# 4. Laws of Motion

**Can you recall?**

1. What are different types of motions?
2. What do you mean by kinematical equations and what are they?
3. Newton's laws of motion apply to most bodies we come across in our daily lives.
4. All bodies are governed by Newton's law of gravitation. Gravitation of the Earth results into weight of objects.
5. Acceleration is directly proportional to force for fixed mass of an object.
6. Bodies possess potential energy and kinetic energy due to their position and motion respectively which may change. Their total energy is conserved in absence of any external force.

## 4.1. Introduction:

If an object continuously changes its position, it is said to be in motion. Mechanics is a branch of Physics that deals with motion. There are basically two branches of mechanics (i) Statics, where we deal with objects at rest or in equilibrium under the action of balanced forces and (ii) Kinetics, which deals with actual motion.

Kinetics can be further divided into two branches **(i) Kinematics:** In kinematics, we describe various motions without discussing their cause. Various parameters discussed in kinematics are distance, displacement, speed, velocity and acceleration. **(ii) Dynamics:** In dynamics we describe the motion along with its cause, which is force and/or torque. Parameters discussed in dynamics are momentum, force, energy, power, etc. in addition to those in kinematics.

It must be understood that motion is strictly a relative concept, i.e., it should always be described in context to a reference frame. For example, if you are in a running bus, neither you nor your co-passengers sitting in the bus are in motion in your reference, i.e., moving bus. However, from the ground reference, bus, you and all the passengers are in motion.

If not random, motions in real life may be understood separately as linear, circular or rotational, oscillatory, etc., or some combinations of these. While describing any of these, we need to know the corresponding forces responsible for these motions. Trajectory of any motion is decided by acceleration $\vec{a}$ and the initial velocity $\vec{u}$.

a) Linear motion: Initial velocity may be zero or non-zero. If initial velocity is zero (starting from rest), acceleration in any direction will result into a linear motion.

   If initial velocity is not zero, the acceleration must be in line with the initial velocity (along the same or opposite direction to that of the initial velocity) for resultant motion to be linear.

b) Circular motion: If initial velocity is not zero and acceleration is throughout perpendicular to the velocity, the resultant motion will be circular.

c) Parabolic motion: If acceleration is constant and initial velocity is *not* in line with the acceleration, the motion is parabolic, e.g., trajectory of a projectile motion.

d) Other combinations of $\vec{u}$ and $\vec{a}$ will result into different more complicated motion.

## 4.2. Aristotle's Fallacy:

Aristotle (384BC-322BC) stated that "an external force is required to keep a body in uniform motion". This was probably based on a common experience like a ball rolling on a surface stops after rolling through some distance. Thus, to keep the ball moving with constant velocity, we have to continuously apply a force on it. Similar examples can be found elsewhere, like a paper plane flying through air or a paper boat propelled with some initial velocity.

Correct explanation to Aristotle's fallacy was first given by Galileo (1564-1642), which was later used by Newton (1643-1727) in formulating *laws of motion*. Galileo showed that all the objects stop moving because of some resistive or opposing forces like friction, viscous drag, etc. In these examples such forces are frictional force for rolling ball, viscous drag or viscous force of air for paper plane and viscous force of water for the boat.

Thus, in reality, for an uninterrupted motion of a body an additional external force is required for overcoming these opposing forces.

**Can you tell?**

1. Was Aristotle correct?
2. If correct, explain his statement with an illustration.
3. If wrong, give the correct modified version of his statement.

## 4.3. Newton's Laws of Motion:

**First law:** Every inanimate object continues to be in its state of rest or of uniform *unaccelerated* motion unless and until it is acted upon by an *external, unbalanced* force.

**Second law:** Rate of change of linear momentum of a rigid body is directly proportional to the applied force and takes place in the direction of the applied force. On selecting suitable units, it takes the form $\vec{F} = \frac{d\vec{p}}{dt}$ (where $\vec{F}$ is the force and $\vec{p} = m\vec{v}$ is the linear momentum.

**Third law:** To every action (force), there is an equal and opposite reaction (force).

**Discussion:** From Newton's second law of motion, $\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v})$. For a *given* body, mass *m* is constant.

$$\therefore \vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a} \ldots \text{(for constant mass)}$$

Thus, if $\vec{F} = 0$, $\vec{v}$ is constant. Hence if there is no force, velocity will not change. This is nothing but Newton's first law of motion.

**Can you tell?**

What is then special about Newton's first law if it is derivable from Newton's second law?

### 4.3.1. Importance of Newton's First Law of Motion:

(i) It shows an equivalence between '*state of rest*' and '*state of uniform motion along a straight line*' as both need a net unbalanced force to change the state. Both these are referred to as '*state of motion*'. The distinction between state of rest and uniform motion lies in the choice of the '*frame of reference*'.

(ii) It defines *force* as an entity (or a physical quantity) that brings about a change in the '*state of motion*' of a body, i.e., force is something that initiates a motion or controls a motion. Second law gives its quantitative understanding or its mathematical expression.

(iii) It defines *inertia* as a fundamental property of every physical object by which the object *resists* any change in its state of motion. Inertia is measured as the mass of the object. More specifically it is called inertial mass, which is the ratio of net force ($|\vec{F}|$) to the corresponding acceleration ($|\vec{a}|$).

### 4.3.2. Importance of Newton's Second Law of Motion:

(i) It gives mathematical formulation for quantitative measure of force as rate of change of linear momentum.

**Do you know ?**

Mathematical expression for force must be remembered as $\vec{F} = \frac{d\vec{p}}{dt}$ and ***not*** as $\vec{F} = m\vec{a}$

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt} = \frac{dm}{dt}(\vec{v}) + m\left(\frac{d\vec{v}}{dt}\right)$$

$$= \frac{dm}{dt}(\vec{v}) + m\vec{a}$$

For a *given* body, mass is constant, i.e., $\frac{dm}{dt} = 0$ and only in this case, $\vec{F} = m\vec{a}$

In the case of a rocket, both the terms are needed as both mass and velocity are varying.

(ii) It defines *momentum* $\left(\vec{p} = m\vec{v}\right)$ instead of velocity as the fundamental quantity related to motion. What is changed by a force is the momentum and not necessarily the velocity.

(iii) Aristotle's fallacy is overcome by considering *resultant unbalanced* force.

### 4.3.3 Importance of Newton's Third Law of Motion:

(i) It defines *action* and *reaction* as a pair of equal and opposite forces acting along the same line.

(ii) Action and reaction forces are always on *different* objects.

**Consequences:**

*Action force* exerted by a body *x* on body *y*, conventionally written as $\vec{F}_{yx}$, is the force experienced by *y*.

As a result, body *y* exerts *reaction force* $\vec{F}_{xy}$ on body *x*.

In this case, body *x* experiences the force $\vec{F}_{xy}$ only while the body *y* experiences the force $\vec{F}_{yx}$ only.

Forces $\vec{F}_{xy}$ and $\vec{F}_{yx}$ are equal in magnitude and opposite in their directions, but there is no question of cancellation of these forces as those are experienced by *different* objects.

Forces $\vec{F}_{xy}$ and $\vec{F}_{yx}$ *need not* be contact forces. Repulsive forces between two magnets is a pair of action-reaction forces. In this case the two magnets are not in contact. Gravitational force between Earth and moon or between Earth and Sun are also similar pairs of non-contact action-reaction forces.

**Example 4.1:** A hose pipe used for gardening is ejecting water horizontally at the rate of 0.5 m/s. Area of the bore of the pipe is 10 cm$^2$. Calculate the force to be applied by the gardener to hold the pipe *horizontally* stationary.

**Solution:** If ejecting water horizontally is considered as action force on the water, the water exerts a backward force (called recoil force) on the pipe as the reaction force.

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d\left(m\vec{v}\right)}{dt} = \frac{dm}{dt}\vec{v} + m\frac{d\vec{v}}{dt}$$

As v, the velocity of ejected water is constant, $F = \frac{dm}{dt}\vec{v}$, where $\frac{dm}{dt}$ is the rate at which mass of water is ejected by the pipe.

As the force is in the direction of velocity (horizontal), we can use scalars. $\therefore F = \frac{dm}{dt}v$

$$\frac{dm}{dt} = \frac{d\left(V\rho\right)}{dt} = \frac{d\left(Al\rho\right)}{dt} = A\rho\frac{dl}{dt} = A\rho v$$

where $V$ = volume of water ejected

$A$ = area of cross section of bore = 10 cm$^2$

$\rho$ = density of water = 1 g/cc

$l$ = length of the water ejected in time $t$

$\frac{dl}{dt} = v =$ velocity of water ejected

$= 0.5$ m/s $= 50$ cm/s

$$F = \frac{dm}{dt}v = \left(A\rho v\right)v = A\rho v^2 = 10\times1\times50^2$$

$= 25000$ dyne $= 0.25$ N

Equal and opposite force must be applied by the gardener.

## 4.4. Inertial and Non-Inertial Frames of Reference

Consider yourself standing on a railway platform or a bus stand and you see a train or bus moving. According to you, that train or bus is moving or is in motion. As per the experience of the passengers in the train or bus, they are at rest and you are moving (in backward direction). Hence motion itself is a relative concept. To know or describe a motion you need to describe or define some reference. Such a reference is called a *frame of reference*. In the example discussed above, if you consider the platform as the reference, then the passengers and the train are moving. However, if the train is considered as the reference, you and platform, etc. are moving.

Usually a set of coordinates with a suitable origin is enough to describe a frame of reference. If position coordinates of an object are continuously changing with time in a frame of reference, then *that object* is in motion in *that* frame of reference. Any frame of reference in which Newton's first law of motion is applicable is the simplest understanding of an inertial frame of reference. It means, if there is no net force, there is no acceleration. Thus in an inertial frame, a body will move with constant velocity (which may be zero also) if there is no net force acting upon it. In the absence of a net force, if an object suffers an acceleration, that frame of reference is ***not*** an inertial frame and is called as non-inertial frame of reference.

Measurements in one inertial frame can be converted to measurements in another inertial frame by a simple transformation, i.e., by simply using some velocity vectors (relative velocity between the two frames of reference).

**Illustration:** Imagine yourself inside a car with all windows opaque so that you can not see anything outside. Also consider that there is a pendulum tied inside the car and not set into oscillations . If the car just starts its motion (with reference to outside or ground), you will experience a jerk, i.e., acceleration inside the car even though there is no force acting upon you. During this time, the string of the pendulum may be steady, but ***not*** vertical. During time of acceleration, the car can be considered to be a non-inertial frame of reference. Later on if the car is moving with constant velocity (with reference to the ground), you will not experience any jerky motion within the car and the car can be considered as an inertial frame of reference. In this case, the pendulum string will be vertical, when not oscillating.

**Do you know ?**

The situations/phenomena that can be explained using Newton's laws of motion fall under Newtonian mechanics. So far as our daily life situations are considered, Newtonian mechanics is perfectly applicable. However, under several extreme conditions we need to use some other theories.

**Limitation of Newton's laws of motion**

(i) Newton's laws are applicable only in the inertial frames of reference (discussed later). If the body is in a frame of reference of acceleration ($a$), we need to use a *pseudo* force $\left(-m\bar{a}\right)$ in addition to all the other forces while writing the force equations.

(ii) Newton's laws are applicable for *point* objects.

(iii) Newton's laws are applicable to rigid bodies. A body is said to be *rigid* if the relative distances between its particles do not change for any deforming force.

(iv) For objects moving with speeds comparable to that of light, Newton's laws of motion do not give results that match with the experimental results and Einstein's special theory of relativity has to be used.

(v) Behaviour and interaction of objects having atomic or molecular sizes cannot be explained using Newton's laws of motion, and quantum mechanics has to be used.

A rocket in intergalactic space (gravity free space between galaxies) with all its engines shut is closest to an ideal inertial frame. However, Earth's acceleration in the reference frame of the Sun is so small that any frame attached to the Earth can be used as an inertial frame for any day-to-day situation or in our laboratories.

## 4.5 Types of Forces:

### 4.5.1. Fundamental Forces in Nature:

All the forces in nature are classified into following **four** interactions that are termed as fundamental forces.

(i) Gravitational force: It is the attractive force between two (point) masses separated by a distance. Magnitude of gravitational force between point masses $m_1$ and $m_2$ separated by distance $r$ is given by $F = \frac{Gm_1m_2}{r^2}$

where G = $6.67\times10^{-11}$ SI units. Between two point masses (particles) separated by a given distance, this is the weakest force having infinite range. This force is always attractive. Structure of the universe is governed by this force.

Common experience of this force for us is gravitational force exerted by Earth on us, which we call as our weight W.

$$\therefore W = \frac{GMm}{R^2} = m\left(\frac{GM}{R^2}\right) = mg$$,

where $M$ and $R$ represent respectivily mass and radius of the Earth. Distance between ourselves and Earth is taken as radius of the Earth when we are on the surface of the Earth because our size is negligible as compared to radius of the Earth ($6.4\times10^6$ m).

$$\left(\frac{GM}{R^2}\right) = \frac{6.67\times10^{-11}\times6\times10^{24}}{\left(6.4\times10^6\right)^2}$$

$\cong 9.8$ m/s$^2$ = $g$ = gravitational acceleration or gravitational field intensity.

We ***feel*** this force only due to normal reaction from the surface of our contact with Earth.

All individual bodies also exert gravitation force on each other but it is too small compared to that by the Earth. For example, mutual gravitational force between two SUMO wrestlers, each of mass 300 kg, assuming the distance between them is 0.5 m, will be

$$F = \frac{6.67\times10^{-11}\times300\times300}{0.5^2}$$

$\cong 2{,}4\times10^{-5}\,\text{N} = 24\ \mu\text{N}$.

This force is negligibly small in comparison to the weight of each SUMO wrestler $\cong$ 3000 N.

