

2. FUNCTIONS



- Function, Domain, Co-domain, Range
- Types of functions – One-one, Onto
- Representation of function
- Evaluation of function
- Fundamental types of functions
- Some special functions



2.1 FUNCTION

Definition : A function f from set A to set B is a relation which associates each element x in A , to a unique (exactly one) element y in B .

Then the element y is expressed as $y = f(x)$.

y is the image of x under f

If such a function exists, then A is called the **domain** of f and B is called the **co-domain** of f .

For example,

Examine the following relations which are given by arrows of line segments joining elements in A and elements in B .

1)

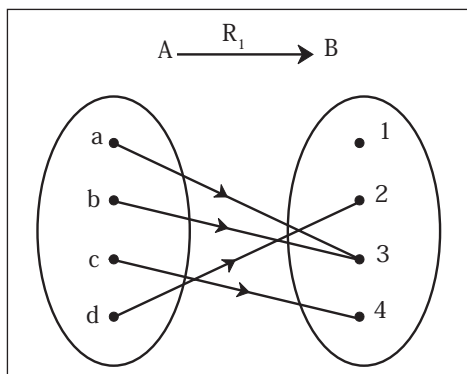


Fig. 2.1

R_1 is a well defined function.

2)

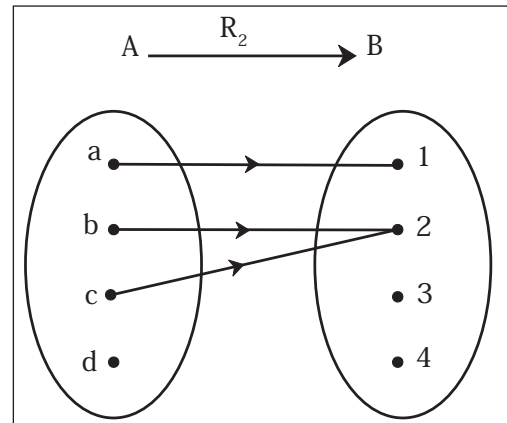


Fig. 2.2

R_2 is not a function because element d in A is not associated to any element in B .

3)

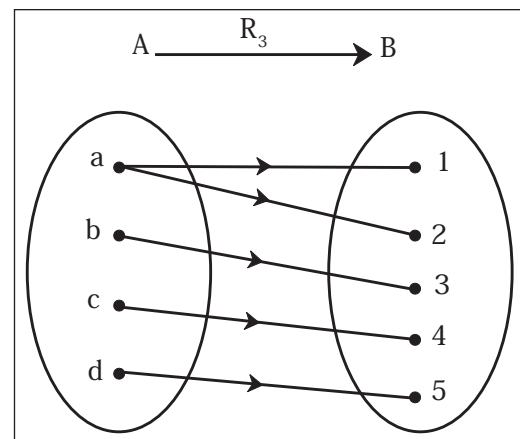


Fig. 2.3

R_3 is not a function because element a in A is associated to two elements in B .

The relation which defines a function f from domain A to co-domain B is often given by an algebraic rule.

2.1.1 Types of function

One-one or One to one or Injective function

Definition : A function $f: A \rightarrow B$ is said to be one-one if distinct elements in A have distinct images

in B. The condition is also expressed as $a \neq b \Rightarrow f(a) \neq f(b)$

Onto or Surjective function

Definition: A function $f: A \rightarrow B$ is said to be onto if every element y in B is an image of some x in A

The image of A can be denoted by $f(A)$.
 $f(A) = \{y \in B \mid y = f(x) \text{ for some } x \in A\}$
 $f(A)$ is also called the **range** of A.
 Note that $f: A \rightarrow B$ is onto if $f(A) = B$.

For example,

1)

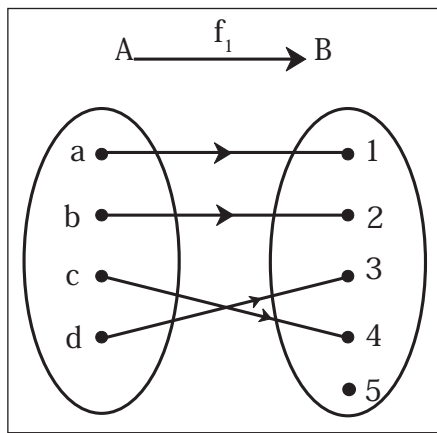


Fig. 2.4

f_1 is one-one, not onto

2)

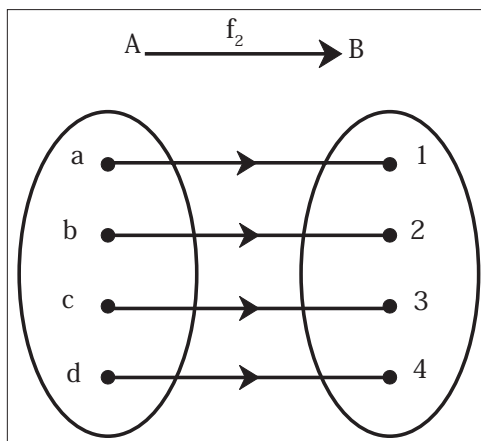


Fig. 2.5

f_2 is one-one, onto

3)

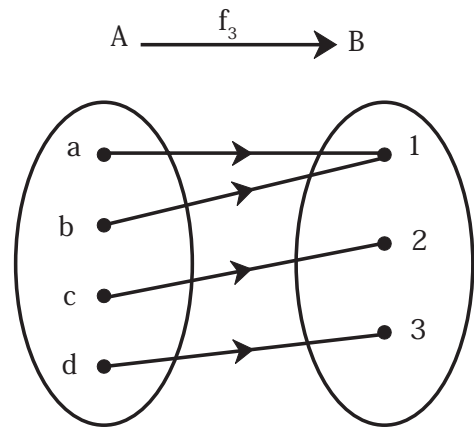


Fig. 2.6

f_3 is not one-one, onto

4)

