers then they work for her.

Thererfore Shruti loves her brothers.

(S, W, V, B)

Q. 11. Complete the following table

Left component	Right component	conjunction •	disjunction ∨	implication ⊃	equivalence ≡
T	T				T
T	F	F			
F	T		T		
F	F			T	

❖ ❖ ❖