(ii) Electromagnetic (EM) force: It is an attractive or repulsive force between electrically charged particles. Earlier, electric and magnetic forces were thought to be independent. After the demonstrations by Michael Faraday (1791-1867) and James Clerk Maxwell (1831-1879), electric and magnetic forces were unified through the theory of electromagnetism. These forces are stronger than the gravitational force. Our life is practically governed by these forces. Majority of forces experienced in our daily life, such as force of friction, normal reaction, tension in strings, collision forces, elastic forces, viscosity (fluid friction), etc. are EM in nature. Under the action of these forces, there is *deformation of objects* that changes intermolecular distances thereby resulting into reaction forces.

(iii) Strong (nuclear) force: This is the strongest force that binds the nucleons together inside a nucleus. Though strongest, it is a short range ($< 10^{-14}$ m) force. Therefore is very strong attractive force and is charge independent.

(iv) Weak (nuclear) force: This is the interaction between subatomic particles that is responsible for the radioactive decay of atoms, in particular beta emission. The weak nuclear force is not as weak as the gravitational force, but much weaker than the strong nuclear and EM forces. The range of weak nuclear force is exceedingly small, of the order of $10^{-16}$ m.

**Weak interaction force:**

The radioactive isotope $C^{13}$ is converted into $N^{14}$ in which a neutron is converted into a proton. This property is used in *carbon dating* to determine the age of a sample.

In radioactive beta decay, the nucleus emits an electron (or positron) and an uncharged particle called neutrino. There are two types of β-decay, $\beta^+$ and $\beta^-$. During $\beta^+$decay, a proton is converted into a neutron (accompanied by positron emission) and during $\beta^-$ decay a neutron is converted into a proton (accompanied by electron emission).

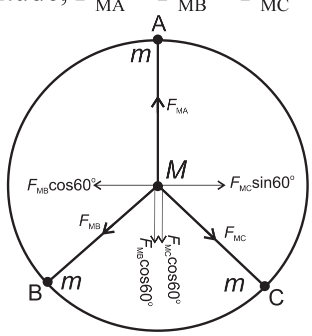
Another most interesting illustration of weak forces is fusion reaction in the core of the Sun. During this, protons are converted into neutrons and a neutrino is emitted due to energy balance. In general, emission of a neutrino is the evidence that there is conversion of a proton into a neutron or a neutron into a proton. This is possible *only* due to weak forces.

**Example 4.2.** Three identical point masses are fixed symmetrically on the periphery of a circle. Obtain the resultant gravitational force on any point mass $M$ at the centre of the circle. Extend this idea to more than three identical masses symmetrically located on the periphery. How far can you extend this concept?

**Solution:**

(i) Figure below shows three identical point masses $m$ on the periphery of a circle of radius $r$. Mass $M$ is at the centre of the circle. Gravitational forces on $M$ due to these masses are attractive and are directed as shown.

In magnitude, $F_{MA} = F_{MB} = F_{MC} = \dfrac{GMm}{r^2}$




Forces $F_{MB}$ and $F_{MC}$ are resolved along $F_{MA}$ and perpendicular to $F_{MA}$ as shown. Components perpendicular to $F_{MA}$ cancel each other. Components along $F_{MA}$ are $F_{MB}\cos 60^0 = F_{MC}\cos 60^0 = \dfrac{1}{2}F_{MA}$ each. Magnitude of their resultant is $F_{MA}$ and its direction is opposite to that of $F_{MA}$. Thus, the resultant force on mass $M$ is zero.

(ii) For any even number of equal masses, the force due to any mass $m$ is balanced (cancelled) by diametrically opposite mass. For any odd number of masses, as seen for 3, the components perpendicular to one of them cancel each other while the components parallel to one of these add up in such a way that the resultant is zero for any number of identical masses $m$ located symmetrically on the periphery.

(iii) As the number of masses tends to infinity, their collective shape approaches circumference of the circle, which is nothing but a ring. Thus, the gravitational force exerted by a ring mass on any other mass at its centre is zero.

In three-dimensions, we can imagine a uniform hollow sphere to be made up of infinite number of such rings with a common diameter. Thus, the gravitational force for any mass kept at the centre of a hollow sphere is zero.

**Do you know ?**

**Unification of forces:** Newton unified terrestrial (related to Earth and hence to our daily life) and celestial (related to universe) domains under a common law of gravitation. The experimental discoveries of Oersted (1777-1851) and Faraday showed that electric and magnetic phenomena are in general inseparable leading to what is called 'EM phenomenon'. Electromagnetism and optics were unified by Maxwell with the proposition that light is an EM wave. Einstein attempted to unify gravity and electromagnetism under general relativity but could not succeed. The EM and the weak nuclear force have now been unified as a single 'electro-weak' force.

### 4.5.2. Contact and Non-Contact Forces:

For some forces, like gravitational force, electrostatic force, magnetostatic force, etc., physical contact is not an essential condition. These forces exist even if the objects are distant or physically separated. Such forces are non-contact forces.

Forces resulting *only* due to contact are called contact forces. All these are EM in nature, arising due to some deformation. Normal reaction, forces occurring during collision, force of friction, etc., are contact forces. There are two common categories of contact forces. Two objects in contact, while exerting mutual force, try to push each other away along their common normal. Quite often we call it as 'normal reaction' force or 'normal' force. While standing on a table, we *push* the table away from us (downward) and the table *pushes* us away from it (upward) both being equal in magnitude and acting along the same '*normal*' line.

Force of friction is also a contact force that arises whenever there is a relative motion or tendency of relative motion between surfaces in contact. This is the parallel (or tangential) component of the reaction force. In this case, the molecules of surfaces in contact, which have developed certain equilibrium, are required to be separated.

### 4.5.3 Real and Pseudo Forces:

Consider ourselves inside a lift (or elevator). When the lift just starts moving up (accelerates upward), we feel a bit heavier as if someone is pushing us down. This is not imaginary or not just a feeling. If we are standing on a weighing scale inside this lift, during this time the weighing scale records an increase in weight. During travelling with uniform upward velocity no such change is recorded. While stopping at some upper floor, the lift undergoes downward acceleration for decreasing the upward velocity. In this case the weighing scale records loss in weight and we also feel lighter. These *extra* upward or downward forces are (i) Measurable, means they are not imaginary, (ii) not accountable as per Newton's second law in the inertial frame and (iii) not among any of the four fundamental forces.

When we are inside a bus such forces are experienced when the bus starts to move (forward acceleration), when the bus is about to stop (backward acceleration) or takes a turn (centripetal acceleration). In all these cases we are inside an accelerated system (which is our frame of reference). If a force measuring device is suitably used – like the weighing scale recording the change in weight – these forces can be recorded and will be found to be always opposite to the acceleration of your frame of reference. They are also exactly equal to $-m\vec{a}$, where $m$ is our mass and $\vec{a}$ is acceleration of the system (frame of reference).

We have already defined or described real forces to be those which obey Newton's laws of motion and are one of the four fundamental forces. Forces in above illustrations do not satisfy this description and cannot be called real forces. Hence these are called ***pseudo*** forces. ***Pseudo*** in this case does not mean imaginary (because these are measurable with instruments) but just means *non-real*. These forces are measured to be $\left(-m\vec{a}\right)$. Hence, a term $\left(-m\vec{a}\right)$ added to resultant force enables us to apply Newton's laws of motion to a non-inertial frame of reference of acceleration $\vec{a}$. Negative sign refers to their direction, which is opposite to that of the acceleration of the reference frame.

As per the illustration of the lift with downward acceleration $\vec{a}$, the weight experienced will be $\vec{W} = m\vec{g} + \left(-m\vec{a}\right)$

As $\vec{g}$ and $\vec{a}$ are along the same direction in this case, $W = mg - ma$. This explains the *feeling* of a loss in weight.

During upward acceleration, say $\vec{a_1}$, we have, $\vec{W_1} = m\vec{g} + \left(+m\vec{a}_1\right)$

In this case, $\vec{g}$ and $\vec{a_1}$ are oppositely directed. $\therefore W_1 = mg + \left(+ma_1\right) = mg + ma_1$ that explains gain in weight or existence of extra downward force.

> In mathematics we define a number to be real if its square is zero or positive. Solution set of equations like $x^2 - 6x + 10 = 0$ does not satisfy the criterion to be a real number. Such numbers are *complex* numbers which include $i = \sqrt{-1}$ along with some real part. It means every *non-real* need not be *imaginary* as in literal verbal sense.

**Example 4.3:** A car of mass 1.5 ton is running at 72 kmph on a straight horizontal road. On turning the engine off, it stops in 20 seconds. While running at the same speed, on the same road, the driver observes an accident 50 m in front of him. He immediately applies the brakes and just manages to stop the car at the accident spot. Calculate the braking force.

**Solution:** On turning the engine off,

$u = 20\,\text{m s}^{-1},\ \text{v} = 0,\ t = 20\,s$

$$\therefore a = \frac{\text{v} - u}{t} = -1\,\text{m s}^{-2}$$

This is frictional retardation (negative acceleration).

After seeing the accident,

$u = 20\,\text{m s}^{-1}$, $v = 0$, $s = 50\,\text{m}$

$$\therefore a_1 = \frac{v^2 - u^2}{2s} = -4\,\text{m s}^{-2}$$

This retardation is the combined effect of braking and friction. Thus, braking retardation $= 4 - 1 = 3\,\text{m s}^{-2}$.

$\therefore$ Braking force = mass × braking retardation = 1500 × 3 = 4500 N

### 4.5.4. Conservative and Non-Conservative Forces and Concept of Potential Energy:

Consider an object lying on the ground is lifted and kept on a table. Neglecting air resistance, the amount of work done is the work done *against* gravitational force and it is *independent* of the actual path chosen (Remember, as there is no air resistance, gravitational force is the only force). Similarly, while keeping the same object back on the ground from the table, the work is done ***by*** the gravitational force. In either case the amount of work done is the same ***and*** is independent of the actual path chosen. The work done by force $\vec{F}$ in moving the object through a distance d*x* can be mathematically represented as $dW = \vec{F}.d\vec{x}$ $= -dU$ or $dU = -\vec{F}.d\vec{x}$.

If work done by or against a force is independent of the actual path, the force is said to be a ***conservative force***. During the work done by a conservative force, the mechanical energy (sum of kinetic and potential energy) is conserved. In fact, we define the concept of potential at a point or potential energy (in the topic of gravitation) with the help of conservative forces only. The work done by or against conservative forces reflects an equal amount of change in the *potential energy*. The corresponding work done is used in changing the position or in achieving the new position in the gravitational field. Hence, potential energy is often referred to as the energy possessed on account of position.

In the illustration given above, the work done is reflected as increase in the gravitational potential energy when the displacement is *against* the (gravitational) force. Same amount of potential energy is decreased when the displacement is in the direction of force. In either case it is independent of the actual path but depends only upon the initial and final positions. This change in the potential energy takes place in such a way that the mechanical energy is conserved.

As discussed above, increase in the potential energy, $dU = -\vec{F}.d\vec{x}$ or $U = -\int \vec{F}.d\vec{x}$ where $\vec{F}$ is a conservative force. This concept, will be described in details in Chapter 5 on Gravitation in context of gravitational potential energy and gravitational potential.

During this process, if friction or air resistance is present, additional work is necessary against the frictional force (for the same displacement). This work is strictly path dependent and not recoverable. Such forces (like friction, air drag, etc.) are called *non-conservative* forces. Work done against these forces appears as heat, sound, light, etc. The work done against non-conservative forces is not recoverable even if the path is exactly reversed.

### 4.5.5. Work Done by a Variable Force:

The popular formula for calculating work done is $W = \vec{F} \cdot \vec{s} = F\,s\cos\theta$ where θ is the angle between the applied force $\vec{F}$ and the displacement $\vec{s}$.

This formula is applicable only if both force $\vec{F}$ and displacement $\vec{s}$ are constant and finite. In several real-life situations, the force is not constant. For example, while lifting an object through several thousand kilometres, the gravitational force is not constant. The viscous forces like fluid resistance depend upon the speed, hence, quite often are not constant with time. In order to calculate the work done by such variable forces we use integration.

**Illustration:** Figure 4.1(a) shows variation of a force $\vec{F}$ plotted against corresponding displacements in its direction $\vec{s}$. As the displacement is in the direction of the applied force, vector nature is not used. We need to calculate the work done by this force during displacement from $s_1$ to $s_2$. As the force is variable, using $W = F\left(s_2 - s_1\right)$ directly is not possible. In order to use integration, let us divide the displacement into a large number of infinitesimal (infinitely small) displacements. One of such displacements is *ds*. It is so small that the force *F* is practically constant for this displacement. Practically constant means the change in the force is so small that the change can not be recorded. The shaded strip shows one of such displacements. As the force is constant, the area of this strip *F.ds* is the work done *dW* for this displacement. Total work done *W* for displacement $\left(s_2 - s_1\right)$ can then be obtained by using integration.

$$\therefore W = \int_{s_1}^{s_2} F.ds$$

Method of integration is applicable if the exact way of variation in $\vec{F}$ with $\vec{s}$ is known and that function is integrable.

The area under the curve between $s_1$ and $s_2$ also gives the work done *W* (if the force axis necessarily starts with zero), as it consists of all the strips of *ds* between $s_1$ and $s_2$. In Fig. 4.1(b), the variation in the force is linear. In this case, the area of the trapezium $AS_1S_2B$ gives total work done *W*.

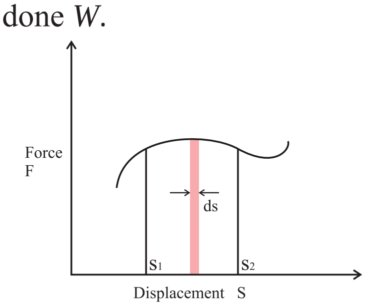



Fig 4.1 (a): Work done by nonlinearly varying force.

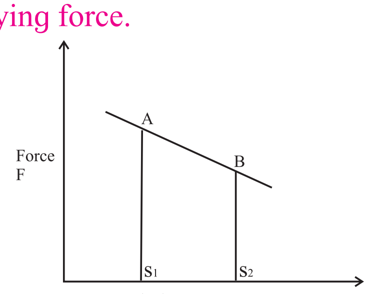



Fig 4.1 (b): Work done by linearly varying force.