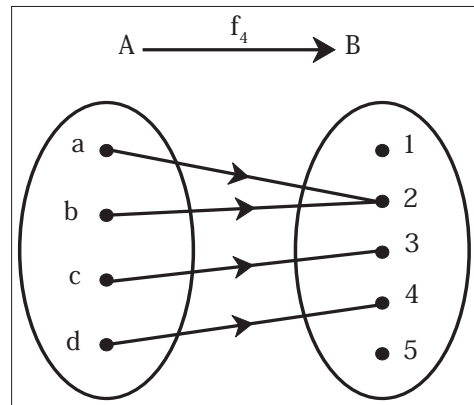
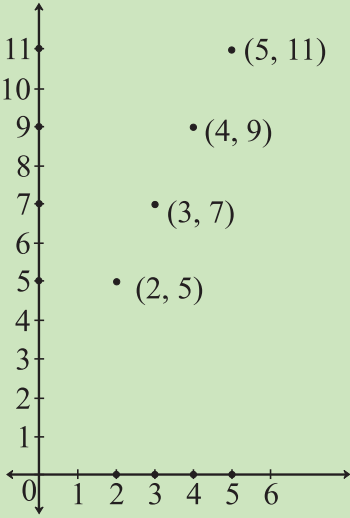


Fig. 2.7

f_4 is not one-one, not onto

Representation of Function

Verbal form	Output is 1 more than twice the input Domain : Set of inputs Range : Set of outputs
Arrow form/ Venn Diagram Form	<p style="text-align: center;">Fig. 2.7(A) Domain : Set of pre-images Range: Set of images</p>

Ordered Pair (x, y)	$f = \{(2,5), (3,7), (4,9), (5,11)\}$ Domain : Set of 1 st components from ordered pair = $\{2, 3, 4, 5\}$ Range : Set of 2 nd components from ordered pair = $\{5, 7, 9, 11\}$										
Rule / Formula	$y = f(x) = 2x + 1$ Where $x \in N, 1 < x < 6$ $f(x)$ read as 'f of x' or 'function of x' Domain : Set of values of x, Range : Set of values of y for which $f(x)$ is defined										
Tabular Form	<table border="1"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr> <td>2</td> <td>5</td> </tr> <tr> <td>3</td> <td>7</td> </tr> <tr> <td>4</td> <td>9</td> </tr> <tr> <td>5</td> <td>11</td> </tr> </tbody> </table> Domain : x values Range: y values	x	y	2	5	3	7	4	9	5	11
x	y										
2	5										
3	7										
4	9										
5	11										
Graphical form	 Fig. 2.7(B) Domain: Extent of graph on x-axis. Range: Extent of graph on y-axis.										

2.1.2 Graph of a function:

If the domain of function is R , we can show the function by a graph in xy plane. The graph consists of points (x,y) , where $y = f(x)$.

Evaluation of a function:

- 1) **Ex:** Evaluate $f(x) = 2x^2 - 3x + 4$ at $x = 7$ and $x = -2t$

Solution : $f(x)$ at $x = 7$ is $f(7)$

$$\begin{aligned} f(7) &= 2(7)^2 - 3(7) + 4 \\ &= 2(49) - 21 + 4 \\ &= 98 - 21 + 4 \\ &= 81 \end{aligned}$$

$$\begin{aligned} f(-2t) &= 2(-2t)^2 - 3(-2t) + 4 \\ &= 2(4t^2) + 6t + 4 \\ &= 8t^2 + 6t + 4 \end{aligned}$$

- 2) Using the graph of $y = g(x)$, find $g(-4)$ and $g(3)$

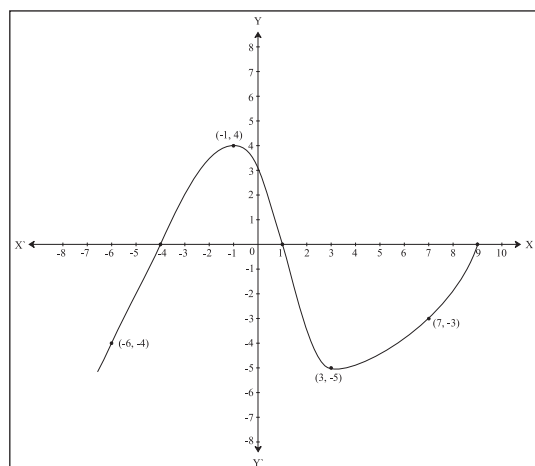


Fig. 2.8

From graph when $x = -4$, $y = 0$ so $g(-4) = 0$

From graph when $x = 3$, $y = -5$ so $g(3) = -5$

- 3) If $f(x) = 3x^2 - x$ and $f(m) = 4$, then find m

Solution : As

$$f(m) = 4$$

$$3m^2 - m = 4$$

$$3m^2 - m - 4 = 0$$

$$3m^2 - 4m + 3m - 4 = 0$$

$$m(3m - 4) + 1(3m - 4) = 0$$

$$(3m - 4)(m + 1) = 0$$

Therefore, $m = \frac{4}{3}$ or $m = -1$

- 4) From the graph below find x for which $f(x) = 4$

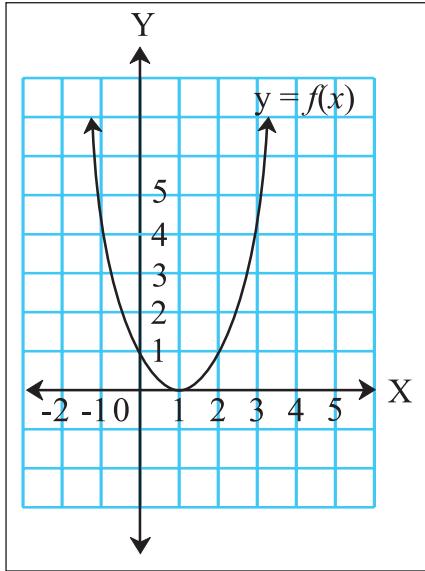


Fig. 2.9

Solution : To solve $f(x) = 4$

find the values of x where graph intersects line $y = 4$

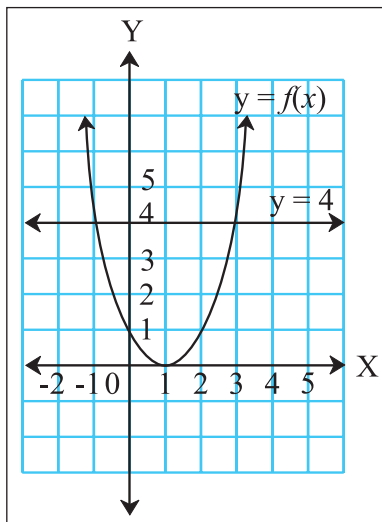


Fig. 2.10

Therefore, $x = -1$ and $x = 3$ are the values of x for which $f(x) = 4$

- 5) From the equation $4x + 7y = 1$ express
i) y as a function of x
ii) x as a function of y

Solution : Given relation is $4x + 7y = 1$

- i) From the given equation
 $7y = 1 - 4x$
 $y = \frac{1-4x}{7}$ = function of x
So $y = f(x) = \frac{1-4x}{7}$

- ii) From the given equation
 $4x = 1 - 7y$
 $x = \frac{1-7y}{4}$ = function of y
So $x = g(y) = \frac{1-7y}{4}$

Some Fundamental Functions

1) Constant Function :

$f: \mathbb{R} \rightarrow \mathbb{R}$ given by

$f(x) = k, x \in \mathbb{R}, k$ is constant is called the constant function.

Example: $f(x) = 3, x \in \mathbb{R}$

Graph of $f(x) = 3$

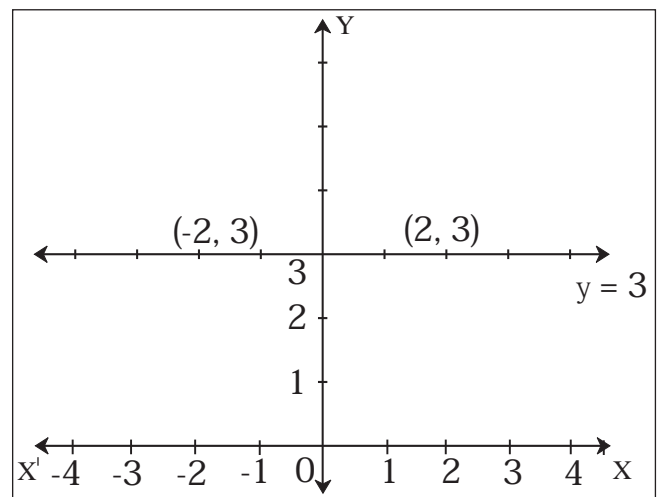


Fig. 2.11

Domain : \mathbb{R} or $(-\infty, \infty)$ and **Range :** $\{3\}$

2) Identity function

If the domain and co-domain are same as A , then we define a special function as identity function defined by $f(x) = x$, for every $x \in A$.

Let $A = \mathbb{R}$. Then identity function $f: \mathbb{R} \rightarrow \mathbb{R}$, where $y = x$ is shown in the graph in fig. 2.12.

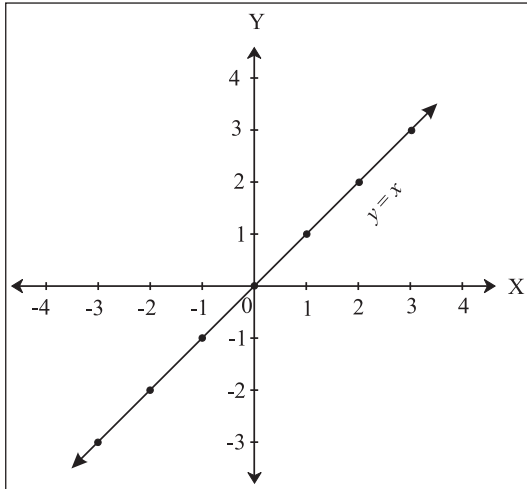


Fig. 2.12

Graph of $f(x) = x$

Domain : \mathbb{R} or $(-\infty, \infty)$ and **Range :** \mathbb{R} or $(-\infty, \infty)$

3) Linear Function :

Example : A function $f: \mathbb{R} \rightarrow \mathbb{R}$ of the form $f(x) = ax + b$

For example, $f(x) = -2x + 3$, the graph of which is shown in Fig. 2.13

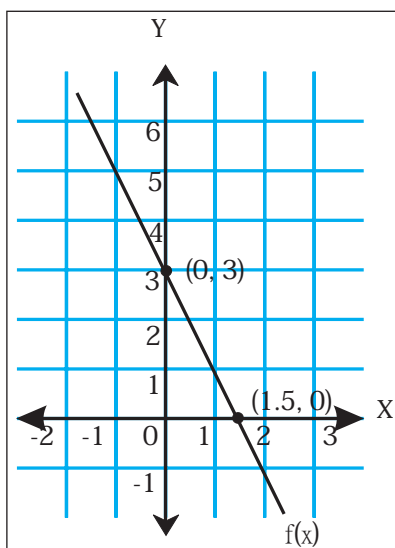


Fig. 2.13

Domain : \mathbb{R} or $(-\infty, \infty)$ and **Range :** \mathbb{R} or $(-\infty, \infty)$

Properties :

- 1) Graph of $f(x) = ax + b$ is a line with slope ' a ', y-intercept ' b ' and x-intercept $\left(-\frac{b}{a}\right)$.
- 2) Graph is increasing when slope is positive and decreasing when slope is negative.

4) Quadratic Function

Function of the form $f(x) = ax^2 + bx + c$ ($a \neq 0$)

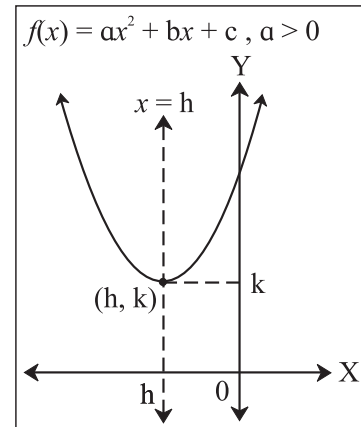


Fig. 2.14

Domain : \mathbb{R} or $(-\infty, \infty)$ and **Range :** $[k, \infty)$

5) Function of the form $f(x) = ax^n$, $n \in \mathbb{N}$
(Note that this function is a multiple of power of x)