**Example 4.4:** Over a given region, a force (in newton) varies as $F = 3x^2 - 2x + 1$. In this region, an object is displaced from $x_1 = 20$ cm to $x_2 = 40$ cm by the given force. Calculate the amount of work done.

**Solution:**

$$W = \int_{s_1}^{s_2} F.ds = \int_{0.2}^{0.4} (3x^2 - 2x + 1)dx$$

$$= \left[x^3 - x^2 + x\right]_{0.2}^{0.4}$$

$$= \left[0.4^3 - 0.4^2 + 0.4\right] - \left[0.2^3 - 0.2^2 + 0.2\right]$$

$$= 0.304 - 0.168 = 0.136\,\text{J}$$

## 4.6. Work Energy Theorem:

If there is a decrease in the potential energy (like a body falling down) due to a conservative force, it is entirely converted into kinetic energy. Work done by the force then appears as kinetic energy. Vice versa if an object is moving against a conservative force its kinetic energy decreases by an amount equal to the work done against the force. This principle is called *work-energy theorem* for conservative forces.

**Case I:** Consider an object of mass *m* moving with velocity $\vec{u}$ experiencing a constant opposing force $\vec{F}$ which slows it down to $\vec{v}$ during displacement $\vec{s}$. As $\vec{u}$ and $\vec{F}$ are oppositely directed, the entire motion will be along the same *line*. In this case we need not use the vector form, just ± signs should be good enough.

If $a = \frac{F}{m}$ is the acceleration, we can write the relevant equation of motion as $v^2 - u^2 = 2(-a)s$ (negative acceleration for opposing force)

Multiplying throughout by m/2, we get

$$\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = \left(ma\right).s = F.s$$

Left-hand side is decrease in the kinetic energy and the right-hand side is the work done by the force. Thus, change in kinetic energy is equal to work done by the conservative force, which is in accordance with work-energy theorem.

**Case II:** Accelerating conservative force along with a retarding non-conservative force:

An object dropped from some point at height *h* falls down through air. While coming down its potential energy decreases. Equal amount of work is done in this case also. However, this time the work is not entirely converted into kinetic energy but some part of it is used in overcoming the air resistance. This part of energy appears in some other forms such as heat, sound, etc. In this case, the work-energy theorem can mathematically be written as $\Delta PE = \Delta KE + W_{air\ resistance}$

(Decrease in the gravitational P.E. = Increase in the kinetic energy + work done against non-conservative forces). Magnitude of air resistance force is not constant but depends upon the speed hence it can be written as $\int \vec{F}.\vec{ds}$ as seen during work done by (or against) a variable force.

## 4.7. Principle of Conservation of Linear Momentum:

According to Newton's second law, resultant force is equal to the rate of change of linear momentum or $\vec{F} = \frac{d\vec{p}}{dt}$

In other words, if there is no resultant force, the linear momentum will not change or will remain constant or will be conserved. Mathematically, if $\vec{F} = 0, \frac{d\vec{p}}{dt} = 0$ or $\vec{p}$ is constant

**Always remember**

*Isolated system* means absence of any external force. A *system* refers to a set of particles, colliding objects, exploding objects, etc. *Interaction* refers to collision, explosion, etc. During any interaction among such objects the *total* linear momentum of the *entire system* of these particles/objects is constant. Remember, forces during collision or during explosion are *internal* forces for that *entire system*.

During collision of two particles, the two particles exert forces on *each other*. If these particles are discussed independently, these are external forces. However, for the *system of the two particles* together, these forces are internal forces.

(or conserved). This leads us to the *principle of conservation of linear momentum* which can be stated as **"The total momentum of an isolated system is conserved during any interaction."**

**Systems and free body diagrams:**

Mathematical approach for application of Newton's second law:

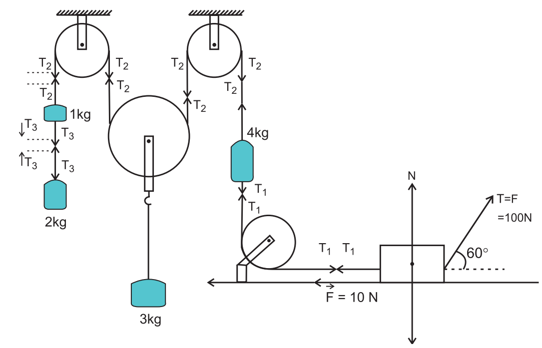



Fig 4.2 (a): System for illustration of free body diagram.

Consider the arrangement shown in Fig.4.2 (a). Pulleys $P_1$, $P_3$ and $P_4$ are fixed, while $P_2$ is movable. Force $F$ = 100 N, applied at an angle 60° with the horizontal is responsible for the motion, if any. Contact surface of the 5 kg mass offers a constant opposing force $F$ = 10 N. Except this, there are no resistive forces anywhere.

**Discussion:** Until 1 kg mass reaches the pulley $P_1$, the motion of 1 kg and 2 kg masses is identical. Thus, these two can be considered to be a *single system* of mass 3 kg except for knowing the tension $T_3$. The forces due to tension in the string joining them are internal forces for this system.

All masses *except* the 3 kg mass are travelling same distance in the same time. Thus, their accelerations, if any, have same magnitudes. If the string S connecting 1 kg and 4 kg masses moves by *x*, the lower string $S_1$ holding the 3 kg mass moves through *x*/2.

**Free body diagrams (FBD):** A free body diagram refers to forces acting on only one body at a time, and its acceleration.

Free body diagram of 2 kg mass: Let *a* be its upward acceleration. According to Newton's second law, it must be due to the resultant vertical force on this mass. To know this force, we need to know all the individual forces acting on this mass. The agencies exerting forces on this mass are Earth (downward force 1g) and force due to the tension $T_3$.

In this case, the lower half of the string is connected to the 2 kg mass. The direction of $T_3$ for lower part of the string is upwards as shown in the Fig. 4.2 (b). Upper part of the string is connected to the 1 kg mass. Thus, the direction of $T_3$ for 1 kg mass will be downwards. However, it will appear only for the free body diagram of the 1 kg mass and will not appear in the free body diagram of 2 kg mass. Hence, the free body diagram of the 2 kg mass will be as follows: Its force equation, according to Newton's second law will then be $T_3 - 2g = 2a$.

**Practical tip:** Easiest way to know the direction of forces due to tension is to put an X-mark on the string. Two halves of this cross indicate the directions of the forces exerted *by the string* on the bodies connected to either parts of the string.

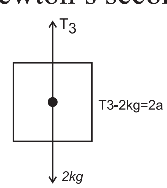


Fig 4.2 (b): Free body diagram for 2 kg mass.

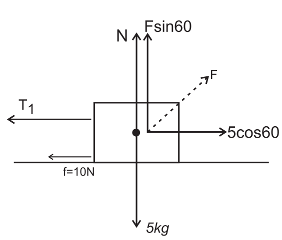


Fig 4.2 (c): Free body diagram for 5 kg mass.

Free body diagram of mass 5 kg: Its horizontal acceleration is also *a*, but towards right. The force exerting agencies are Earth (force 5g downwards), contact surface (normal force *N*, vertically upwards ***and*** opposing force *F* = 10 N, towards left), and the two strings on either side (Forces due to their tensions *T* and $T_1$). All these are shown in its free body diagrams in Fig. 4.2 (c). On resolving the force *F* along the vertical and horizontal directions, the free body diagram of 5 kg mass can be drawn as explained below.

As this mass has only horizontal motion, the vertical forces must cancel. Therefore, along the vertical direction,

$$\text{N} + \text{F} \sin 60° = 5\text{g}$$

Along the horizontal direction,

$$T_1 + 10(\text{opposing force}) = F\cos 60^0 = T\cos 60^0$$

Similar equations can be written for all the masses and also for the movable pully. On solving these equations simultaneously, we can obtain all the necessary quantities.

**Example 4.5:** Figure (a) shows a fixed pulley. A massless inextensible string with masses $m_1$ and $m_2 > m_1$ attached to its two ends is passing over the pulley. Such an arrangement is called an Atwood machine. Calculate accelerations of the masses and force due to the tension along the string assuming axle of the pulley to be frictionless.

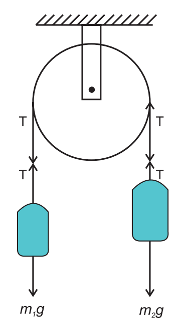


Fig. (a).

**Solution: Method I:** Direct method: As $m_2 > m_1$, mass $m_2$ is moving downwards and mass $m_1$ is moving upwards.

Net downward force

$$= F = (m_2)g - (m_1)g = (m_2 - m_1)g$$

As the string is inextensible, both the masses travel the same distance in the same time. Thus, their accelerations are numerically the same (one upward, other downward). Let it be *a*.

Thus, total mass in motion, $M = m_2 + m_1$

$$\therefore a = \frac{F}{M} = \left(\frac{m_2 - m_1}{m_2 + m_1}\right)g$$

For mass $m_1$, the upward force is the force due to tension *T* and downward force is *mg*. It has upward acceleration *a*. Thus, $T - m_1 g = ma$

$\therefore T = m_1(g + a)$

Using the expression for *a*, we get

$$T = \left(\frac{2m_1m_2}{m_1+m_2}\right)g$$

**Method II:** (Free body equations)

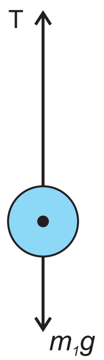



Fig. (b)

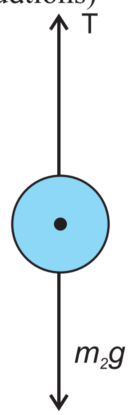



Fig. (b)

Free body diagrams of $m_1$ and $m_2$ are as shown in Figs. (b) and (c).

Thus, for the first body, $T - m_1g = m_1a$ --- (I)

For the second body, $m_2g - T = m_2a$ --- (II)

Adding (I) and (II), and solving for *a*,

we get, $a = \left(\frac{m_2 - m_1}{m_2 + m_1}\right)g$ --- (III)

Solving Eqs. (II) and (III) for *T*, we get,

$$T = m_2(g - a) = \left(\frac{2m_1m_2}{m_1+m_2}\right)g$$

## 4.8. Collisions:

During collisions a number of objects come together, interact (exert forces on *each other*) and scatter in different directions.

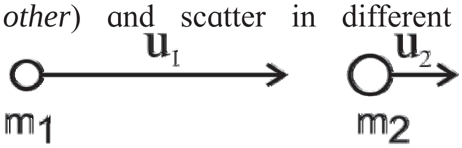



Fig. 4.3 (a): Head on collision-before collision.

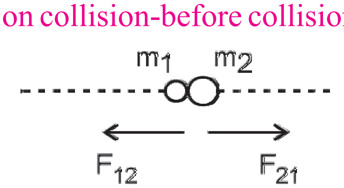



Fig. 4.3 (b): Head on collision-during impact.

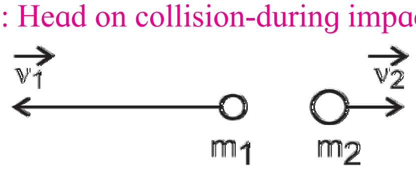



Fig. 4.3 (c): Head on collision-after collision.

### 4.8.1. Elastic and inelastic collisions:

During a collision, the linear momentum of the entire system of particles is always conserved as there is no external force acting on the *system* of particles. However, the individual momenta of the particles change due to mutual forces, which are internal forces.

$\therefore \sum \vec{p}_{initial} = \sum \vec{p}_{final}$, during ***any*** collision (or explosion), where $\vec{p}$'s are the linear momenta of the particles.

However, kinetic energy of the entire system may or may not conserve.

Collisions can be of two types: ***elastic collisions*** and ***inelastic collisions.***

**Elastic collision:** A collision is said to ***elastic*** if kinetic energy of the entire system is conserved during the collision (along with the linear momentum). Thus, during an elastic collision,

$$\sum K.E._{initial} = \sum K.E._{final}$$

An elastic collision is *impossible* in daily life. However, in many situations, the interatomic or intermolecular collisions are considered to be elastic (like in kinetic theory of gases, to be discussed in the next standard).

**Inelastic Collision:** A collision is said to be ***inelastic*** if there is a loss in the kinetic energy during collision, but linear momentum is conserved. The loss in kinetic energy is either due to internal friction or vibrational motion of atoms causing heating effect. Thus, during an inelastic collision,

$$\sum K.E._{initial} > \sum K.E._{final}.$$

During an explosion as energy is supplied internally. Thus,

$$\sum K.E._{final} > \sum K.E._{initial}.$$

As stated earlier, $\sum \vec{p}_{initial} = \sum \vec{p}_{final}$ for inelastic collisions or explosion also. In fact, this is always the first equation for discussing these interactions or while solving numerical questions.

### 4.8.2. Perfectly Inelastic Collision:

This is a special case of inelastic collisions. If colliding bodies join together after collision, it is said to be a ***perfectly inelastic collision***.

In other words, the colliding bodies have a common final velocity after a perfectly inelastic collision. Being an inelastic collision, obviously there is a loss in the kinetic energy of the system during a perfectly inelastic collision. In fact, the loss in kinetic energy is maximum in perfectly inelastic collision.

**Illustrations:**

(i) Consider a bullet fired towards a block kept on a smooth surface. Collision between bullet and the block will be elastic if the bullet rebounds with exactly the same initial speed and the block remains stationory. If the bullet gets embedded into the block and the two move jointly, it is perfectly inelastic collision. If the bullet rebounds with smaller speed or comes out of the block on the other side with some speed, it is an inelastic (or partially inelastic) collision. Remember, there is nothing called a partially elastic collision Elastic collisions are always perfectly elastic. An inelastic collision however, may be partially or perfectly inelastic.

(ii) Visualise a ball dropped from some height on a hard surface, the entire system being in an evacuated space. If the ball rebounds exactly to the same height from where it was dropped, the collision between the ball and the surface (in turn, with the Earth) is elastic. As you know, the ball never reaches the same initial height or a height greater than the initial height. Rebounding to smaller height refers to inelastic collision. Instead of ball, if mud or clay is dropped, it sticks to the surface. This is perfectly inelastic collision.