i) Square Function

Example : $f(x) = x^2$

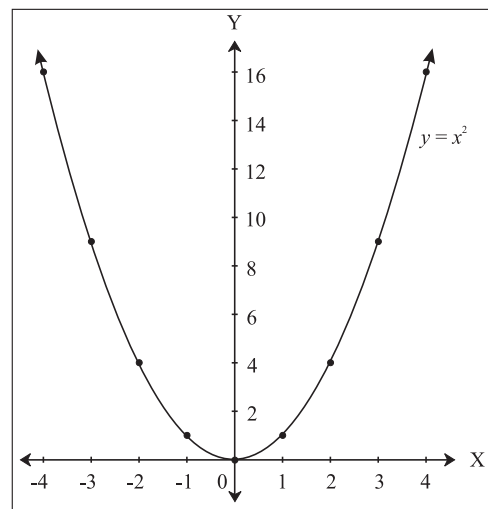


Fig. 2.15

Domain : \mathbb{R} or $(-\infty, \infty)$ and **Range :** $[0, \infty)$

ii) Cube Function

Example : $f(x) = x^3$

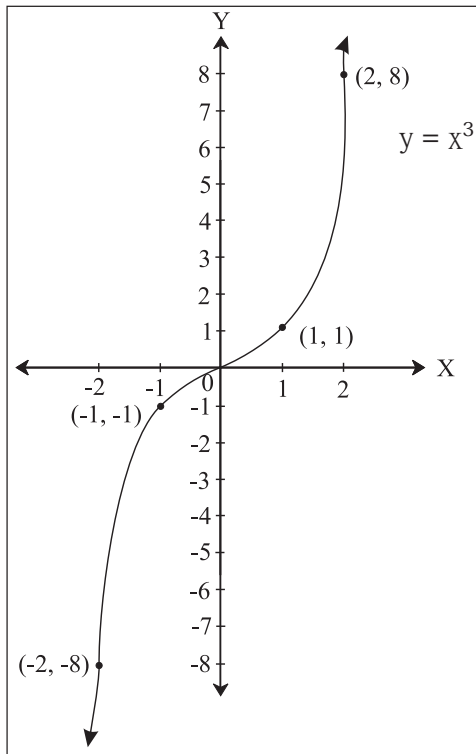


Fig. 2.16

Domain : \mathbb{R} or $(-\infty, \infty)$ and **Range :** \mathbb{R} or $(-\infty, \infty)$

6) Polynomial Function : A functions of the form

$$f(x) = a_0 x^n + a_1 x^{n-1} + \dots + a_{n-1} x + a_n$$

is polynomial function of degree n , if $a_0 \neq 0$, and $a_0, a_1, a_2, \dots, a_n$ are real. The graph of a general polynomial is more complicated and depends upon its individual terms.

Graph of $f(x) = x^3 - 1$

$f(x) = (x - 1)(x^2 + x + 1)$ cuts x -axis at only one point $(1,0)$, which means $f(x)$ has one real root and two complex roots.

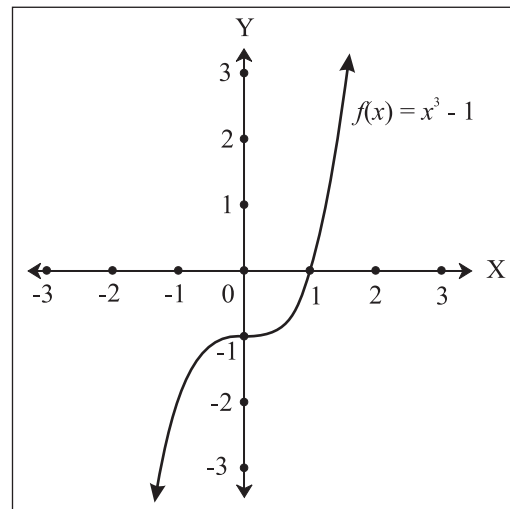


Fig. 2.17

Note that, any polynomial of odd degree must have at least one real root, because the complex roots appear in conjugate pairs.

7) Rational Function

Definition: Given polynomials $p(x), q(x)$

$f(x) = \frac{p(x)}{q(x)}$ ($q(x) \neq 0$) is called a rational function.

For example, $f(x) = \frac{1}{x}$

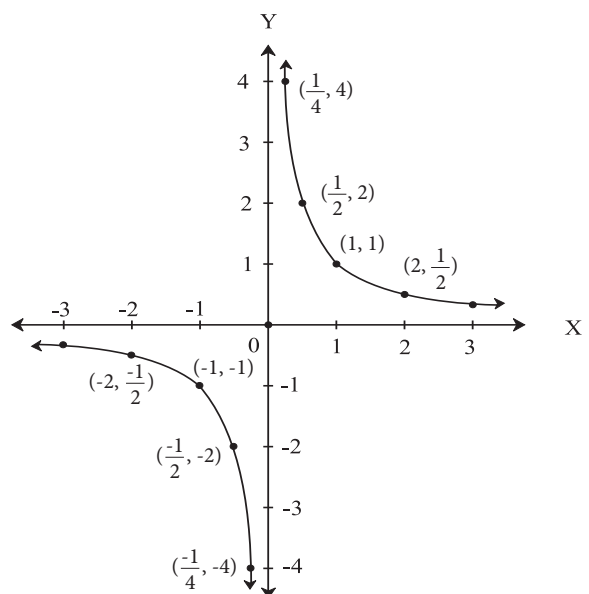


Fig. 2.18

Domain : $\mathbb{R} - \{0\}$ and **Range :** $\mathbb{R} - \{0\}$

Example : Domain of function $f(x) = \frac{2x+9}{x^3-25x}$

i.e. $f(x) = \frac{2x+9}{x(x-5)(x+5)}$ is $\mathbb{R} - \{0, 5, -5\}$ as for

$x = 0, x = -5$ and $x = 5$, denominator becomes 0 .

8) Exponential Function : A function $f: \mathbb{R} \rightarrow \mathbb{R}^+$ given by $f(x) = a^x$ is an exponential function with base a and exponent (or index) $x, a \neq 1, a > 0$ and $x \in \mathbb{R}$.

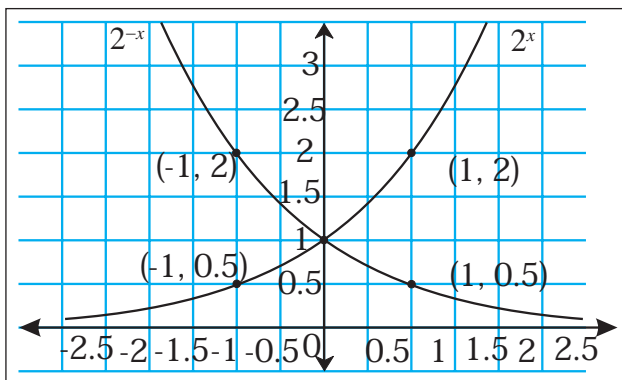


Fig. 2.19

Domain: \mathbb{R} and **Range :** $(0, \infty) = \mathbb{R}^+$

9) Logarithmic Function: A function $f: \mathbb{R}^+ \rightarrow \mathbb{R}$, defined as $f(x) = \log_a x, (a > 0, a \neq 1, x > 0)$ where $y = \log_a x$ if and only if $x = a^y$ is called Logarithmic Function.

$$\underbrace{y = \log_a x}_{\text{logarithmic form}} \text{ is equivalent to } \underbrace{a^y = x}_{\text{exponential form}} .$$



- 1) $y = \log_a x \Leftrightarrow a^y = x \Leftrightarrow \text{anti } \log_a y = x$
- 2) As $a^0 = 1$, so $\log_a 1 = 0$ and as $a^1 = a$, so $\log_a a = 1$

- 3) As $a^x = a^y \Leftrightarrow x = y$ so $\log_a x = \log_a y \Leftrightarrow x = y$
- 4) For natural base $e, \log_e x = \ln x (x > 0)$ is called as Natural Logarithm Function.

Domain : $(0, \infty)$

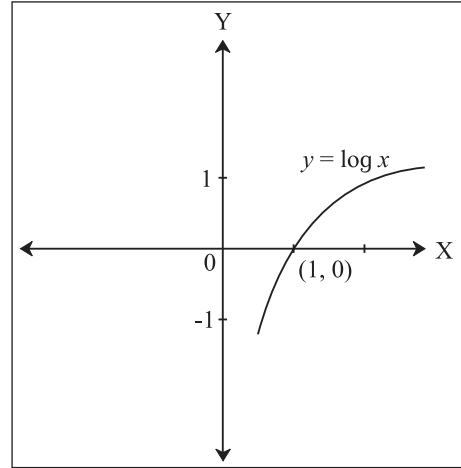


Fig. 2.20

Range : $(-\infty, \infty)$

Rules of Logarithm :

- 1) $\log_m ab = \log_m a + \log_m b$
- 2) $\log_m \frac{a}{b} = \log_m a - \log_m b$
- 3) $\log_m a^b = b \cdot \log_m a$
- 4) Change of base formula:

$$\text{For } a, x, b > 0 \text{ and } a, b \neq 1, \log_a x = \frac{\log_b x}{\log_b a}$$

Note: $\log_a x = \frac{\log_x x}{\log_x a} = \frac{1}{\log_x a}$

Algebra of functions:

If f and g are functions from $X \rightarrow \mathbb{R}$, then the functions $f + g, f - g, fg, \frac{f}{g}$ are defined as follows.

Operations
$(f + g)(x) = f(x) + g(x)$
$(f - g)(x) = f(x) - g(x)$
$(f \cdot g)(x) = f(x) \cdot g(x)$
$\frac{f}{g}(x) = \frac{f(x)}{g(x)}$ where $g(x) \neq 0$

Ex. If $f(x) = x^2 + 2$ and $g(x) = 5x - 8$, then find

- $(f + g)(1)$
- $(f - g)(-2)$
- $(f \cdot g)(3m)$
- $\frac{f}{g}(0)$

i) As $(f + g)(x) = f(x) + g(x)$

$$\begin{aligned} (f + g)(1) &= f(1) + g(1) \\ &= (1)^2 + 2 + 5(1) - 8 \\ &= 1 + 2 + 5 - 8 \\ &= 8 - 8 \\ &= 0 \end{aligned}$$

ii) As $(f - g)(x) = f(x) - g(x)$

$$\begin{aligned} (f - g)(-2) &= f(-2) - g(-2) \\ &= [(-2)^2 + 2] - [5(-2) - 8] \\ &= [4 + 2] - [-10 - 8] \\ &= 6 + 18 \\ &= 24 \end{aligned}$$

iii) As $(fg)(x) = f(x)g(x)$

$$\begin{aligned} (f \cdot g)(3m) &= f(3m)g(3m) \\ &= [(3m)^2 + 2][5(3m) - 8] \\ &= [9m^2 + 2][15m - 8] \\ &= 135m^3 - 72m^2 + 30m - 16 \end{aligned}$$

iv) As $\frac{f}{g}(x) = \frac{f(x)}{g(x)}$

$$\frac{f}{g}(0) = \frac{f(0)}{g(0)}$$

$$\begin{aligned} &= \frac{2}{-8} \\ &= -\frac{1}{4} \end{aligned}$$

2.1.3 Composite function:

Let $f:A \rightarrow B$ and $g: B \rightarrow C$ where $f(x) = y$ and $g(y) = z$, $x \in A$, $y \in B$, $z \in C$. We define a function $h:A \rightarrow C$ such that $h(x) = z$ then the function h is called composite function of f and g and is denoted by $g \circ f$.

$$\therefore g \circ f: A \rightarrow C$$

$$g \circ f(x) = g[f(x)]$$

It is also called function of a function.

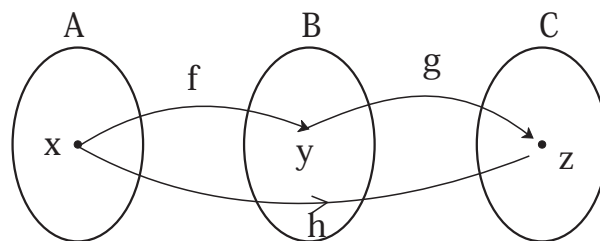


Fig. 2.21

e.g.