### 4.8.3. Coefficient of Restitution *e*:

For collision of two objects, the negative of ratio of relative velocity of separation to relative velocity of approach is defined as the coefficient of restitution *e*.

**One dimensional or head-on collision:** A collision is said to be head-on if the colliding objects move along the same straight line, before and after the collision. Here, we use $u_1$, $u_2$, $v_1$, $v_2$ as symbols.

Consider such a head-on collision of two bodies of masses $m_1$ and $m_2$ with respective initial velocities $u_1$ and $u_2$. As the collision is head on, the colliding masses are along the same line before and after the collision. Hence, vector treatment is not necessary. (**However, velocities must be substituted with proper signs in actual calculation**). Relative velocity of approach is then $u_a = u_2 - u_1$

Let $v_1$ and $v_2$ be their respective velocities after the collision. The relative velocity of recede (or separation) is then $v_s = v_2 - v_1$

$$\therefore \text{Coefficient of restitution, } e = -\frac{v_s}{u_a}$$

$$= \frac{-)v_2 - v_1(}{u_2 - u_1} = \frac{v_1 - v_2}{u_2 - u_1}$$

$$= \frac{\text{Relative speed of separation}}{\text{Relative speed of approach}} \quad \text{---(4.1)}$$

For a perfectly inelastic collision, the colliding bodies move jointly after the collision, i.e., $v_2 = v_1$ or $v_2 - v_1 = 0$. Hence, for a perfectly inelastic collision, $e$ =0. In other words, if $e$ = 0, the head-on collision is perfectly inelastic collision.

**Coefficient of restitution during a head-on, elastic collision:**

Consider the collision described above to be elastic. According to the principle of conservation of linear momentum,

Total initial momentum = Total final momentum.

$$\therefore m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad \text{--- (4.2)}$$

$$\therefore m_1\left(u_1 - v_1\right) = m_2\left(v_2 - u_2\right) \quad \text{--- (4.3)}$$

As the collision is elastic, total kinetic energy of the system is also conserved.

$$\therefore \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad \text{--- (4.4)}$$

$$\therefore m_1\left(u_1^2 - v_1^2\right) = m_2\left(v_2^2 - u_2^2\right)$$

$$\therefore m_1\left(u_1 + v_1\right)\left(u_1 - v_1\right)$$

$$= m_2\left(u_2 + v_2\right)\left(v_2 - u_2\right) \quad \text{--- (4.5)}$$

Dividing Eq. (4.5) by Eq. (4.3), we get

$$u_1 + v_1 = u_2 + v_2 \quad \text{--- (4.6)}$$

$$\therefore u_2 - u_1 = v_1 - v_2$$

For an elastic collision,

$$e = \frac{v_1 - v_2}{u_2 - u_1} = 1 \quad \text{--- (4.7)}$$

Thus, for an elastic collision, coefficient of restitution, $e$ =1. For a perfectly inelastic collision, $e$ =0 (by definition). Thus, for any collision, the coefficient of restitution lies between 1 and 0.

**Above expressions (Eq. (4.1) Eq. (4.7)) are general. While substituting the values of $u_1$, $u_2$, $v_1$, $v_2$, their algebraic values must be used in actual calculation.**

For example referring to the

Fig. 4.3 (a), (b) and (c)

Eq (4.1) gives

$$e = -\frac{v_s}{u_a}$$

Here $u_a = u_1 - u_2$ since $u_1 > u_2$

and $v_s = v_1 + v_2$ since the objects go in opposite directions.

$$\therefore e = -\frac{v_1 + v_2}{u_1 - u_2} \quad \text{--- (a)}$$

Using Eq (4.6),

$u_1 + v_1 = u_2 + v_2$

$\therefore$ According to Fig. 4.3,

$u_1 - v_1 = u_2 + v_2$

$\therefore v_1 + v_2 = u_2 - u_1$ --- (b)

By substituting in (a),

$$e = -\frac{(u_2 - u_1)}{(u_1 - u_2)} = 1,$$

which is the case of a perfectly elastic collision.

### 4.8.4. Expressions for final velocities after a head-on, elastic collision:

From Eq. (4.6), $v_2 = u_1 + v_1 - u_2$

Using this in Eq. (4.2), we get

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 \left(u_1 + v_1 - u_2\right)$$

$$\therefore v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) u_1 + \left(\frac{2m_2}{m_1 + m_2}\right) u_2 \quad \text{--- (4.8)}$$

Subscripts 1 and 2 were arbitrarily chosen. Thus, just interchanging 1 with 2 gives us $v_2$ as

$$v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) u_2 + \left(\frac{2m_1}{m_1 + m_2}\right) u_1 \quad \text{--- (4.9)}$$

Equation (4.9) can also be obtained by substituting $v_1$ from Eq. (4.8) in Eq. (4.6).

**Particular cases:**

(i) If the bodies are of equal masses (or identical), $m_1 = m_2$, Eqs. (4.8) and (4.9) give

$v_1 = u_2$ and $v_2 = u_1$.

Thus, the bodies just exchange their velocities.

(ii) If colliding body is much heavier and the struck body is initially at rest, i.e.,

$$m_1 \gg m_2 \text{ and } u_2 = 0,$$

we can use

$$m_1 \pm m_2 \cong m_1 \text{ and } \frac{m_2}{m_1 + m_2} \cong 0$$

$\therefore v_1 \cong u_1$ and $v_2 \cong 2u_1$, i.e., the massive striking body is practically unaffected and the tiny body which is struck, travels with double the speed of the massive striking body.

(iii) The body which is struck is much heavier than the colliding body and is initially at rest, i.e., $m_1 \ll m_2$ and $u_2 = 0$.

Using similar approximations, we get, $v_1 \cong -u_1$ and $v_2 \cong 0$, i.e., the tiny (lighter) object rebounds with same speed while the massive object is unaffected. This is as good as dropping an elastic object on hard surface of the Earth .

**Do you know ?**

Are you aware of elasticity of materials? Is there any connection between elasticity of materials and elastic collisions?

**Example 4.6:** One marble collides head-on with another identical marble at rest. If the collision is partially inelastic, determine the ratio of their final velocities in terms of coefficient of restitution $e$.

**Solution:** According to conservation of momentum, $m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

As $m_1 = m_2$, we get, $u_1 + u_2 = v_1 + v_2$

$\therefore$ If $u_2 = 0$, we get, $v_1 + v_2 = u_1$ ... (I)

Coefficient of restitution,

$$e = \frac{v_2 - v_1}{u_1 - u_2} \quad \therefore v_2 - v_1 = e u_1 \text{ ... (II)}$$

Dividing Eq. (I) by Eq. (II),

$$\frac{v_1 + v_2}{v_2 - v_1} = \frac{1}{e}$$

Using componendo and dividendo, we get,

$$\frac{v_2}{v_1} = \frac{1+e}{1-e}$$

### 4.8.5. Loss in the kinetic energy during a perfectly inelastic head-on collision:

Consider a perfectly inelastic, head on collision of two bodies of masses $m_1$ and $m_2$ with respective initial velocities $u_1$ and $u_2$. As the collision is perfectly inelastic, they move jointly after the collision, i.e., their final velocity is the same. Let it be v.

According to conservation of linear momentum, $m_1u_1 + m_2u_2 = (m_1 + m_2)v$

$$\therefore v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2} \quad \text{--- (4.10)}$$

This is the common velocity after a perfectly inelastic collision

Loss in K.E. = Δ (K.E.)

= Total initial K.E. - Total final K.E.

$$\therefore \Delta(\text{K.E.}) = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - \frac{1}{2}(m_1 + m_2)v^2$$

$$= \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - \frac{1}{2}(m_1 + m_2)\left(\frac{m_1u_1 + m_2u_2}{m_1 + m_2}\right)^2$$

$$\therefore \Delta(\text{K.E.}) = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - \frac{1}{2}\frac{(m_1u_1 + m_2u_2)^2}{(m_1 + m_2)}$$

On simplifying, we get,

$$\Delta(\text{K.E.}) = \frac{1}{2}\left(\frac{m_1m_2}{m_1 + m_2}\right)(u_1 - u_2)^2 \quad \text{--- (4.11)}$$

Masses are always positive and $(u_1 - u_2)^2$ is also positive. Hence, there is always a loss in the kinetic energy in a perfectly inelastic collision.

**Final velocities and loss in K.E. in an inelastic head-on collision:**

If $e$ is the coefficient of restitution, using Eq. (4.2), the expressions for final velocities after an inelastic collision can be derived as

$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1 + \left(\frac{[1+e]m_2}{m_1 + m_2}\right)u_2$$

$$= \frac{em_2(u_2 - u_1) + m_1u_1 + m_2u_2}{m_1 + m_2}$$

and

$$v_2 = \left(\frac{m_2 - em_1}{m_1 + m_2}\right)u_2 + \left(\frac{[1+e]m_1}{m_1 + m_2}\right)u_1$$

$$= \frac{em_1(u_1 - u_2) + m_1u_1 + m_2u_2}{m_1 + m_2}$$

Loss in the kinetic energy is given by

$$\Delta(\text{K.E.}) = \frac{1}{2}\left(\frac{m_1m_2}{m_1 + m_2}\right)(u_1 - u_2)^2(1 - e^2).$$

As $e < 1$, $(1 - e^2)$ is always positive. Thus, there is always a loss of K.E. in an inelastic collision. Also, for a perfectly inelastic collision, $e = 0$. Hence, in this case, the loss is maximum. Using $e = 1$, these equations lead us to an elastic collision and for $e = 0$ they lead us to a perfectly inelastic collision. Verify that they give the same expressions that are derived earlier.

The quantity $\mu = \frac{m_1m_2}{m_1 + m_2}$ is the reduced mass of the system.

**Impulse or change in momenta of the bodies:**

During collision, the linear momentum delivered by first body (particle) to the second body must be equal to change in momentum or *impulse* of the second body, and vice versa.

$$\therefore \text{Impulse}, |J| = |\Delta p_1| = |\Delta p_2|$$

$$= |m_1v_1 - m_1u_1| = |m_2v_2 - m_2u_2|$$

On substituting the values of $v_1$ and $v_2$ and solving, we get

$$|J| = \left(\frac{m_1m_2}{m_1 + m_2}\right)(1+e)(|u_1 - u_2|)$$

$$= \mu(1+e)u_{\text{relative}}$$

$|u_1 - u_2| = u_{\text{relative}}$ = velocity of approach

### 4.8.6. Collision in two dimensions, i.e., a nonhead-on collision:

In this case, the direction of at least one initial velocity is NOT along the line of impact. In order to discuss such collisions mathematically, it is convenient to use two mutually perpendicular directions as shown in Fig. 4.4. One of them is the common tangent at the point of impact, along which there is no force (or along this direction, there is no change in momentum). The other direction is perpendicular to this common tangent through the point of impact, in the two-dimensional plane of initial and final velocities. This is called the line of impact. Internal mutual forces exerted during impact, which are responsible for change in the momenta, are acting along this line. From Fig. (4.4), $\vec{u}_1$ and $\vec{u}_2$, initial velocities make angles $\alpha_1$ and $\alpha_2$ respectively with the line of impact while $\vec{v}_1$ and $\vec{v}_2$, final velocities make angles $\beta_1$ and $\beta_2$ respectively with the line of impact.

According to conservation of linear momentum along the line of contact,

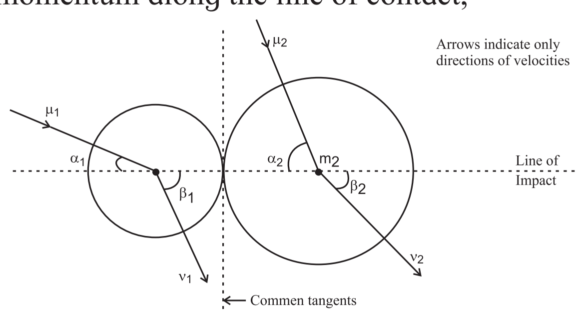



Fig. 4.4: Oblique or non head-on collision.

$$m_1u_1\cos\alpha_1 + m_2u_2\cos\alpha_2$$
$$= m_1v_1\cos\beta_1 + m_2v_2\cos\beta_2 \qquad \text{--- (4.12)}$$

As there is no force along the common tangent (perpendicular to line of impact),

$$m_1u_1\sin\alpha_1 = m_1v_1\sin\beta_1 \qquad \text{--- (4.13)}$$

and $m_2u_2\sin\alpha_2 = m_2v_2\sin\beta_2$ --- (4.14)

For coefficient of restitution, along the line of impact,

$$e = -\left(\frac{v_2\cos\beta_2 - v_1\cos\beta_1}{u_2\cos\alpha_2 - u_1\cos\alpha_1}\right)$$
$$= \frac{v_2\cos\beta_2 - v_1\cos\beta_1}{u_1\cos\alpha_1 - u_2\cos\alpha_2} \qquad \text{--- (4.15)}$$

Equations (4.12), (4.13), (4.14) and (4.15) are to be solved for the four unknowns $v_1$, $v_2$, $\beta_1$ and $\beta_2$

Magnitude of the impulse, along the line of impact,

$$|J| = \left(\frac{m_1m_2}{m_1+m_2}\right)(1+e)\left(|u_1\cos\alpha_1 - u_2\cos\alpha_2|\right)$$
$$= \mu(1+e)u_{\text{relative}}$$

along line of impact.

Loss in the kinetic energy = Δ (K.E.)

$$\frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)(u_1\cos\alpha_1 - u_2\cos\alpha_2)^2(1-e^2)$$
$$= \frac{1}{2}\mu\left(u^2_{\text{relative}}\right)(1-e^2)$$

**Example 4.7:** A shell of mass 3 kg is dropped from some height. After falling freely for 2 seconds, it explodes into two fragments of masses 2 kg and 1 kg. Kinetic energy provided by the explosion is 300 J. Using $g$ = 10 m/s², calculate velocities of the fragments. Justify your answer if you have more than one options.