Let $A = \{1, 2, 3, 4\}$ and $B = \{1, 3, 5, 7, 10\}$

$C = \{-2, 6, 22, 46, 97, 100\}$ where

$f = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$ and

$g = \{(1, -2), (3, 6), (5, 22), (7, 46), (10, 97)\}$

$\therefore g \circ f = \{(1, -2), (2, 6), (3, 22), (4, 46)\}$

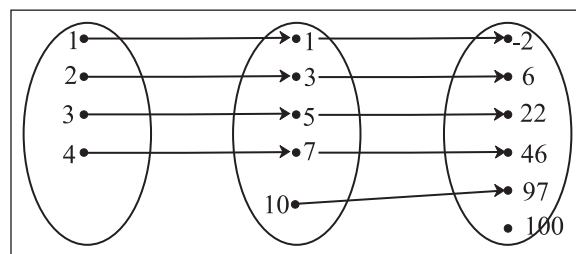


Fig. 2.22

Since

$$f = \{(x, y) / x \in A, y \in B, \text{ and } y = 2x - 1\}$$

$$g = \{(y, z) / y \in B, z \in C, \text{ and } z = y^2 - 3\}$$

then

$$g \circ f(x) = \{(x, z) / x \in A, z \in C\} \text{ and}$$

$$z = (2x - 1)^2 - 3$$

Ex 1. If $f(x) = \frac{2}{x+5}$ and $g(x) = x^2 - 1$, then find

i) $(f \circ g)(x)$ ii) $(g \circ f)(3)$

Solution :

i) As $(f \circ g)(x) = f[g(x)]$ and $f(x) = \frac{2}{x+5}$

Replace x from $f(x)$ by $g(x)$, to get

$$\begin{aligned} (f \circ g)(x) &= \frac{2}{g(x)+5} \\ &= \frac{2}{x^2-1+5} \\ &= \frac{2}{x^2+4} \end{aligned}$$

ii) As $(g \circ f)(x) = g[f(x)]$ and $g(x) = x^2 - 1$

Replace x by $f(x)$ in $g(x)$, to get

$$\begin{aligned} (g \circ f)(x) &= [f(x)]^2 - 1 \\ &= \left(\frac{2}{x+5}\right)^2 - 1 \end{aligned}$$

Now let $x = 3$

$$\begin{aligned} (g \circ f)(3) &= \left(\frac{2}{3+5}\right)^2 - 1 \\ &= \left(\frac{2}{8}\right)^2 - 1 \\ &= \left(\frac{1}{4}\right)^2 - 1 \\ &= \frac{1-16}{16} \\ &= -\frac{15}{16} \end{aligned}$$

Ex 2. If $f(x) = x^2$, $g(x) = x + 5$, and $h(x) = \frac{1}{x}$, find $(g \circ f \circ h)(x)$

→ As $(g \circ f \circ h)(x) = g\{f[h(x)]\}$ and

$$g(x) = x + 5$$

Replace x in $g(x)$ by $f[h(x)] + 5$ to get

$$(g \circ f \circ h)(x) = f[h(x)] + 5$$

Now $f(x) = x^2$, so replace x in $f(x)$ by $h(x)$, to get

$$(g \circ f \circ h)(x) = f[h(x)]^2 + 5$$

Now $h(x) = \frac{1}{x}$ therefore,

$$\begin{aligned} (g \circ f \circ h)(x) &= \left(\frac{1}{x}\right)^2 + 5 \\ &= \frac{1}{x^2} + 5 \end{aligned}$$

2.1.4 : Inverse functions:

Let $f: A \rightarrow B$ be one-one and onto function and $f(x) = y$ for $x \in A$. The inverse function

$f^{-1}: B \rightarrow A$ is defined as $f^{-1}(y) = x$ for $y \in B$.

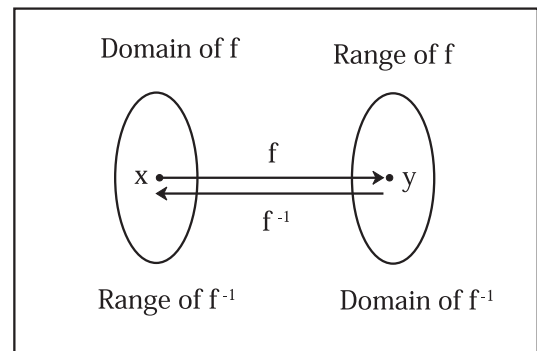


Fig. 2.23

Note: As f is one-one and onto every element $y \in B$ has a unique element $x \in A$ associated with y . Also note that $f \circ f^{-1}(x) = x$

Ex. 1 If f is one-one onto function with $f(3) = 7$ then $f^{-1}(7) = 3$.

Ex. 2. If f is one-one onto function with

$$f(x) = 9 - 5x, \text{ find } f^{-1}(-1).$$

→ Let $f^{-1}(-1) = m$, then $-1 = f(m)$

Therefore,

$$-1 = 9 - 5m$$

$$5m = 9 + 1$$

$$5m = 10$$

$$m = 2$$

That is $f(2) = -1$, so $f^{-1}(-1) = 2$.

Ex.3 Verify that $f(x) = \frac{x-5}{8}$ and $g(x) = 8x + 5$ are inverse functions.

As $f(x) = \frac{x-5}{8}$, replace x in $f(x)$ with $g(x)$

$$f[g(x)] = \frac{g(x)-5}{8}$$

$$= \frac{8x+5-5}{8}$$

$$= \frac{8x}{8}$$

$$= x$$

and $g(x) = 8x + 5$, replace x in $g(x)$ with $f(x)$

$$g[f(x)] = 8f(x) + 5$$

$$= 8\left[\frac{x-5}{8}\right] + 5$$

$$= x - 5 + 5$$

$$= x$$

As $f[g(x)] = x$ and $g[f(x)] = x$, f and g are inverse functions.

Ex. 4: Determine whether the function

$$f(x) = \frac{2x+1}{x-3} \text{ has inverse, if it exists find it.}$$

(f^{-1} exists only if f is one-one and onto.)

Solution : Consider $f(x_1) = f(x_2)$,

Therefore,

$$\frac{2x_1+1}{x_1-3} = \frac{2x_2+1}{x_2-3}$$

$$(2x_1+1)(x_2-3) = (2x_2+1)(x_1-3)$$

$$2x_1x_2 - 6x_1 + x_2 - 3 = 2x_1x_2 - 6x_2 + x_1 - 3$$

$$-6x_1 + x_2 = -6x_2 + x_1$$

$$7x_2 = 7x_1$$

$$x_2 = x_1$$

Hence, f is one-one function.

Let $f(x) = y$, so $y = \frac{2x+1}{x-3}$

Express x as function of y , as follows

$$y = \frac{2x+1}{x-3}$$

$$y(x-3) = 2x+1$$

$$xy - 3y = 2x+1$$

$$xy - 2x = 3y+1$$

$$x(y-2) = 3y+1$$

$$x = \frac{3y+1}{y-2}$$

Here the range of $f(x)$ is $\mathbb{R} - \{2\}$.

x is defined for all y in the range.

Therefore $f(x)$ is onto function.

As function is one-one and onto, so f^{-1} exists.

As $f(x) = y$, so $f^{-1}(y) = x$

Therefore, $f^{-1}(y) = \frac{3y+1}{y-2}$

Replace x by y , to get

$$f^{-1}(x) = \frac{3x+1}{x-2}.$$

2.1.5 :Some Special Functions

i) Signum function : The function $f: \mathbb{R} \rightarrow \mathbb{R}$

$$f(x) = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$$

is called the signum function. It is denoted as $\text{sgn}(x)$

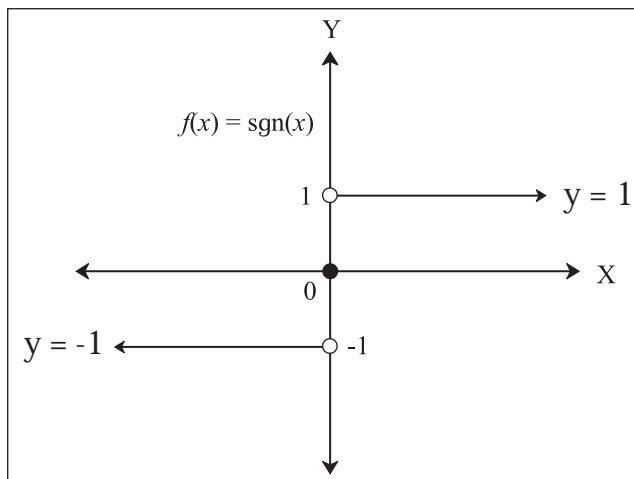


Fig. 2.24

Domain: \mathbb{R} **Range:** $\{-1, 0, 1\}$

ii) Absolute value function (Modulus function):

Definition: Let $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = |x|$, $\forall x \in \mathbb{R}$

$$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

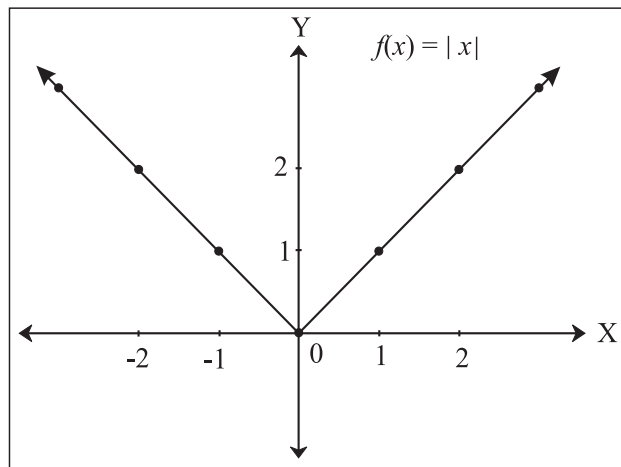


Fig. 2.25

Domain : \mathbb{R} or $(-\infty, \infty)$ and **Range :** $[0, \infty)$

iii) Greatest Integer Function (Step Function):

Definition: For every real x , $f(x) = [x]$ = The greatest integer less than or equal to x .

$$f(x) = n, \text{ if } n \leq x < n + 1, x \in [n, n + 1)$$

We have

$$f(x) = \begin{cases} -2 & \text{if } -2 \leq x < -1 \text{ or } x \in [-2, -1) \\ -1 & \text{if } -1 \leq x < 0 \text{ or } x \in [-1, 0) \\ 0 & \text{if } 0 \leq x < 1 \text{ or } x \in [0, 1) \\ 1 & \text{if } 1 \leq x < 2 \text{ or } x \in [1, 2) \\ 2 & \text{if } 2 \leq x < 3 \text{ or } x \in [2, 3) \end{cases}$$

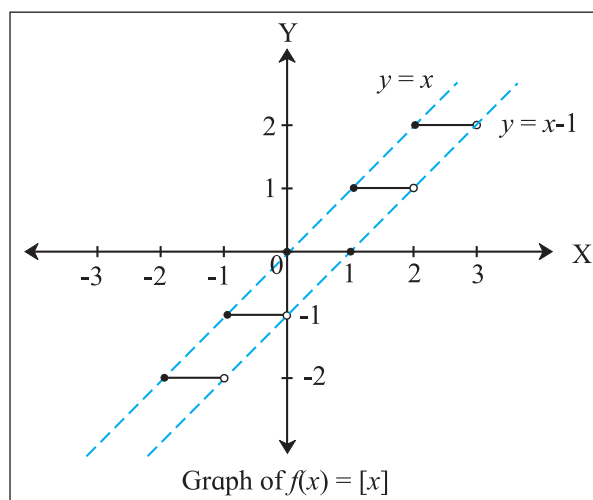


Fig. 2.26

Domain = \mathbb{R} and **Range** = \mathbb{I} (Set of integers)

EXERCISE 2.1

- 1) Check if the following relations are functions.
 - (a)

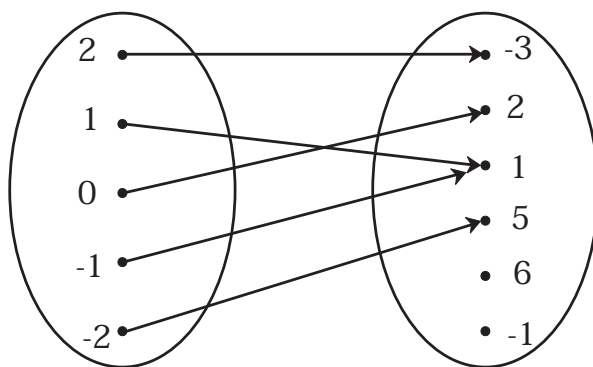


Fig. 2.27

(b)

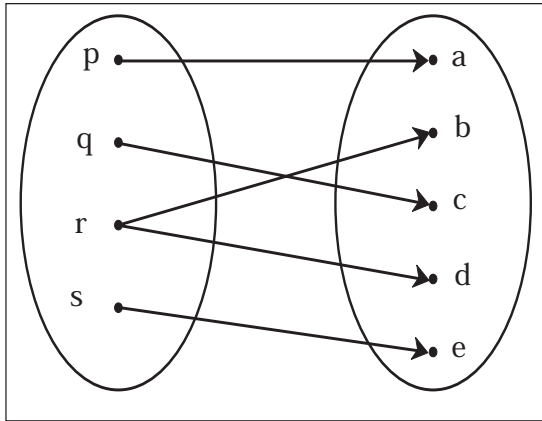


Fig. 2.28

(c)