**Solution:** $m_1 + m_2 = 3$ kg.

After falling freely for 2 seconds,

$$v = u + at = 0 + 10(2) = 20\,m\,s^{-1} = u_1 = u_2$$

According to conservation of linear momentum, $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$$\therefore 3\times 20 = 2v_1 + 1v_2 \quad \therefore v_2 = 60 - 2v_1 \quad \text{--- (I)}$$

K.E. provided = Final K.E. – Initial K.E. $= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \frac{1}{2}(m_1+m_2)u^2$

$$\therefore \frac{1}{2}2v_1^2 + \frac{1}{2}v_2^2 - \frac{1}{2}3(20)^2 = 300\,\text{J}$$

$$\text{or } 2v_1^2 + v_2^2 = 1800$$

$$\text{or } 2v_1^2 + (60-2v_1)^2 = 1800 \text{ using Eq. (I)}$$

$$\therefore 3600 - 240v_1 + 6v_1^2 = 1800$$

$$\therefore v_1^2 - 40v_1 + 300 = 0$$

$$\therefore v_1 = 30\,m\,s^{-1} \text{ or } 10\,m\,s^{-1} \text{ and}$$

$$v_2 = 0 \text{ or } 40\,m\,s^{-1}$$

There are two possible answers since the positions of two fragments can be different as explained below.

If $v_1 = 30$ m s$^{-1}$ and $v_2 = 0$, lighter fragment 2 should be above. On the other hand, if $v_1 = 10$ m s$^{-1}$ and $v_2 = 40$ m s$^{-1}$, lighter fragment 2 should be below, both moving downwards.

**Example 4.8:** Bullets of mass 40 g each, are fired from a machine gun at a rate of 5 per second towards a firmly fixed hard surface of area 10 cm$^2$. Each bullet hits normal to the surface at 400 m/s and rebounds in such a way that the coefficient of restitution for the collision between bullet and the surface is 0.75. Calculate average force and average pressure experienced by the surface due to this firing.

**Solution:** For the collision,

$u_1 = 400\,\text{m s}^{-1}$, $e = 0.75$, $v_1 = ?$

For the firmly fixed hard surface, $u_2 = v_2 = 0$

$$e = 0.75 = \frac{v_1 - v_2}{u_2 - u_1} = \frac{v_1 - 0}{0 - 400} \therefore v_1 = -300 \text{ m/s.}$$

-ve sign indicates that the bullet rebounds in exactly opposite direction.

Change in momentum of each bullet $= m(v_1 - u_1)$

Equal and opposite will be the momentum transferred to the surface, per collision.

∴ Momentum transferred to the surface, per collision

$$p = m(u_1 - v_1) = 0.04(400 - [-300]) = 28 \text{ N s}$$

The rate of collision is same as rate of firing.

∴ Momentum received by the surface per second, $\frac{dp}{dt} = 28 \times 5 = 140$ N

This must be the *average* force experienced by the surface of area $A = 10 \text{ cm}^2 = 10^{-3} \text{ m}^2$

∴ Average pressure experienced,

$$P = \frac{F}{A} = \frac{140}{10^{-3}} = 1.4 \times 10^5 \text{ N m}^{-2}$$

≈ 1.4 times the atmospheric pressure.

## 4.9. Impulse of a force:

According to Newton's first law of motion, any unbalanced force changes linear momentum of the system, i.e., basic effect of an unbalanced force is to change the momentum.

According to Newton's second law of motion, $\vec{F} = \frac{d\vec{p}}{dt}$

$$\therefore d\vec{p} = \vec{F}.dt$$

The quantity 'change in momentum' is separately named as *Impulse of the force* $\vec{J}$.

If the force is constant, and is acting for a finite and measurable time, we can write

The change in momentum in time $t$

$$\vec{J} = d\vec{p} = \vec{p}_2 - \vec{p}_1 = \vec{F}.t \qquad \text{---(4.16)}$$

For a given body of mass $m$, it becomes

$$\vec{J} = \vec{p}_2 - \vec{p}_1 = m(\vec{v}_2 - \vec{v}_1) = \vec{F}.t \qquad \text{---(4.17)}$$

If $\vec{F}$ is not constant but we know how it varies with time, then

$$\vec{J} = \Delta\vec{p} = \int d\vec{p} = \int \vec{F}.dt \qquad \text{--- (4.18)}$$

**Always remember:**

1) Colliding objects experience forces along the line of impact which changes their momenta. For their system, these forces are internal forces. These forces form an action-reaction pair, which are equal and opposite, and act on different objects.
2) There is no force along the common tangent, i.e., perpendicular to the line of contact.
3) In reality, the impact is followed by emission of sound and heat and occasionally light. Thus, in general, part of mechanical energy- kinetic energy - is lost (i.e., converted into some other non-recoverable forms). However, total energy of the system is conserved.
4) In reality, velocity of separation (relative final velocity) is less than velocity of approach (relative initial velocity) along the line of impact. Thus coefficient of restitution e < 1.
5) Only during elastic collisions (atomic and molecular level only, never possible in real life), the kinetic energy is conserved and the velocity of separation is equal to the velocity of approach or the initial relative velocity is equal to the final relative velocity.

### 4.9.1. Necessity of defining impulse:

As discussed above, if a force is constant over a given interval of time or if we know how it varies with time, we can calculate the corresponding change in momentum directly by multiplying the force and time.

However, in many cases, an appreciable force acts for an extremely small interval of time (too small to measure the force and the time independently). However, change in the momentum due to this force is noticeable and can be measured. This change is defined as *impulse* of the force.

**Real life illustrations:** While (i) hitting a ball with a bat, (ii) giving a kick to a foot-ball, (iii) hammering a nail, (iv) bouncing a ball from a hard surface, etc., appreciable amount of force is being exerted. In such cases the time for which these forces act on respective objects is negligibly small, mostly not easily recordable. However, the effect of this force is a recordable change in the momentum of that object. Thus, it is convenient to define the change in momentum itself as a physical quantity.

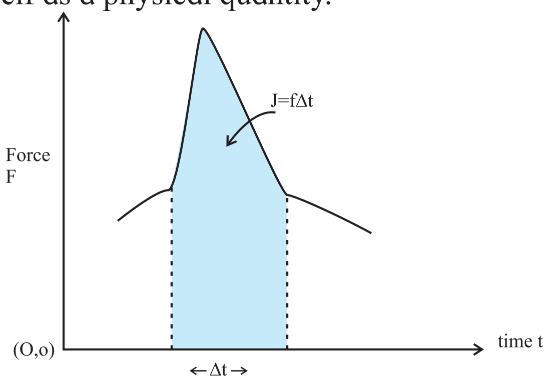



Fig. 4.5: Graphical representation of impulse of a force.

Figure 4.5 shows variation of a force as a function of time e.g., for a collision between bat and ball with the *force axis starting with zero*. The shaded area or the area under the curve gives the product of force against corresponding time (in this case, $\Delta t$ ), hence gives the impulse. For a constant force it is obviously a rectangle. Generally, force is zero before the impact, rises to a maximum and decreases to zero after the impact. For softer tennis ball, the collision time is larger and the maximum force is less. The area under the ($F$ - $t$) graph is the same. Wicket keeper eases off (by increasing the time of collision) while catching a fast ball. As mentioned earlier, it is absolutely necessary that the force axis must start from zero.

Recall from Chapter 3, analogues concepts using area under a curve are (i) Obtaining displacement in a given time interval as area under the curve for *v-t* graph, with zero origin for velocity axis. (ii) Obtaining work done by a force as the area under the curve for *F-s* graph, with zero origin for force axis.

**Example 4.9:** Mass of an Oxygen molecule is $5.35 \times 10^{-26}$ kg and that of a Nitrogen molecule is $4.65 \times 10^{-26}$ kg. During their Brownian motion (random motion) in air, an Oxygen molecule travelling with a velocity of 400 m/s collides elastically with a nitrogen molecule travelling with a velocity of 500 m/s in the exactly opposite direction. Calculate the impulse received by each of them during collision. Assuming that the collision lasts for 1 ms, how much is the average force experienced by each molecule?

Let

$$m_1 = m_O = 5.35\times10^{-26} \text{ kg},$$

$$m_2 = m_N = 4.65\times10^{-26} \text{kg}.$$

$$\therefore u_1 = 400 \text{m s}^{-1} \; and \; u_2 = -500 \text{m s}^{-1}$$

taking direction of motion of Oxygen molecule as the positive direction.

For an elastic collision,

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2 \quad \text{and}$$

$$v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \left(\frac{2m_1}{m_1 + m_2}\right)u_1$$

$$\therefore v_1 = -437 \text{m s}^{-1} \; and \; v_2 = 463 \text{m s}^{-1}$$

$$\therefore J_O = m_O\left(v_1 - u_1\right) = -4.478\times10^{-23} \text{ N s},$$

$$J_N = m_N\left(v_2 - u_2\right) = +4.478\times10^{-23} \text{ N s}$$

As expected, the net impulse or net change in momentum is zero.

$$F_{ON} = \frac{dp_O}{dt} = \frac{J_O}{\Delta t} = \frac{-4.478\times10^{-23}}{10^{-3}}$$

$$= -4.478\times10^{-20} \text{ N}$$

and $F_{NO} = -F_{ON} = 4.478\times10^{-20}$ N

## 4.10. Rotational analogue of a force - moment of a force or torque:

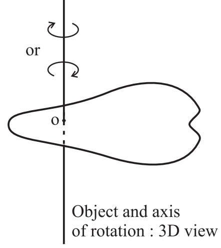
While opening a door fixed to a frame on hinges, we apply the force away from the hinges and perpendicular to the door to open it with ease. In this case we are interested in achieving some angular displacement for the door. If the force is applied near the hinges or nearly parallel to the door, it is very difficult to open the door. Similarly, if the door is heavier (made up of iron instead of wood or plastic), we need to apply proportionally larger force for the same angular displacement.

It shows that rotational ability of a force not only depends upon the mass (greater force for greater mass), but also upon the point of application of the force (the point should be as away as possible from the axis of rotation) and the angle between direction of the force and the line joining the axis of rotation with the point of application (effect is maximum, if this angle is $90^0$).

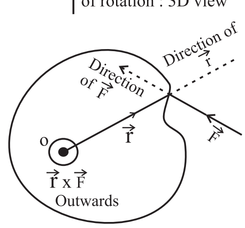
Taking into account all these factors, the quantity **moment of a force** or **torque** is defined as the rotational analogue of a force. As rotation refers to direction (sense of rotation), torque must be a vector quantity. In its mathematical form, torque or moment of a force is given by

$$\vec{\tau} = \vec{r} \times \vec{F} \qquad \text{--- (4.17)}$$

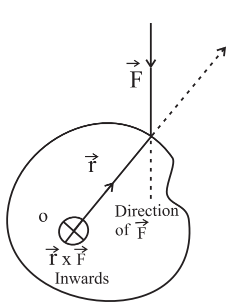
where $\vec{F}$ is the applied force and $\vec{r}$ is the position vector of the point of application of the force from the axis of rotation, as shown in the Figs. 4.6 (b) and 4.6 (c).




Figs. 4.6(a): Illustration of moment of force with object and axis of rotation in 3D view.




Figs. 4.6(b): Top view for moment of force $\vec{F}$ in anticlockwise rotation with $\vec{F}$ and $\vec{r}$ in the plane of paper.




Figs. 4.6(c): Top view of moment of force $\vec{F}$ in clockwise rotation $\vec{F}$ and $\vec{r}$ in the plane of paper.

Figures 4.6(a), 4.6(b) and 4.6(c) illustrate the directions involved. Figure 4.6(a) is a 3D drawing indicating the laminar (plane or two dimensional) object rotating about a (*fixed*) axis of rotation AOB, the axis being perpendicular to the object and passing through it. Figure 4.6(b) indicates the top view of the object when the rotation is in anticlockwise direction and Fig. 4.6(c) shows the view from the top, if rotation is in clockwise direction. (In fact, Figs. 4.6(b) and 4.6(c) are drawn in such a way that the applied force $\vec{F}$ and position vector $\vec{r}$ of the point of application of the force are in the plane of these figures). Direction of the torque is always perpendicular to the plane containing the vectors $\vec{r}$ and $\vec{F}$ and can be obtained from the rule of cross product or by using the right-hand thumb rule. In Fig. 4.6(b), it is perpendicular to the plane of the figure (in this case, perpendicular to the body) and outwards, i.e., coming out of the paper while in the Fig. 4.6(c), it is inwards, i.e., going into the paper.

In order to indicate the directions which are not in the plane of figure, we use a special convention: ⊙ for perpendicular to the plane of figure and outwards and ⊗ for perpendicular to the plane of figure and inwards.

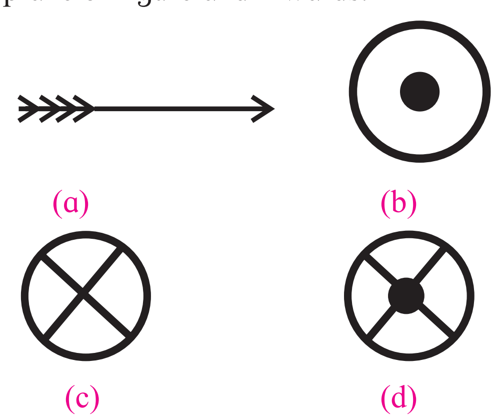



Fig. 4.7: Convention of pictorial representation of vectors as shown in (a) acting in a direction perpendicular to the plane of paper (b) coming out of paper, (c) going in to the paper and (d) perpendicular to the plane of paper.

This convention depends upon a traditional arrow shown in Fig. 4.7 (a). Consider yourself, looking towards the figure from the top. If this arrow approaches you, the tip of the arrow will be prominently seen. Hence circle with a dot in it [Fig. 4.7 (b)] refers to perpendicular and outwards (or towards you). When you are leaving an arrow, i.e., if an arrow is going away from you, the feathers like a cross will be seen. Hence, a circle with a cross [Fig. 4.6 (c)] indicates perpendicular and inwards (or away from you). Circle with cross and dot indicates a line perpendicular to the plane of figure [Fig. 4.6 (d)].

Magnitude of torque, $\tau = r\,F \sin\theta$ --- (4.18)

where $\theta$ is the smaller angle between the *directions* of $\vec{r}$ and $\vec{F}$.