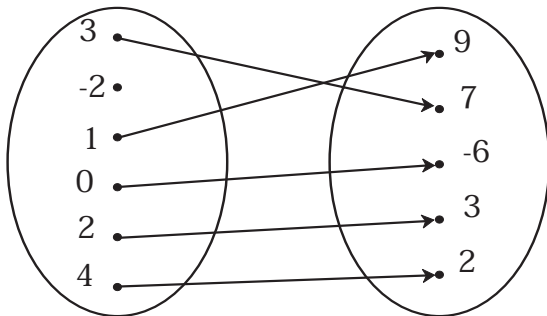


Fig. 2.29

2) Which sets of ordered pairs represent functions from $A = \{1, 2, 3, 4\}$ to $B = \{-1, 0, 1, 2, 3\}$? Justify.

- (a) $\{(1, 0), (3, 3), (2, -1), (4, 1), (2, 2)\}$
- (b) $\{(1, 2), (2, -1), (3, 1), (4, 3)\}$
- (c) $\{(1, 3), (4, 1), (2, 2)\}$
- (d) $\{(1, 1), (2, 1), (3, 1), (4, 1)\}$

3) If $f(m) = m^2 - 3m + 1$, find

- (a) $f(0)$
- (b) $f(-3)$
- (c) $f\left(\frac{1}{2}\right)$
- (d) $f(x + 1)$
- (e) $f(-x)$

4) Find x , if $g(x) = 0$ where

(a) $g(x) = \frac{5x-6}{7}$ (b) $g(x) = \frac{18-2x^2}{7}$

(c) $g(x) = 6x^2 + x - 2$

5) Find x , if $f(x) = g(x)$ where

$f(x) = x^4 + 2x^2, g(x) = 11x^2$

6) If $f(x) = \begin{cases} x^2 + 3, & x \leq 2 \\ 5x + 7, & x > 2 \end{cases}$, then find

- (a) $f(3)$ (b) $f(2)$ (c) $f(0)$

7) If $f(x) = \begin{cases} 4x - 2, & x \leq -3 \\ 5, & -3 < x < 3 \\ x^2, & x \geq 3 \end{cases}$, then find

- (a) $f(-4)$ (b) $f(-3)$
 (c) $f(1)$ (d) $f(5)$

8) If $f(x) = 3x + 5, g(x) = 6x - 1$, then find

- (a) $(f+g)(x)$ (b) $(f-g)(2)$
 (c) $(fg)(3)$ (d) $(f/g)(x)$ and its domain.

9) If $f(x) = 2x^2 + 3, g(x) = 5x - 2$, then find

- (a) $f \circ g$ (b) $g \circ f$
 (c) $f \circ f$ (d) $g \circ g$



Let's remember!

- A Function from set A to the set B is a relation which associates every element of set A to unique element of set B and is denoted by $f: A \rightarrow B$.
- If f is a function from set A to the set B and if $(x, y) \in f$ then y is called the image of x under f and x is called the pre-image of y under f .

- A function $f : A \rightarrow B$ is said to be one-one if distinct elements in A have distinct images in B .
- A function $f : A \rightarrow B$ is said to be onto if every element of B is image of some element of A under the function f .
- A function $f : A \rightarrow B$ is such that there exists atleast one element in B which does not have pre-image in A then f is said to be an into function.
- If $f : A \rightarrow B$ and $g : B \rightarrow C$ with $f(x) = y$ and $g(y) = z$, $x \in A$, $y \in B$, $z \in C$ then define $h : A \rightarrow C$ such that $h(x) = z$, then the function h is called composite function of f and g and is denoted by gof .
- If $f : A \rightarrow B$ is one-one and onto, $g : B \rightarrow A$ is one-one and onto such that $gof : A \rightarrow A$ and $fog : B \rightarrow B$ are both identity functions then f and g are inverse functions of each other.
- Domain of $f = \text{Range of } f^{-1}$
Range of $f = \text{Domain of } f^{-1}$

MISCELLANEOUS EXERCISE -2

- 1) Which of the following relations are functions? If it is a function determine its domain and range.
 - i) $\{(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)\}$
 - ii) $\{(0, 0), (1, 1), (1, -1), (4, 2), (4, -2), (9, 3), (9, -3), (16, 4), (16, -4)\}$
 - iii) $\{(1, 1), (3, 1), (5, 2)\}$
- 2) A function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \frac{3x}{5} + 2$, $x \in \mathbb{R}$. Show that f is one-one and onto. Hence find f^{-1} .
- 3) A function f is defined as follows
 $f(x) = 4x + 5$, for $-4 \leq x < 0$.

Find the values of $f(-1)$, $f(-2)$, $f(0)$, if they exist.

- 4) A function f is defined as follows:
 $f(x) = 5 - x$ for $0 \leq x \leq 4$
Find the value of x such that $f(x) = 3$
- 5) If $f(x) = 3x^2 - 5x + 7$ find $f(x - 1)$.
- 6) If $f(x) = 3x + a$ and $f(1) = 7$ find a and $f(4)$.
- 7) If $f(x) = ax^2 + bx + 2$ and $f(1) = 3$, $f(4) = 42$. find a and b .
- 8) If $f(x) = \frac{2x-1}{5x-2}$, $x \neq \frac{2}{5}$
Verify whether $(f \circ f)(x) = x$.
- 9) If $f(x) = \frac{x+3}{4x-5}$, $g(x) = \frac{3+5x}{4x-1}$
then verify that $(f \circ g)(x) = x$.

ACTIVITIES

Activity 2.1 :

If $f(x) = \frac{x+3}{x-2}$, $g(x) = \frac{2x+3}{x-1}$ Verify whether $f \circ g(x) = g \circ f(x)$.

Activity 2.2 :

$f(x) = 3x^2 - 4x + 2$, $n \in \{0, 1, 2, 3, 4\}$ then represent the function as

- i) By arrow diagram
- ii) Set of ordered pairs
- iii) In tabular form
- iv) In graphical form

Activity 2.3 :

If $f(x) = 5x - 2$, $x > 0$, find $f^{-1}(x)$, $f^{-1}(7)$, for what value of x is $f(x) = 0$.