Consequences: (i) If $r$ or $F$ is greater, the torque (hence the rotational effect) is greater. Thus, it is recommended to apply the force away from the hinges.

(ii) If $\theta = 90^0$, $\tau = \tau_{max} = rF$. Thus, the force should be applied along normal direction for easy rotation.

(iii) If $\theta = 0°$ or $180°$, $\tau = \tau_{min} = 0$. Thus, if the force is applied parallel or anti-parallel to $\vec{r}$, there is no rotation.

(iv) Moment of a force depends not only on the magnitude and direction of the force, but also on the point where the force acts with respect to the axis of rotation. Same force can have different torque as per its point of application.

## 4.11. Couple and its torque:

In the discussion of the torque given above, we had considered rotation of the body about a fixed axis and due to a single force. In real life, quite often we apply two equal and opposite forces acting along different lines of action in order to cause rotation. Common illustrations are turning a bicycle handle, turning the steering wheel, opening a common water tap, opening the lid of a bottle (rotation type), etc. Such a pair of forces consisting of two forces of equal magnitude acting in opposite directions along different lines of action is called a couple. It is used to realise a *purely* rotational motion. Moment of a couple or rotational effect of a couple is also called a *torque*.

It may be noted that in the discussion of rotation of a body about a fixed axis due to a single force, there is a reaction force at the fixed axis. Hence, for rotation one always needs two forces acting in opposite direction along different lines of action.

**Torque or Moment of a couple:** Figure 4.8 shows a couple consisting of two forces $\vec{F_1}$ *and* $\vec{F_2}$ of equal magnitudes and opposite directions acting along different lines of action separated by a distance $r$. Corresponding position vectors should now be defined with reference to the lines of action of forces. Position vector of *any* point on the line of action of force $\vec{F_1}$ from the line of action of force $\vec{F_2}$ is $\vec{r}_{12}$. Similarly, the position vector of *any* point on the line of action of force $\vec{F_2}$ from the line of action of force $\vec{F_1}$ is $\vec{r}_{21}$. Torque or moment of the couple is then given mathematically as

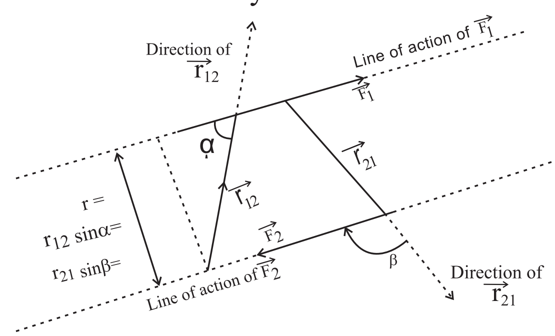



Fig. 4.8: Torque of a couple.

$$\vec{\tau} = \vec{r}_{12} \times \vec{F_1} = \vec{r}_{21} \times \vec{F_2} \quad \text{--- (4.19)}$$

From the figure, it is clear that $r_{12}\sin\alpha = r_{21}\sin\beta = r$

If $\left|\vec{F_1}\right| = \left|\vec{F_2}\right| = F$, the magnitude of torque is given by

$$\tau = r_{12}\,F_1 \sin\alpha = r_{21}\,F_2 \sin\beta = r\,F \quad \text{--- (4.20)}$$

It clearly shows that the torque corresponding to a given couple, i.e., the ***moment of a given couple is constant***, i.e., ***it is independent of the points of application of forces or the position of the axis of rotation***, but depends only upon magnitude of either force and the separation between their lines of action.

The direction of moment of couple can be obtained by using the vector formula of the torque or by using the right-hand thumb rule. For the couple shown in the Fig. 4.8, it is perpendicular to the plane of the figure and inwards. For a given pair of forces, the direction of the torque is fixed.

In many situations the word *couple* is used synonymous to moment of the couple or its torque, i.e., every time we may not say it as torque due to the couple, but say that a *couple* is acting.

| | Moment of a force | Moment of a couple |
|---|---|---|
| 1 | $\vec{\tau} = \vec{r} \times \vec{f}$ | $\vec{\tau} = \vec{r}_{12} \times \vec{F}_1 = \vec{r}_{21} \times \vec{F}_2$ |
| 2 | $\vec{\tau}$ depends upon the axis of rotation and the point of application of the force. | $\vec{\tau}$ depends ***only*** upon the two forces, i.e., it is independent of the axis of rotation or the points of application of forces. |
| 3 | It can produce translational acceleration also, if the axis of rotation is not fixed or if friction is not enough. | Does not produce any translational acceleration, but produces only rotational or angular acceleration. |
| 4 | Its rotational effect can be balanced by a proper single force or by a proper couple. | Its rotational effect can be balanced *only* by another couple of equal and opposite torque. |

### 4.11.1. To prove that the moment of a couple is independent of the axis of rotation:

Figure 4.9 shows a rectangular sheet (any object would do) free to rotate only about a fixed axis of rotation, perpendicular to the plane of figure, as shown. A couple consisting of forces $\vec{F}$ and -$\vec{F}$ is acting on the sheet at different locations.

Here we are considering the torque of a couple to be two torques due to individual forces causing rotation about the axis of rotation. In Fig. 4.9(a), the axis of rotation is between the lines of action of the two forces constituting the couple. Perpendicular distances of the axis of rotation from the forces $\vec{F}$ and -$\vec{F}$ are *x* and *y* respectively. Rotation due to the pair of forces *in this case* is anticlockwise (from top view), i.e., directions of individual torques due to the two forces are the same.

$$\therefore \tau = \tau_+ + \tau_- = xF + yF = (x+y)F = rF \quad (4.21)$$

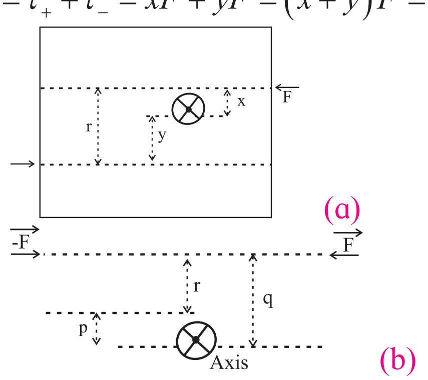



Fig. 4.9: Same couple on same object with fixed axis of rotation at different locations in (a) and (b).

In the Fig. 4.9 (b), lines of action of both the forces are on the same side of the axis of rotation. Thus, *in this case,* the rotation of $+\vec{F}$ is anticlockwise, while that of -$\vec{F}$ is clockwise (from the top view). As a result, their individual torques are oppositely directed. Perpendicular distance of the forces *F* and -*F* from the axis of rotation are *q* and *p* respectively.

$$\therefore \tau = \tau_+ - \tau_- = qF - pF$$

$$= (q-p)F = rF \quad \text{--- (4.22)}$$

From equations (4.21) and (4.22), it is clear that the torque of a couple is independent of the axis of rotation.

## 4.12. Mechanical equilibrium:

As a consequence of Newton's second law, the momentum of a system is constant in the absence of an external unbalanced force. This state is called *mechanical equilibrium.*

A particle is said to be in mechanical equilibrium, if no net force is acting upon it. For a system of bodies to be in mechanical equilibrium, the net force acting on *any part* of the system should be zero. In other words, velocity or linear momentum of all parts of the system must be constant or (zero) for the system to be in mechanical equilibrium. Also, there is no acceleration in any part of the system.

Mathematically, $\sum \vec{F} = 0$, for any part of the system for mechanical equilibrium.

### 4.12.1 Stable, unstable and neutral equilibrium:

Figures 4.10 (a), (b) and (c) show a ball at rest in three situations under the action of balanced forces. In all these cases, it is under equilibrium. However, potential energy-wise, the three cases differ.

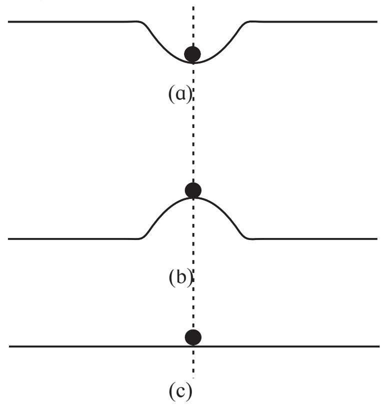



Fig. 4.10: states of mechanical equilibrium (a) stable, (b) unstable and (c) neutral.

**Stable equilibrium:** In Fig. 4.9(a), the ball is most stable and is said to be in *stable* equilibrium. If it is disturbed slightly from its equilibrium position and released, it tends to recover its position. In this case, potential energy of the system is at its local minimum.

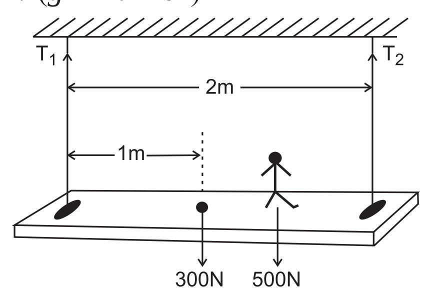
**Unstable equilibrium:** In Fig. 4.9(b), the ball is said to be in *unstable* equilibrium. If it is slightly disturbed from its equilibrium position, it moves farther from that position. This happens because initially, potential energy of the system is at its local maximum. If disturbed, it tries to achieve the configuration of minimum potential energy.

If potential energy function is known for the system, mathematically, the three equilibria can be explained with the help of derivatives of that function. At any equilibrium position, the first derivative of the potential energy function is zero $\left(\frac{dU}{dx}=0\right)$.

The sign of the second derivative $\left(\frac{d^2U}{dx^2}\right)$ decides the type of equilibrium. It is positive at stable equilibrium (or vice versa), negative at unstable equilibrium and zero (or does not exist) at neutral equilibrium configuration.

**Neutral equilibrium:** In Fig. 4.9(c), potential energy of the system is constant over a plane and remains same at any position. Thus, even if the ball is disturbed, it still remains in equilibrium at practically any position. This is described as *neutral* equilibrium.

**Example 4.10:** A uniform wooden plank of mass 30 kg is supported symmetrically by two light identical cables; each can sustain a tension up to 500 N. After tying, the cables are exactly vertical and are separated by 2 m. A boy of mass 50 kg, standing at the centre of the plank, is interested in walking on the plank. How far can he walk? ($g$ = 10 ms$^{-2}$)




**Solution:** Let $T_1$ and $T_2$ be the tensions along the cables, both acting vertically upwards.

Weight of the plank 300 N is acting vertically downwards through the centre, 1 m from either cable. Weight of the boy, 500 N is vertically downwards at the point where he is standing.

$$\therefore T_1 + T_2 = 300 + 500 = 800 \text{ N}$$

Suppose that the boy is able to walk $x$ m towards the right. Obviously, the tension in the right side cable goes on increasing as he walks towards the cable.

Moments of 300 N and 500 N forces about left end A are clockwise, while that of $T_2$ is anticlockwise.

As the cable can sustain 500 N, $(T_2)_{max}$ = 500 N

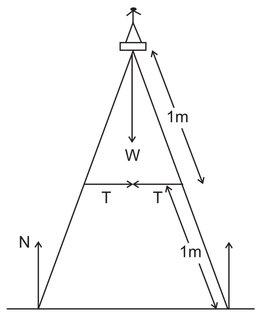
Thus, for the equilibrium about A, we can write,

$$300\times 1 + 500\times(1+x) = 500\times 2 \quad \therefore x = 0.4 \text{ m}$$

Thus, the boy can walk up to 40 cm on either side of the centre.

**Example 4.11:** A ladder of negligible mass having a cross bar is resting on a frictionless horizontal floor with angle between its legs to be 40$^0$. Each leg is 1 m long. Calculate the force experienced by the cross bar when a person of mass 50 kg is standing on the ladder. (g = 10 m s$^{-2}$)

**Solution:** Tension $T$ along the cross bar is horizontal. Let $L$ be the length of each leg, which is 1 m.




As there is no friction, there is no horizontal reaction at the floor. Reaction $N$ given by the floor at the base of the ladder will then be only vertical. Thus, along the vertical, two such reactions balance weight $W = mg$ of the person.

$$\therefore N = \frac{mg}{2} = 250 \text{ N}$$

At the left leg, about the upper end, the torque due to $N$ is clockwise and that due to the tension $T$ is anticlockwise. For equilibrium, these two torques should have same magnitude.

$$\therefore N.L\sin 20^0 = T.\left(\frac{L}{2}\right)\cos 20^0$$

$$\therefore T = 2N\tan 20^0 = 2\times 250\times 0.364 = 182 \text{ N}$$

## 4.13. Centre of mass:

As discussed earlier, Newton's laws of motion and many other laws are applicable for point masses only. However, in real life, we always come across finite objects (objects of measurable sizes). Concept of centre of mass (c.m.) helps us in considering these objects to be point objects at a particular location, thereby allowing us to apply Newton's laws of motion.

### 4.13.1. Mathematical understanding of centre of mass:

(i) System of $n$ particles: Consider a system of $n$ particles of masses $m_1$, $m_2$ ... $m_n$.

$$\therefore \sum_1^n m_i = M \text{ the total mass.}$$

Let $\vec{r}_1, \vec{r}_2, \ldots \vec{r}_n$ be their respective position vectors from a given origin O (Fig. 4.11) .

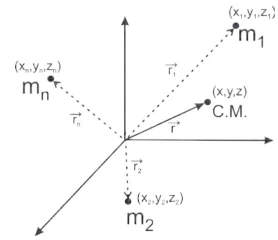



Fig. 4.11: Centre of mass for n particles.

Position vector $\vec{r}$ of their centre of mass from the same origin is then given by

$$\vec{r} = \frac{\sum_1^n m_i\vec{r}_i}{\sum_1^n m_i} = \frac{\sum_1^n m_i\vec{r}_i}{M}$$

If the origin itself is at the centre of mass,

$\vec{r} = 0 \;\therefore \sum_1^n m_i\vec{r}_i = 0$, then

$\sum_1^n m_i\vec{r}_i$ gives the moment of masses (similar to moment of force) about the centre of mass.

*Thus, centre of mass is a point about which the summation of moments of masses in the system is zero.*

If $x_1$, $x_2$, ... $x_n$ are the respective $x$-coordinates of $r_1, r_2, \ldots r_n$, the $x$-coordinate of the centre of mass is given by

$$x = \frac{\sum_1^n m_i x_i}{\sum_1^n m_i} = \frac{\sum_1^n m_i x_i}{M}$$

Similarly, $y$ and $z$-coordinates of the centre of mass are respectively given by

$$y = \frac{\sum_1^n m_i y_i}{\sum_1^n m_i} = \frac{\sum_1^n m_i y_i}{M}$$

and

$$z = \frac{\sum_1^n m_i z_i}{\sum_1^n m_i} = \frac{\sum_1^n m_i z_i}{M}$$

(i) Continuous mass distribution: For a continuous mass distribution ***with uniform density***, we need to use integration instead of summation. In this case, the position vector of the centre of mass is given by

$$\vec{r} = \frac{\int \vec{r}\,dm}{\int dm} = \frac{\int \vec{r}\,dm}{M},$$

where $\int dm = M$ is the total mass of the object. Then the Cartesian coordinates of c.m. are

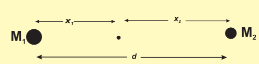
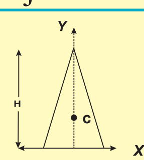
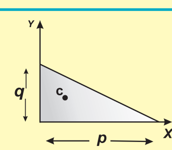
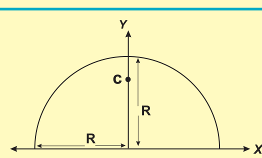
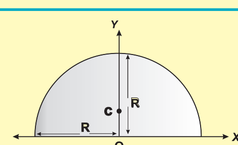
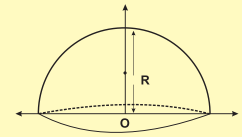
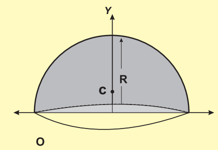
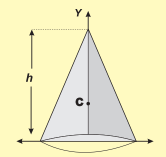
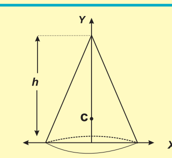
$$x = \frac{\int x\, dm}{\int dm} = \frac{\int x\, dm}{M}$$

$$y = \frac{\int y\, dm}{\int dm} = \frac{\int y\, dm}{M}$$

$$z = \frac{\int z\, dm}{\int dm} = \frac{\int z\, dm}{M}$$

Using the expressions given above, the centres of mass of uniform symmetric objects can be obtained. Some of these are listed in the Table 4.1 given below:

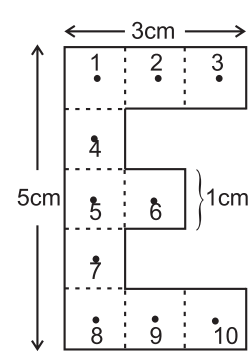
**Table 4.1: Coordinate of the centre of mass (c.m.) for some symmetrical objects**

| Coordinates of c.m. | Uniform Symmetric Objects |
|---|---|
| System of two point masses: c.m. divides the distance in inverse proportion of the masses |  |
| Any geometrically symmetric object of uniform density. | Centre of mass at a geometrical centre of the object |
| Isosceles triangular plate $x_c = 0, y_c = \frac{H}{3}$ |  |
| Right angled triangular plate $x_c = \frac{p}{3},\ y_c = \frac{q}{3}$ |  |
| Thin semicircular ring of radius $R$ $x_c = 0,\ y_c = \frac{2R}{\pi}$ |  |
| Thin semicircular disc of radius $R$ $x_c = 0,\ y_c = \frac{4R}{3\pi}$ |  |
| Hemispherical shell of radius $R$ $x_c = 0,\ y_c = \frac{R}{2}$ |  |
| Solid hemisphere of radius $R$ $x_c = 0,\ y_c = \frac{3R}{8}$ |  |
| Hollow right circular cone of height $h$ $x_c = 0,\ y_c = \frac{h}{3}$ |  |
| Solid right circular cone of height $h$ $x_c = 0,\ y_c = \frac{h}{4}$ |  |

**Example 4.12:** A letter 'E' is prepared from a uniform cardboard with shape and dimensions as shown in the figure. Locate its centre of mass.

**Solution:** As the sheet is uniform, each square can be taken to be equivalent to mass $m$ concentrated at its respective centre. These masses will then be at the points labelled with numbers 1 to 10, as shown in figure. Let us select the origin to be at the left central mass $m_5$, as shown and all the co-ordinates to be in cm.

By symmetry, the centre of mass of $m_1$, $m_2$ and $m_3$ will be at $m_2(1, 2)$ having effective mass $3m$. Similarly, effective mass $3m$ due to $m_8$, $m_9$ and $m_{10}$ will be at $m_9(1, -2)$. Again, by symmetry, the centre of mass of these two ($3m$ each) will have co-ordinates (1, 0). Mass $m_6$ is also having co-ordinates (1, 0). Thus, the effective mass at (1, 0) is $7m$.



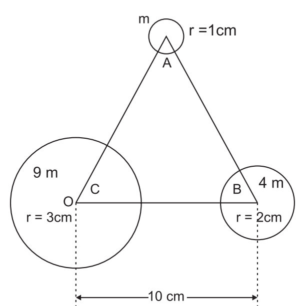

Using symmetry for $m_4$, $m_5$ and $m_7$, there will be effective mass $3m$ at the origin (0, 0).

Thus, effectively, $3m$ and $7m$ are separated by 1 cm along $x$-direction. $y$-coordinate is not required.

$$x_c = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{3\times 0 + 7\times 1}{3+7} = 0.7\,\text{cm}$$

Alternately, for two point masses, the centre of mass divides the distance between them in the inverse ratio of their masses. Hence, 1 cm is divided in the ratio 7:3. $\therefore x_c = \frac{7}{7+3}\times 1 = 0.7\,\text{cm}$ from $3m$, i.e., from the origin at $m_c$

**Example 4.13:** Three thin walled uniform hollow spheres of radii 1cm, 2 cm and 3 cm are so located that their centres are on the three vertices of an equilateral triangle ABC having each side 10 cm. Determine centre of mass of the system.




**Solution:** Mass of a thin walled uniform hollow sphere is proportional to its surface area, (as density is constant) hence proportional to $r^2$. Thus, if mass of the sphere at A is $m_A = m$, then $m_B = 4m$ and $m_C = 9m$. By symmetry of the spherical surface, their centres of mass are at their respective centres, i.e., at A, B and C.

Let us choose the origin to be at C, where the largest mass $9m$ is located and the point B with mass $4m$ on the positive $x$-axis. With this, the co-ordinates of C are (0, 0) and that of B are (10, 0). (Locating the origin at the larger mass here save our efforts of calculations like multiplications with larger numbers). If A of mass $m$ is taken in the first quadrant, its co-ordinates will be $\left[5, \frac{10\sqrt{3}}{2}\right]$

$$x_c = \frac{m_A x_A + m_B x_B + m_C x_C}{m_A + m_B + m_C}$$

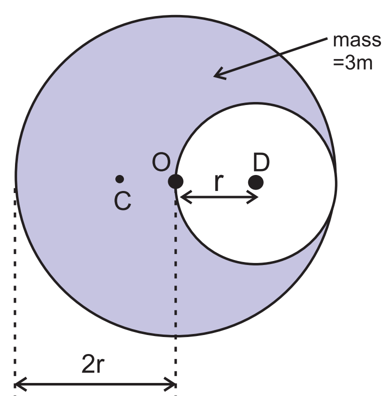
$$= \frac{m\times 5 + 4m\times 10 + 9m\times 0}{m + 4m + 9m} = \frac{45}{14}\,\text{cm}$$

$$y_c = \frac{m_A y_A + m_B y_B + m_C y_C}{m_A + m_B + m_C}$$

$$= \frac{m\times\frac{10\sqrt{3}}{2} + 4m\times 0 + 9m\times 0}{m + 4m + 9m} = \frac{10\sqrt{3}}{28}\,\text{cm}$$

**Example 4.14:** A hole of radius $r$ is cut from a uniform disc of radius $2r$. Centre of the hole is at a distance $r$ from centre of the disc. Locate centre of mass of the remaining part of the disc.

**Solution: Method I:** (Using entire disc): Before cutting the hole, c.m. of the full disc was at its centre. Let this be our origin O. Centre of mass of the cut portion is at its centre D. Thus, it is at a distance $x_1 = r$ form the origin. Let C be the centre of mass of the remaining disc. Obviously, it should be on the extension of the line DO. Let it be at a distance $x_2 = x$ from the origin. As the disc is uniform, mass of any of its part is proportional to the area of that part.




Thus, if $m$ is the mass of the cut disc, mass of the entire disc must be $4m$ and mass of the remaining disc will be $3m$.

$$x_c = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

As centre of mass of the full disc is at the origin, we can write,

$$0 = \frac{m\times r + 3m\times(x)}{m + 3m} \quad \therefore x = \frac{-r}{3}$$

**Method II:** (Using negative mass): Let $\vec{R}$ be the position vector of the centre of mass of the uniform disc of mass $M$. Mass $m$ is with centre of mass at position vector $\vec{r}$ from the centre of the disc. Position vector of the centre of mass of the remaining disc is then given by

$$\vec{r}_c = \frac{M\vec{R} - m\vec{r}}{M - m}$$ ….. (as if there is a negative mass, i.e., $m_2 = -m$)

With our description, $M = 4m$, $m = m$, $R = 0$ and $r = r$ $\therefore r_c = \frac{-mr}{3m} = \frac{-r}{3}$ ... Same as method I.

### 4.13.2. Velocity of centre of mass:

Let $v_1, v_2, \ldots v_n$ be the velocities of a system of point masses $m_1, m_2, \ldots m_n$. Velocity of the centre of mass of the system is given by

$$\vec{v}_{cm} = \frac{\sum_1^n m_i \vec{v}_i}{\sum_1^n m_i} = \frac{\sum_1^n m_i \vec{v}_i}{M}$$

$$= \frac{\text{resultant linear momentum}}{\text{total mass}}$$

= weighted average of momenta

$x$, $y$ and $z$ components of $\vec{v}$ can be obtained similarly.

For continuous distribution, $\vec{v}_{cm} = \frac{\int \vec{v}\, dm}{M}$

### 4.13.3. Acceleration of the centre of mass:

Let $a_1, a_2, \ldots a_n$ be the accelerations of a system of point masses $m_1, m_2, \ldots m_n$. Acceleration of the centre of mass of the system is given by

$$\vec{a}_{cm} = \frac{\sum_1^n m_i \vec{a}_i}{\sum_1^n m_i} = \frac{\sum_1^n m_i \vec{a}_i}{M}$$

$$= \frac{\text{resultant force}}{\text{total mass}}$$

= weighted average of forces

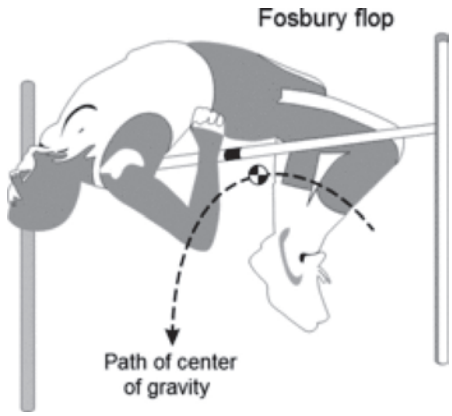
$x$, $y$ and $z$ components of $\vec{a}$ can be obtained similarly.

For continuous distribution, $\vec{a}_{cm} = \frac{\int \vec{a}\, dm}{M}$

### 4.13.4. Characteristics of centre of mass:

1. Centre of mass is a hypothetical point at which entire mass of the body can be assumed to be concentrated.
2. Centre of mass is a location, and not a physical quantity.
3. Centre of mass is particle equivalent of a given object for applying laws of motion.
4. Centre of mass is the point at which, if a force is applied, it causes only linear acceleration and not angular acceleration.
5. Centre of mass is located at the centroid, for a rigid body ***of uniform density***.
6. Centre of mass is located at the geometrical centre, for a symmetric rigid body ***of uniform density***.
7. Location of centre of mass can be changed ***only*** by an external unbalanced force.
8. Internal forces (like during collision or explosion) ***never*** change the location of centre of mass.
9. Position of the centre of mass depends only upon the distribution of mass, however, to describe its location we may use a coordinate system with a *suitable* origin. In statistical terms the centre of mass is decided by the *weighted* average of individual masses. This is obtained by giving proper mass *weightage* to the distance. This should be clear from the mathematical expression for the location of the centre of mass.
10. For a system of particles, the centre of mass *need not* coincide with any of the particles.
11. While balancing an object on a pivot, the line of action of *weight* must pass through the centre of mass and the pivot. Quite often, this is an unstable equilibrium.
12. Centre of mass of a system of only two particles divides the distance between the particles in an inverse ratio of their masses, i.e., it is closer to the heavier mass.
13. Centre of mass is a point about which the summation of moments of masses in the system is zero.
14. If there is an axial symmetry for a given object, the centre of mass lies on the axis of symmetry.
15. If there are multiple axes of symmetry for a given object, the centre of mass is at their point of intersection.
16. Centre of mass need not be within the

body (See the photograph given below: Picture 4.1). Other examples are a ring, a horse shoe, etc.



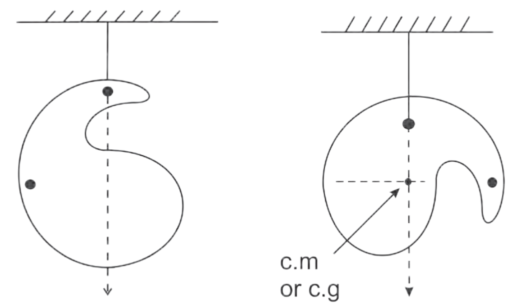

Picture 4.1: Courtesy Wikipedia: Estimated center of mass/gravity of a high jumper doing a Fosbury Flop. Note that it is below the bar in this position. This is possible because our head and legs are much heavier than the fleshy part. Increase in the gravitational potential energy of the high jumper depends upon this point.

### 4.14. Centre of gravity

Centre of gravity (c.g.) of a body is the point around which the resultant torque due to force of gravity on the body is zero. Analogous to centre of mass, it is the weighted average of the gravitational forces (weights) on individual particles.

For uniform gravitational field (in simple words, if $g$ is constant), c.g. always coincides with the c.m. Obviously it is true for all the objects *on the Earth* in our daily life. Thus, in common usage, the terms c.g. and c.m. are used for same purpose. This property can be used to determine the c.g. (or c.m.) of a laminar (laminar means like a leaf – two dimensional) object.

In Fig. 4.12, a laminar object is suspended from a rigid support at two orientations. Lines are to be drawn on the object parallel to the plumb line shown. Plumb line is always vertical, i.e., parallel to the line of action of gravitational force. Intersection of the lines drawn is then the point through which line of action of the gravitational force passes for any orientation. Thus, it gives the location of the c.g. or c.m.




Centre of mass is a fixed property for a given rigid body in spite of any orientation. The centre of gravity may depend upon non-uniformity of the gravitational field, in turn, will depend upon the orientation. For objects on the Earth , this will be possible only if the size of an object is comparable to that of the Earth (size at least few thousand km). In such cases, the c.g. will be slightly lower than the c.m. as on the lower side of an object the gravitational field is stronger. Of course, we shall not come across such an object.

## Exercises

**1. Choose the correct answer.**

i) Consider following pair of forces of equal magnitude and opposite directions:
   (P) Gravitational forces exerted on each other by two point masses separated by a distance.
   (Q) Couple of forces used to rotate a water tap.
   (R) Gravitational force and normal force experienced by an object kept on a table.
   For which of these pair/pairs the two forces do NOT cancel each other's translational effect?
   (A) Only P (B) Only P and Q
   (C) Only R (D) Only Q and R

ii) Consider following forces: (w) Force due to tension along a string, (x) Normal force given by a surface, (y) Force due to air resistance and (z) Buoyant force or upthrust given by a fluid.
   Which of these are electromagnetic forces?
   (A) Only w, y and z

(B) Only w, x and y
(C) Only x, y and z
(D) All four.

iii) At a given instant three point masses *m*, 2*m* and 3*m* are equidistant from each other. Consider only the gravitational forces between them. Select correct statement/s for this instance only:
(A) Mass *m* experiences maximum force.
(B) Mass 2*m* experiences maximum force.
(C) Mass 3*m* experiences maximum force.
(D) All masses experience force of same magnitude.

iv) The rough surface of a horizontal table offers a definite *maximum* opposing force to initiate the motion of a block along the table, which is proportional to the resultant normal force given by the table. Forces $F_1$ and $F_2$ act at the same angle θ with the horizontal and both are just initiating the sliding motion of the block along the table. Force $F_1$ is a pulling force while the force $F_2$ is a pushing force. $F_2 > F_1$, because
(A) Component of $F_2$ adds up to weight to increase the normal reaction.
(B) Component of $F_1$ adds up to weight to increase the normal reaction.
(C) Component of $F_2$ adds up to the opposing force.
(D) Component of $F_1$ adds up to the opposing force.

v. A mass 2*m* moving with some speed is *directly* approaching another mass *m* moving with double speed. After some time, they collide with coefficient of restitution 0.5. Ratio of their respective speeds after collision is
(A) 2/3 (B) 3/2
(C) 2 (D) ½

vi. A uniform rod of mass 2*m* is held horizontal by two sturdy, practically inextensible vertical strings tied at its ends. A boy of mass 3*m* hangs himself at one third length of the rod. Ratio of the tension in the string close to the boy to that in the other string is
(A) 2 (B) 1.5
(C) 4/3 (D) 5/3

vii. Select WRONG statement about centre of mass:
(A) Centre of mass of a 'C' shaped uniform rod can never be a point on that rod.
(B) If the line of action of a force passes through the centre of mass, the moment of that force is zero.
(C) Centre of mass of our Earth is not at its geometrical centre.
(D) While balancing an object on a pivot, the line of action of the gravitational force of the earth passes through the centre of mass of the object.

viii. For which of the following objects will the centre of mass NOT be at their geometrical centre?
(I) An egg
(II) a cylindrical box full of rice
(III) a cubical box containing assorted sweets
(A) Only (I)
(B) Only (I) and (II)
(C) Only (III)
(D) All, (I), (II) and (III).

**2. Answer the following questions.**

i) In the following table, every entry on the left column can match with any number of entries on the right side. Pick up ***all*** those and write respectively against A, B, C and D.

| Name of the force | | Type of the force | |
|---|---|---|---|
| A | Force due to tension in a string | P | EM force |
| B | Normal force | Q | Reaction force |
| C | Frictional force | R | Conservative force |
| D | Resistive force offered by air or water for objects moving through it. | S | Non-conservative force |

ii) In real life objects, never travel with uniform velocity, even on a horizontal surface, unless *something* is done? Why is it so? What is to be done?

iii) For the study of any kind of motion, we never use Newton's first law of motion directly. Why should it be studied?

iv) Are there any situations in which we cannot apply Newton's laws of motion? Is there any alternative for it?

v) You are inside a closed capsule from where you are not able to see anything about the outside world. Suddenly you feel that you are pushed towards your right. Can you explain the possible cause (s)? Is it a feeling or a reality? Give at least one more situation like this.

vi) Among the four fundamental forces, only one force governs your daily life almost entirely. Justify the statement by stating that force.

vii) Find the odd man out: (i) Force responsible for a string to become taut on stretching (ii) Weight of an object (iii) The force due to which we can hold an object in hand.

viii) You are sitting next to your friend on ground. Is there any gravitational force of attraction between you two? If so, why are you not coming together naturally? Is any force other than the gravitational force of the earth coming in picture?

ix) Distinguish between: (A) Real and pseudo forces, (B) Conservative and non-conservative forces, (C) Contact and non-contact forces, (C) Inertial and non-inertial frames of reference.

x) State the formula for calculating work done by a force. Are there any conditions or limitations in using it directly? If so, state those clearly. Is there any mathematical way out for it? Explain.

xi) Justify the statement, "Work and energy are the two sides of a coin".

xii) From the terrace of a building of height $H$, you dropped a ball of mass $m$. It reached the ground with speed v. Is the relation $mgH = \frac{1}{2}m\text{v}^2$ applicable exactly? If not, how can you account for the difference? Will the ball bounce to the same height from where it was dropped?

xiii) State the law of conservation of linear momentum. It is a consequence of which law? Given an example from our daily life for conservation of momentum. Does it hold good during burst of a cracker?

xiv) Define coefficient of restitution and obtain its value for an elastic collision and a perfectly inelastic collision.

xv) Discuss the following as special cases of elastic collisions and obtain their exact or approximate final velocities in terms of their initial velocities.

(i) Colliding bodies are identical.

(ii) A veru heavy object collides on a lighter object, initially at rest.

(iii) A very light object collides on a comparatively much massive object, initially at rest.

xvi) A bullet of mass $m_1$ travelling with a velocity $u$ strikes a stationary wooden block of mass $m_2$ and gets embedded into it. Determine the expression for loss in the kinetic energy of the system. Is this violating the principle of conservation of energy? If not, how can you account for this loss?

xvii) One of the effects of a force is to change the momentum. Define the quantity related to this and explain it for a variable force. Usually when do we define it instead of using the force?

xviii) While rotating an object or while opening a door or a water tap we apply a force or forces. Under which conditions is this process easy for us? Why? Define the vector quantity concerned. How does it differ for a single force and for two opposite forces with different lines of action?

xix) Why is the moment of a couple independent of the axis of rotation even if the axis is fixed?

xx) Explain balancing or mechanical equilibrium. Linear velocity of a rotating fan as a whole is generally zero. Is it in mechanical equilibrium? Justify your answer.

xxi) Why do we need to know the centre of mass of an object? For which objects, its position may differ from that of the centre of gravity?

***Use g = 10 m s$^{-2}$, unless, otherwise stated.***

**3. Solve the following problems.**

i) A truck of mass 5 ton is travelling on a horizontal road with 36 km hr $^{-1}$ stops on traveling 1 km after its engine fails suddenly. What fraction of its weight is the frictional force exerted by the road?

If we assume that the story repeats for a car of mass 1 ton i.e., can moving with same speed stops in similar distance same how much will the fraction be?

[Ans: $\frac{1}{200}$ in the both]

ii) A lighter object *A* and a heavier object *B* are initially at rest. Both are imparted the same linear momentum. Which will start with greater kinetic energy: *A* or *B* or both will start with the same energy?

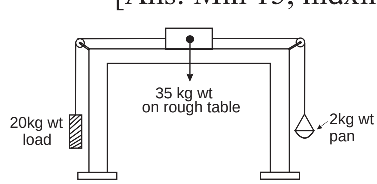
[Ans: *A*]

iii) As I was standing on a weighing machine inside a lift it recorded 50 kg wt. Suddenly for few seconds it recorded 45 kg wt. What must have happened during that time? Explain with complete numerical analysis.

[Ans: Lift must be coming down with acceleration $\frac{g}{10} = 1 \text{m s}^{-2}$ ]

iv) Figure below shows a block of mass 35 kg resting on a table. The table is so rough that it offers a self adjusting resistive force 10% of the weight of the block for its sliding motion along the table. A 20 kg wt load is attached to the block and is passed over a pulley to hang freely on the left side. On the right side there is a 2 kg wt pan attached to the block and hung freely. Weights of 1 kg wt each, can be added to the pan. Minimum how many and maximum how many such weights can be added into the pan so that the block does not slide along the table?

[Ans: Min 15, maximum 21].




v) Power is rate of doing work or the rate at which energy is supplied to the system. A constant force *F* is applied to a body of mass *m*. Power delivered by the force at time *t* from the start is proportional to

(a) $t$ (b) $t^2$

(c) $\sqrt{t}$ (d) $t^0$

Derive the expression for power in terms of *F*, *m* and *t*.

[Ans: $p = \frac{F^2 t}{m}, \therefore p \alpha t$ ]

vi) 40000 litre of oil of density 0.9 g cc is pumped from an oil tanker ship into a storage tank at 10 m higher level than the ship in half an hour. What should be the power of the pump?

[Ans: 2 kW]

vii) Ten identical masses (*m* each) are connected one below the other with 10 strings. Holding the topmost string, the system is accelerated upwards with acceleration g/2. What is the tension in the $6^{th}$ string from the top (Topmost string being the first string)?

[Ans: 6 *mg*]

viii) Two galaxies of masses 9 billion solar mass and 4 billion solar mass are 5 million light years apart. If, the Sun has to cross the line joining them, without being attracted by either of them, through what point it should pass?

[Ans: 3 million light years from the 9 billion solar mass]

ix) While decreasing linearly from 5 N to 3 N, a force displaces an object from 3 m to 5 m. Calculate the work done by this force during this displacement.

[Ans: 8 N]

x) Variation of a force in a certain region is given by $F = 6x^2 - 4x - 8$. It displaces an object from $x = 1$ m to $x = 2$ m in this region. Calculate the amount of work done. [Ans: Zero]

xi) A ball of mass 100 g dropped on the ground from 5 m bounces repeatedly. During every bounce 64% of the potential energy is converted into kinetic energy. Calculate the following:

(a) Coefficient of restitution.

(b) Speed with which the ball comes up from the ground after third bounce.

(c) Impulse given by the ball to the ground during this bounce.

(d) Average force exerted by the ground

if this impact lasts for 250 ms.

(e) Average pressure exerted by the ball on the ground during this impact if contact area of the ball is 0.5 $cm^2$.

[Ans: 0.8, 5.12 m/s, 1.152N s, 4.608 N, $9.216\times10^4$ $N/m^2$]

xii) A spring ball of mass 0.5 kg is dropped from some height. On falling freely for 10 s, it explodes into two fragments of mass ratio 1:2. The lighter fragment continues to travel downwards with speed of 60 m/s. Calculate the kinetic energy supplied during explosion.

[Ans: 200 J]

xiii) A marble of mass 2*m* travelling at 6 cm/s is directly followed by another marble of mass *m* with double speed. After collision, the heavier one travels with the average initial speed of the two. Calculate the coefficient of restitution.

[Ans: 0.5]

xiv) A, 2 m long wooden plank of mass 20 kg is pivoted (supported from below) at 0.5 m from either end. A person of mass 40 kg starts walking from one of these pivots to the farther end. How far can the person walk before the plank topples?

[Ans: 1.25 m]

xv) A 2 m long ladder of mass 10 kg is kept against a wall such that its base is 1.2 m away from the wall. The wall is smooth but the ground is rough. Roughness of the ground is such that it offers a maximum horizontal resistive force (for sliding motion) half that of normal reaction at the point of contact. A monkey of mass 20 kg starts climbing the ladder. How far can it climb ***along*** the ladder? How much is the horizontal reaction at the wall?

[Ans: 1.5 m, 15 N]

xvi) Four uniform solid cubes of edges 10 cm, 20 cm, 30 cm and 40 cm are kept on the ground, touching each other in order. Locate centre of mass of their system.

[Ans: 65 cm, 17.7 cm]

xvii) A uniform solid sphere of radius *R* has a hole of radius *R*/2 drilled inside it. One end of the hole is at the centre of the sphere while the other is at the boundary. Locate centre of mass of the remaining sphere.

[Ans: -R/14 ]

xviii) In the following table, every item on the left side can match with any number of items on the right hand side. Select all those.

***

| Types of collision | Illustrations |
|---|---|
| (a) Elastic collision<br>(b) Inelastic collision<br>(c) Perfectly inelastic collision<br>(d) Head on collision | (i) A ball hit by a bat.<br>(ii) Molecular collisions responsible for pressure exerted by a gas.<br>(iii) A stationary marble A is hit by marble B and the marble B comes to rest.<br>(iv) A blob of clay dropped on the ground sticks to the ground.<br>(v) Out of anger, giving a kick to a wall.<br>(vi) A striker hits the boundary of a carrom board in a direction perpendicular to the boundary and rebounds. |